Diffraction of Oblique Shock Wave Past a Bend of Small Angle (Subsonic Case)

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ABSTRACT

Lighthill considered the diffraction of normal shock wave past a small bend of angle δ. Srivastava and Srivastava and Chopra extended the work of Lighthill to the case of diffraction of oblique shock waves (Consisting of incident and reflected shock wave). Srivastava obtained the curvature of reflected diffracted shock wave when the relative outflow behind reflected shock is sonic. In the present investigation a qualitative estimate of curvature of the reflected diffracted shock wave is obtained when the relative outflow behind reflected shock wave is subsonic.

Keyword:Curvature, Diffraction, Reflection, Subsonic.

I. INTRODUCTION

Lighthill (1949) considered the diffraction of a normal shock wave past a bend of small angle δ. The analogous problem of a plane shock wave hitting the wall obliquely together with the associated reflected shock has been considered by Srivastava (1968) and Srivastava and Chopra (1970). Srivastava (1968) has developed the mathematical theory of oblique shock wave diffraction when the relative outflow behind the reflected shock wave before diffraction is subsonic and sonic. Srivastava and Chopra (1970) completed the theory when the relative outflow behind the reflected shock before diffraction is supersonic. Srivastava (2016) gave the results concerning curvature of the reflected diffracted shock when the relative outflow behind reflected shock before diffraction is sonic. In the present paper we have obtained curvature results for the reflected diffracted shock when the relative outflow behind reflected shock before diffraction is subsonic. Reference may be made to the book by Srivastava (1994).

II. MATHEMATICAL FORMULATION

Let the velocity, pressure, density and sound speed ahead of the shock wave be denoted by $0, p_0, \rho_1, a_0$, in the intermediate region by q_1, p_1, ρ_1, a_1 and behind the reflected shock wave by q_2, p_2, ρ_2, a_2. Let U denote the velocity of the point of intersection of the incident and reflected shock, δ the angle of bend, α_0 is the angle of incidence and α_2 is the angle of reflection. The Rankine-Hugoniot equations across the incident and reflected shock for $\gamma = 1.4$ (γ being the ratio of specific heats) are given as follows (Srivastava 1995, 2003).

Across the incident shock

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$$q_1 = \frac{5}{6} U \sin\alpha_0 \left(1 - \frac{a_0^2}{U^2 \sin^2 \alpha_0}\right) \quad - (1)$$

$$p_1 = \frac{5}{6} \rho_0 \left(U^2 \sin^2 \alpha_0 - \frac{a_0^2}{7}\right) \quad - (2)$$

$$\rho_1 = \frac{6\rho_0}{\left(1 + \frac{5a_0^2}{U^2 \sin^2 \alpha_0}\right)} \quad - (3)$$

$$a_0 = \sqrt{\frac{\gamma\, p_0}{\rho_0}}$$

Across the reflected shock

$$\bar{q}_2 = \bar{q}_1 + \frac{5}{6}\left(U^* - \bar{q}_1\right)\left(1 - \frac{a_1^2}{\left(U^* - \bar{q}_1\right)^2}\right) \quad - (4)$$

$$p_2 = \frac{5}{6} \rho_1 \left(\left(U^* - \bar{q}_1\right)^2 - \frac{a_1^2}{7}\right) \quad - (5)$$

$$\rho_2 = \frac{6\rho_1}{1 + \frac{5a_1^2}{\left(U^* - \bar{q}_1\right)^2}} \quad - (6)$$

Where $\bar{q}_2 = q_2 \sin\alpha_2, \; \bar{q}_1 = -q_1 \cos(\alpha_0 + \alpha_2)$

$$U^* = U \sin\alpha_2, \quad a_1 = \sqrt{\frac{\gamma\, p_1}{\rho_1}}$$

Also we have

$$q_1 \cos\theta' = q_2 \cos\alpha_2, \quad \theta' = \alpha_0 + \alpha_2 - \frac{\pi}{2}$$

Let the pressure, density, velocity and entropy behind the reflected diffracted shock be denoted by $p_2', \rho_2', \vec{q}_2'$ and S_2. Following Lighthill (1949), by the use of small perturbation theory and conical field transformations, the flow equations can be linearized and they yield a single second order differential equation in *p*, namely

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \quad - (7)$$

where $p = \frac{p_2' - p_2}{a_2 \rho_2 q_2}, \quad x = \frac{X - q_1 t}{a_1 t}, \quad y = \frac{Y}{a_2 t}$ - (8)

$$\vec{q}_2' = q_2 \{(1+u), v\} \quad - (9)$$

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It would be necessary here to discuss about the axes. *X,Y* are the axes in which *X* axis is along the original wall produce, *Y* is the axis normal to the leading edge of the wedge. In (*x,y*) axes, the origin and *x* axis lies on the original wall produced and *y* axis normal to it. The origin with respect to *x,y* coordinates is at $\left(-\frac{q_2}{a_2}, 0\right)$ and the coordinates of the point of intersection of incident and reflected shock is $\left(\frac{U-q_2}{a_2}, 0\right)$.

The characteristics of the differential equation (7) are tangents to the unit circle $x^2 + y^2 = 1$, the region of disturbance will therefore be enclosed by arc of the unit circle $x^2 + y^2 = 1$, reflected diffracted shock and the wall.

Following Srivastava (1968) and Srivastava and Chopra (1970), the equation of the straight portion of the reflected shock after diffraction is given by

$$x = k - y\cot\alpha_2 \quad \text{where } k = \frac{U-q_2}{a_2} \qquad \text{- (10)}$$

The equation of the diffracted shock may therefore be written as

$$x = k - y\cot\alpha_2 + f(y) \qquad \text{- (11)}$$

where $f(y)$ is small.

From the equation (11), curvature κ is given by

$$\kappa = \frac{d^2y/dx^2}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}} = -\sin^3\alpha_2 f''(y) \qquad \text{- (12)}$$

Following Srivastava (1968) and Srivastava and Chopra (1970), on the reflected diffracted shock we have the relation

$$\frac{\partial p}{\partial y} = [B_2 - A_2 G(y)]\, f''(y) \qquad \text{- (13)}$$

where $G(y) = (q_2 \sin\alpha_2 \cos\alpha_2 + a_2 y)$ and B_2, and A_2 are constants.

Combining (12) and (13) we obtain

$$\kappa = -\frac{1}{B_2 - A_2 G(y)} \sin^3\alpha_2 \frac{\partial p}{\partial y} \qquad \text{- (14)}$$

Equation (14) can be written as

$$\kappa = -\frac{1}{B_2 - A_2 G(y)} \sin^3\alpha_2 \frac{\partial p}{\partial x_1} \cdot \frac{\partial x_1}{dy} \qquad \text{- (15)}$$

On the diffracted shock we have the relation (Srivastava 1968)

$$y=\kappa[\cos\alpha_2+\sin\alpha_2\tan\theta],\quad \kappa=\frac{U-q_2}{a_2}\sin\alpha_2 \qquad -(16)$$

In (16) $\tan\theta=\left(\frac{Z^2-1}{Z^2+1}\right)\frac{\kappa'}{\kappa},\quad \kappa'=\sqrt{1-\kappa^2}$

The relationship between Z and z_1 from Srivastava (1968) is

$$z_1=\frac{1}{2}\left[\left(\frac{bz+1}{bz-1}\right)^{\pi/\lambda}+\left(\frac{bz+1}{bz-1}\right)^{-\pi/\lambda}\right] \qquad -(17)$$

where $b=\left(\frac{\kappa'\sin\alpha_2+\kappa\cos\alpha_2}{\kappa'\sin\alpha_2-\kappa\cos\alpha_2}\right)^{1/2}$

From equation (17), we obtain

$$z=-\frac{1}{b}\frac{\left\{1+\left(z_1+\sqrt{z_1^2-1}\right)^{\lambda/\pi}\right\}}{\left\{1-\left(z_1+\sqrt{z_1^2-1}\right)^{-\lambda/\pi}\right.} \qquad -(18)$$

$z_1=x_1+iy_1$ and on the real axis z_1 is replaced by x_1.

From (16) we have

$$y=\kappa\cos\alpha_2+\kappa'\sin\alpha_2\frac{(Z^2-1)}{(Z^2+1)} \qquad -(19)$$

In (19), Z is substituted in terms of z_1 (in terms of x_1 as on the real axis $y_1=0,\, z_1$ being equal to $x_1+iy_1)$ then we obtain $\frac{dy}{dx_1}$.

The values for the calculation are

$\frac{p_0}{p_1}=0,\quad \alpha_0=39.97,\quad \alpha_2=32.97^0$ (obtained through calculation).

These data produces $\frac{U-q_2}{a_2}=0.94699<1$ (subsonic)

Following Srivastava (1968)

$$\frac{\partial p}{\partial x_1}=(x_1-1)^{\beta/\pi}e^{-\frac{x_1}{12\pi}}\left\{(-1.51698-\beta)+\frac{4(-0.05779-\beta)}{(1-0.25x_1)}+\frac{2(-0.05767-\beta)}{(1-0.5x_1)}\right.$$
$$\left.+\frac{4(0.18751-\beta)}{(1-0.75x_1)}+\frac{\pi/2-\beta}{(1-x_1)}\right\}\cdot\frac{C\delta[D(x_1-x_0)-1]}{(x_1-x_0)\sqrt{x_1^2-1}}\cos\beta \qquad -(20)$$

where $\beta' = \left[\tan^{-1}\left(\partial p/\partial y_1\right)/\left(\partial p/\partial x_1\right)\right] x_1 = t = \frac{1}{x} = z_1$ - (21)

and

$$\left(\partial p/\partial y_1\right)/\left(\partial p/\partial x_1\right) = \frac{0.16149 - 0.03072\tan\theta - 0.02355\tan^2\theta + 0.01038\tan^3\theta}{\left[0.73441 - 0.26559\tan^2\theta\right]^{1/2}\left[0.12908 + 0.04215\tan\theta - 0.02592\tan^2\theta\right]}$$

- (22)

$\frac{\partial p}{\partial x_1}$ is given by (20) and $\frac{dy}{dx_1}$ is given by (19) and so κ (curvature) is known from (15). This gives the final expression for curvature.

From (16) we have $\tan\theta = \frac{\kappa'}{\kappa}\frac{\left(Z^2 - 1\right)}{\left(Z^2 + 1\right)}$

where $z \to \infty \tan\theta = \frac{\kappa'}{\kappa}$ so from the relation (16)

we have $y = \kappa\left(\cos\alpha_2 + \sin\alpha_2 \frac{\kappa'}{\kappa}\right)$

So $\frac{y}{\left(\kappa\cos\alpha_2 + \sin\alpha_2\kappa'\right)} = 1$

when $z \to \frac{1}{b}$, then we have

$$\tan\theta = \frac{\kappa'}{\kappa}\frac{\left(1 - b^2\right)}{\left(1 + b^2\right)} = -\cot\alpha_2$$

So from the relation (16)

$$y = \kappa\left(\cos\alpha_2 + \sin\alpha_2\tan\theta\right)$$

We have $y = 0$

or $$\frac{y}{\kappa\cos\alpha_2 + \sin\alpha_2\kappa'} = 0$$

So finally $z \to \infty \left(z_1 \to 1 \text{ ie } x_1 \to 1\right)$, $\frac{y}{\left(\kappa\cos\alpha_2 + \sin\alpha_2\right)} = 1$

and $z \to \frac{1}{b}\left(z_1 \to \infty, x_1 \to \infty\right)$, $\frac{y}{\left(\kappa\cos\alpha_2 + \sin\alpha_2\right)} = 0$

From (15) and (20) it could be seen that

$\frac{\kappa}{\delta} = 0$ at $\frac{y}{\left(\kappa\cos\alpha_2 + \sin\alpha_2\right)} = 1$ ie at the intersection of unit circle and shock.

$\frac{\kappa}{\delta}=\infty$ at $\frac{y}{(\kappa\cos\alpha_2+\sin\alpha_2)}=0$ ie at the wall and shock wave intersection. The variation of $\frac{\kappa}{\delta}$ between $\frac{y}{(\kappa\cos\alpha_2+\sin\alpha_2)}=0$ to $\frac{y}{(\kappa\cos\alpha_2+\sin\alpha_2)}=1$ is expected to be that as proposed in the case $\frac{U-q_2}{a_2}=1$ (sonic case) given by Srivastava (2016).

The point of inflexion in the curvature of the diffracted shock is given by when from (20)

$$D(x_1-x_0)-1=0 \qquad \text{- (23)}$$

or when $x_1=x_0+\frac{1}{D}$ - (24)

From the calculations we have

$$x_0=0.60388 \text{ and } D=0.52589$$

when these values of substituted in (24) we get

$$x_1=2.505418$$

This means that at the point $x_1=2.50548,$ there is a point of inflexion. So in the reflected diffracted shock there is a point of inflexion. The curvature is infinite, then it passes through point of inflexion and finally it becomes zero. This is a qualitative estimate of the curvature.

III. CONCLUSION

The results for curvature distribution $\frac{\kappa}{\delta}$ for the sonic case by Srivastava (2016) and present results for subsonic case are very important contribution on the subject and possibly first attempt in this direction. The results will be useful in the area of aeronautics.

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