Optimizing Inventory Policy for Deteriorating Items with Permissible delay in payment under the Effect of Price Discounting on Lost Sales

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ABSTRACT

In this paper an inventory model is developed with permissible delay in payment including seasonal quadratic demand with time proportional deterioration. Shortages are assumed with partial backlogging. Price discount is offered for reducing the lost sales of back ordered quantity and increasing the profit per unit time. Further a credit period is permitted by the supplier to the retailer to earn interest on the sales revenue till the credit period. The retailer pays interest on the amount not sold till the credit period. The retailer also gets some revenue from sales of deteriorated quantity and it is retained to fulfill daily needs. Numerical illustrations and sensitivity analysis of the optimal solution with respect to various parameters is carried out to describe the model.

Kewords: Deterioration, Permissible delay in payment, Price discounting, Seasonal demand.

I. INTRODUCTION

Inventory System is one of the main streams of the Operations Research which is essential in business enterprises and Industries. Interest in the subject is constantly increasing, and its development in recent years closely parallels the development of operations research in general. Deterioration of things in an inventory control system is a realistic feature in marketing era. Because of deterioration the goods in the inventory system become decay, obsolete, damaged or devalued. In any inventory control problem a major factor is the maintenance of inventory of deteriorating items. Many products decay or deteriorate over time. Product such as fruits, vegetables, food stuffs are subject to direct spoilage during storage. First exponential deteriorating inventory model developed by Ghare and Schrader [1]. Sometime later, Covert and Philip [2] extended Ghare and Schrader's constant exponential deterioration rate with two-parameter of Weibull distribution. Since then there are so many authors presented their models using deterioration factor in different ways like He Y and Huang H [3], Chakrabarty [4], Teng et al[5], Dave and patel [6], Garg et al.[7] manna and chaudhary [8] ,Dye [9], Sana [10], Sarkar [11], Chang [12], Haidar [13], Guchhait [14] and Pandey and vaish [15].

A valuation access where items are initially mentioned at artificially but are then offered for sale to be minimizing cost to the consumer. This process termed as Price discount. Price discount on unit selling price of goods attracts the customers to buy more and more. Thus price discount is important factors which enhance the demand which in turn increases the total profit per unit time. Ardalan [16] introduced an inventory model in form of temporary price discount. Since then there are so many authors presented their models using price discount factor in different ways like. Wee [17], Sana and Chaudhuri [18], Lin [19], Huang [20], Paul [21] Sarkar [22] and Liying [23].

It has been seen many times that stock ends before the arrival of next replenishment and some customers do not want wait up to the next replenishment. This is termed as partial backlogging. There are so many authors presented their models using price discount factor in different ways like Geethal and Udaykumari [24], Chang and Dye [25] and Pandey et al [26].

Normally, the payment for any consignment is done by retailer to the supplier immediately. But in present days, due to tough competition in market and for attracting more customers a credit period has to be allotted to retailer by the suppliers to attain the maximum profit. No interest has to pay during this time credit period but beyond it, the retailer has to be pay an interest on the amount in stock. Before the end of time period of trade credit limit, the retailer can accumulate the revenue and earn so much profit and interest. This is termed as permissible delay in payment. Hence it is more profitable to the retailer to delay his payment till the very last day of the settlement period. First time inventory model with permissible delay in payment was developed by Goyal [27] using suitable conditions. After that many authors presented inventory models using permissible delay factor like Aggarwal and Jaggi [28] Jamal et al. [29] ,Chang and Dye [25], Ouyang et al. [30],Tsao and Sheen [31] and Teng et al [32], Jaggi [33] described an inventory model with effects of inspection on retailer's ordering policy for deteriorating items with time-dependent demand. Jun li [34] introduced an inventory model with credit period. Khanna [35] developed an inventory policy for deteriorating imperfect quality items including permissible delay in payment.

The present paper is based on of Pandey and Vaish (2017) papers. In Pandey and Vaish paper an economic ordered quantity model is developed considering seasonal quadratic type of demand and instantaneous deterioration with time dependent. In the present model introduce permissible delay in payment factor. In both cases profit are considered due to credit period. Shortages are allowed. A fraction of demand is backordered which depends on waiting time up to the next replenishment and a price discount is given on the backordered quantity. Numerical illustration, tables and graphs are presented to describe the model. In addition sensitivity analysis of the optimal solution with respect to various parameters involved in inventory problem is carried out.

II. ASSUMPTIONS & NOTATIONS

1. Demand pattern is D (t) = at (T-t) which is quadratic in nature.

2. Shortages are allowed and are partial backlogged. The backlogged rate is $\frac{1}{1+\delta(T-t)}$ where ($\delta > 0$).

3. d ($0 \le d \le 1$)is the percentage offer on unit selling price on backordered quantity declared at the start of the stock out period. $\alpha = (1-d)^{-n}$. Where $d \to 0, \alpha \to 1$

4. The permissible delay in payments is also assumed in the model. There can be two cases regarding the credit period M: $0 \le M \le t_1 \le T$ and $0 \le t_1 \le M \le T$ the two cases are considered separately.

5. Instantaneous deterioration with linearly time dependent which *is* θ (t) = θt. C Purchasing cost

p Selling price

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

θ Deterioration coefficient, $\theta <<< 1$

T Cycle length

t_1 The time at which inventory level becomes zero

Q_1 Initial inventory level at the beginning of each cycle and

Q_2 Backordered quantity

Q Ordered Quantity (Q_1+Q_2)

DQ Total Deteriorated quantity

p_r Reduced selling price of each deteriorated unit

M trade credit period

I_p Interest paid

Ie Interest earned

h Holding cost

s Shortage cost

l Lost sale cost

A Ordering cost

δ Backlogging Rate

R Sales revenue per replenishment cycle

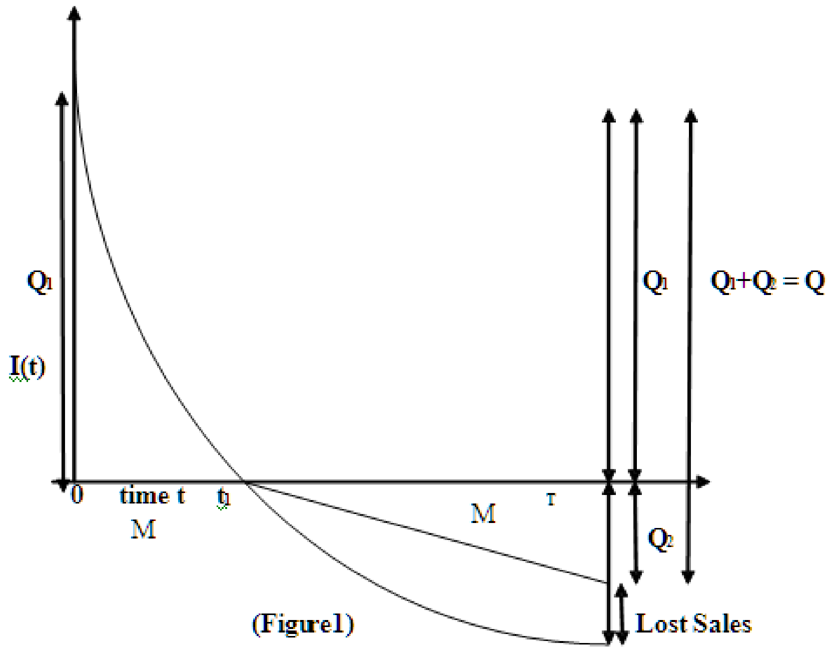
I(t) The inventory level at time t.

F (t_1, T) Profit per unit time

$t_1{}^*$, $F^*(t_1, d)$, Q^* represents the optimal values of t_1, $F(t_1,T)$, Q respectively.

The behavior of the inventory level during cycle T is depicted in figure1.

The permissible delay in payments is also assumed in the model. There can be two cases regarding the credit period M in Fig (1): $0 \le M \le t_1 \le T$ and $0 \le t_1 \le M \le T$ the two cases are considered separately.



(Figure1)

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

III. MATHEMATICAL MODELING

The differential equations showing the fluctuation of inventory with time t are shown as below:

$$\frac{dI(t)}{dt} = -\theta t I(t) - at(T-t) \qquad 0 \le t \le t_1$$

… (1)

$$\frac{dI(t)}{dt} = -\frac{at\alpha(T-t)}{1+\delta(T-t)} \qquad t_1 \le t \le T$$

… (2)

With boundary condition $I(t_1) = 0$ the solutions of these equations are given by:

$$I(t) = a\{(\frac{Tt_1^2}{2} - \frac{t_1^3}{3} + \frac{\theta T t_1^4}{8} - \frac{\theta t_1^5}{10}) - (\frac{T}{2} + \frac{\theta T t_1^2}{4} - \frac{\theta t_1^3}{6})t^2 + \frac{t_1^3}{3} + \frac{\theta T}{8}t^4 - \frac{\theta}{15}t^5\} \quad 0 \le t \le t_1$$

… (3)

$$I(t) = a(1-d)^{-n}\left(\frac{(t_1^2 - t^2)}{2\delta} + \frac{(t_1 - t)}{\delta^2} + \left(\frac{1+\delta T}{\delta^3}\right)\{\log[1+\delta(T-t_1)] - \log[1+\delta(T-t)]\}\right)$$

$t_1 \le t \le T$ …(4)

Sales revenue R= $pQ_1 + p(1-d)Q_2 + p_r DQ$ where

$$Q_1 = p\int_0^{t_1} at(T-t)dt$$

$$Q_1 = \frac{apTt_1^2}{2} - \frac{apt_1^3}{3}$$

… (5)

$$Q_2 = -I(T)$$

$$Q_2 = a(1-d)^{-n}(1-d)\left(\frac{(T^2 - t_1^2)}{2\delta} + \frac{(T-t_1)}{\delta^2} - \left(\frac{1+\delta T}{\delta^3}\right)\log[1+\delta(T-t_1)]\right)$$

…(6) $$R = ap\{\frac{t_1^2 T}{2} - \frac{t_1^3}{3} + (1-d)^{-n}(1-d)\left(\frac{(T^2 - t_1^2)}{2\delta} + \frac{(T-t_1)}{\delta^2} - \left(\frac{1+\delta T}{\delta^3}\right)\log[1+\delta(T-t_1)]\right)\}$$

… (7)

Purchasing cost PC = $(Q_1+Q_2)\, C = C\,(I_1(0) + (-\ I_2(T))$

$$= aC\left(\frac{t_1^2 T}{2} - \frac{t_1^3}{3} + \frac{\theta T t_1^4}{8} - \frac{\theta t_1^5}{10} + (1-d)^{-n}\left(\frac{(T^2 - t_1^2)}{2\delta} + \frac{(T-t_1)}{\delta^2} - \left(\frac{1+\delta T}{\delta^3}\right)\log[1+\delta(T-t_1)]\right)\right)$$

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

… (8)

Deteriorated Quantity DQ $= ap_r\{\frac{\theta T t_1^4}{8} - \frac{\theta t_1^5}{10}\}$

… (9)

Holding Cost HC $= h\int_0^{t_1} I(t)dt = ah\left(\frac{Tt_1^3}{3} - \frac{t_1^4}{4} + \frac{\theta T t_1^5}{15} - \frac{\theta t_1^6}{18}\right)$

… (10)

Shortage cost SC $= as\left(\frac{T^3}{6} - \frac{t_1^2 T}{2} + \frac{t_1^3}{3}\right)$

… (11)

Lost Sale Cost LSC $= al\int_{t_1}^{T} (t(T-t) - \frac{\alpha t(T-t)}{1+\delta(T-t)})dt$

$$= al(1-d)^{-n}\{\left(\frac{t_1}{\delta^2} + \frac{t_1^2}{2\delta} + \left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right] - \frac{T^2}{2\delta} - \frac{T}{\delta^2}\right\} + l(\frac{T^3}{6} - \frac{t_1^2 T}{2} + \frac{t_1^3}{3}\right)$$

… (12)

Ordering Cost OC = A

Now Interest Earned and Interest Paid for two cases of credit period are discussed separately as follows;

Case-1: $0 \le M \le t_1 \le T$ in this case the retailer can use sales revenue to earn interest during the period [0, M] and the annual interest will be paid for the amount of inventory not sold till the credit period M. The total interest earned IE_1 and the total interest paid IP_1 for this case will be as follows:

$$IE_1 = pI_e(\int_0^M at(T-t)tdt + (1-d_2)MQ_2)$$

$$IE_1 = apI_e\{\frac{TM^3}{6} - \frac{M^4}{12} + (1-d)^{-n}(1-d)M\left(\frac{\left(T^2-t_1^2\right)}{2\delta} + \frac{\left(T-t_1\right)}{\delta^2} - \left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]\right)\}$$

… (13)

$$IP_1 = CI_p\int_M^{t_1} I(t)dt$$

$$IP_1 = aCI_p[(\frac{t_1^2 T}{2} - \frac{t_1^3}{3} + \frac{\theta T t_1^4}{8} - \frac{\theta t_1^5}{10})(t_1 - M) - (\frac{T}{2} + \frac{\theta T t_1^2}{4} - \frac{\theta t_1^6}{6})\frac{(t_1^3 - M^3)}{3} + \frac{(t_1^4 - M^4)}{12} + \frac{\theta T(t_1^5 - M^5)}{40} - \frac{\theta(t_1^6 - M^6)}{90}]$$

Now total profit per unit time for case 1 $F(t_1, T)$ can be calculated as: … (14)

$F(t_1,T) = \frac{1}{T}$ [Sales revenue + interest earned – purchasing cost – shortage cost

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

-lost sale cost – holding cost – ordering cost – interest paid]

Total Profit per Unit Time for case 1:

$$F(t_1)=\frac{a}{T}\Bigg[\Bigg\{p\{\frac{t_1^2T}{2}-\frac{t_1^3}{3}+(1-d)^{-n}(1-d)\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\}+p_r\{\frac{\theta Tt_1^4}{8}-\frac{\theta t_1^5}{10}\}$$

$$+pI_e\{\frac{TM^3}{6}-\frac{M^4}{12}+(1-d)^{-n}(1-d)M\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\}-$$

$$CI_p\{(\frac{t_1^2T}{2}-\frac{t_1^3}{3}+\frac{\theta Tt_1^4}{8}-\frac{\theta t_1^5}{10})(t_1-M)-(\frac{T}{2}+\frac{\theta Tt_1^2}{4}-\frac{\theta t_1^6}{6})(\frac{t_1^3-M^3}{3})+\frac{(t_1^4-M^4)}{12}+\frac{\theta T(t_1^5-M^5)}{40}-\frac{\theta(t_1^6-M^6)}{90}\}$$

$$-C\left(\frac{t_1^2T}{2}-\frac{t_1^3}{3}+\frac{\theta Tt_1^4}{8}-\frac{\theta t_1^5}{10})+(1-d)^{-n}\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\right)$$

$$-h\left(\frac{Tt_1^3}{3}-\frac{t_1^4}{4}+\frac{\theta Tt_1^5}{15}-\frac{\theta t_1^6}{18}\right)-s\left(\frac{T^3}{6}-\frac{t_1^2T}{2}+\frac{t_1^3}{3}\right)$$

$$-l\{(1-d)^{-n}\left(\frac{t_1-T}{\delta^2}+\frac{t_1^2-T^2}{2\delta}+\left(\frac{(1+\delta T)\log\left[1+\delta(T-t_1)\right]}{\delta^3}\right)\right)+\frac{1}{(1-d)^{-n}}(\frac{T^3}{6}-\frac{t_1^2T}{2}+\frac{t_1^3}{3})\}-A\Bigg\}\Bigg]$$

… (14)

Case-2: $0\le t_1\le M\le T$ in this in this case the retailer can use sales revenue to earn interest during the period [0, M] and as there is no inventory on hand beyond the credit period M so the retailer has to pay no interest. The total interest earned for this case IE_2 will be calculated as follows:

$$IE_2=pI_e\{\int_0^{t_1}at(T-t)tdt+(M-t_1)\int_0^{t_1}at(T-t)dt+a(1-d)^{-n}(1-d)M\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\}$$

$$IE_2=apI_e\{(\frac{Tt_1^3}{3}-\frac{t_1^4}{4})+(M-t_1)(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})+(1-d)^{-n}(1-d)M\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\}$$

Now total profit per unit time for case 2 $F(t_1,T)$ can be calculated as:

$F(t_1,T)=\frac{1}{T}$ [Sales revenue + interest earned – purchasing cost – shortage cost

-lost sale cost – holding cost – ordering cost]

Total Profit per Unit Time for case 2:

$$F(t_1)=\frac{a}{T}\Bigg\{p\{\frac{t_1^2T}{2}-\frac{t_1^3}{3}+(1-d)^{-n}(1-d)\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\}$$

$$+apI_e\{(\frac{Tt_1^3}{3}-\frac{t_1^4}{4})+(M-t_1)(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})+(1-d)^{-n}(1-d)M\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\}$$

$$-C\left(\frac{t_1^2T}{2}-\frac{t_1^3}{3}+\frac{\theta Tt_1^4}{8}-\frac{\theta t_1^5}{10}+(1-d)^{-n}\left(\frac{(T^2-t_1^2)}{2\delta}+\frac{(T-t_1)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\right)$$

$$+pr\{\frac{\theta Tt_1^4}{8}-\frac{\theta t_1^5}{10}\}-h\left(\frac{Tt_1^3}{3}-\frac{t_1^4}{4}-\frac{\theta Tt_1^5}{15}-\frac{\theta t_1^6}{18}\right)-s\left(\frac{T^3}{6}-\frac{t_1^2T}{2}+\frac{t_1^3}{3}\right)$$

$$-l(1-d)^{-n}\{\left(\frac{t_1}{\delta^2}+\frac{t_1^2}{2\delta}+\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]-\frac{T^2}{2\delta}-\frac{T}{\delta^2}\}+l(\frac{T^3}{6}-\frac{t_1^2T}{2}+\frac{t_1^3}{3}\right)-A\Bigg\}$$

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

IV. SOLUTION PROCEDURE

In the model unit time profit is a function of two variables t_1 and T. To find out the optimal solution

$$\frac{\partial F(t_1,T)}{\partial t_1}=0 \quad \frac{\partial F(t_1,T)}{\partial T}=0$$

… (16)

The optimal values of t_1 and d are obtained by solving these equations simultaneously provided

$$\frac{\partial^2 F(t_1,T)}{\partial t_1^2}.\frac{\partial^2 F(t_1,T)}{\partial T^2}-\left(\frac{\partial^2 F(t_1,T)}{\partial t_1 \partial T}\right)^2>0$$

... (17)

NUMERICAL ILLUSTRATION FOR CASE-1

M= 0.5, θ=0.009, I_p = 0.12 units, I_e = 0.10 units, p= 550, n = 5, p_r = 200, s= 15 rs/unit, l=13 rs/unit, a=40000, A = 100rs/ order

h= 1.5 rs/unit, d = 0.15, C = 120, δ = 2.2.

Applying the solution procedure described above the optimal values obtained is as follows:

t_1*= 0.648923, T^* = 1.13489, $F^*(t_1,T)$ = 386156 rs, Q^* = 11205.7 units

Effects of various parameters on Total Profit per Unit Time For Case-1 …

(15)

Effects of parameter"θ" on Total Profit per Unit Time

%change in θ	θ	t1	T	F
-15%	0.00765	0.649038	1.13499	387012
-10%	0.0081	0.648998	1.13496	386713
-5%	0.00855	0.64896	1.13492	386708
0	0.009	0.648923	1.13489	386156
5%	0.0135	0.648581	1.13461	385849
10%	0.0099	0.648856	1.13484	385656
15%	0.01035	0.648825	1.13481	385430

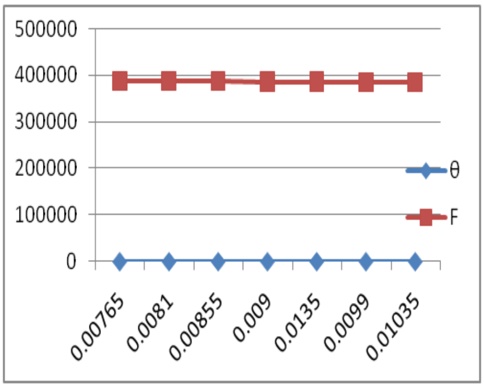

TABLE-1& FIGURE-2

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ISSN (O) 2319 - 8354
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Effects of parameter"d" on Total Profit per Unit Time

%change in d	d	t1	T	F
-15%	0.1275	0.638696	1.12586	144952
-10%	0.135	0.640139	1.12876	224872
-5%	0.1425	0.650299	1.13255	313083
0	0.15	0.648923	1.13489	386156
5%	0.1575	0.636328	1.13644	449574
10%	0.165	0.640842	1.13938	538292
15%	0.1725	0.644979	1.14205	626502

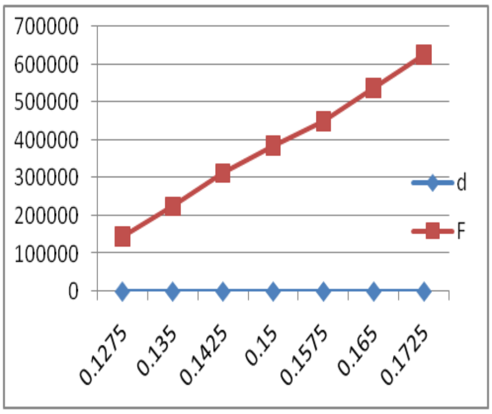

TABLE-2& FIGURE-3

Effects of parameter"n" on Total Profit per Unit Time

%change in n	n	t1	T	F
-15%	4.25	0.638421	1.12352	84750.7
-10%	4.5	0.639244	1.12714	178426
-5%	4.75	0.65058	1.1319	293722
0	5	0.648923	1.13489	386156
5%	5.25	0.637556	1.13724	472603
10%	5.5	0.643023	1.14077	582813
15%	5.75	0.647803	1.14385	691087

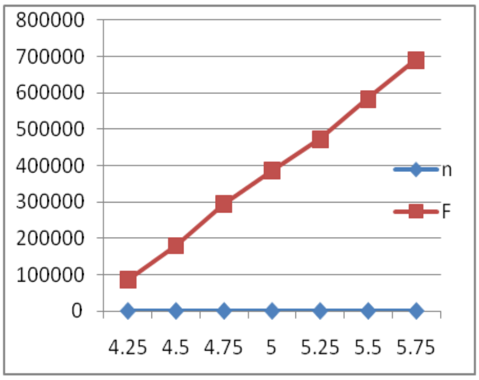

TABLE-3& FIGURE-4

Effects of parameter"a" on Total Profit per Unit Time

%change in a	a	t1	T	F
-15%	34000	0.648923	1.13489	328233
-10%	36000	0.648923	1.13489	347541
-5%	38000	0.648923	1.13489	366848
0	40000	0.648923	1.13489	386156
5%	42000	0.648923	1.13489	405464
10%	44000	0.648923	1.13489	424772
15%	46000	0.648923	1.13489	444080

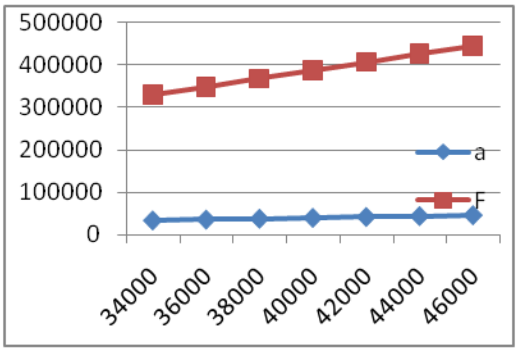

TABLE-4& FIGURE-5

Sensitivity Analysis

parameter	% change	%change t_1	% change T	%change $F(t_1)$
θ	-15	0.0001772	0.0000881	0.002216721
	-5	0.0000577	0.0000264	0.001429474
	5	-0.000527	-0.00024672	-0.000795015
	15	-0.000151	-0.0000715	-0.001880069
d	-15	-0.01576	-0.013289712	-0.624628389
	-5	-0.01329	-0.002061874	-0.189231813
	5	0.0114112	0.001365771	0.164228964
	15	0.0247425	0.006308981	0.006308981
n	-15	-0.016184	-0.010018592	-0.7805273
	-5	-0.012857	-0.002634617	-0.239369581
	5	0.0025165	0.002070685	0.223865484
	15	0.0136842	0.007895038	0.789657548
a	-15	0	0	-0.149998964
	-5	0	0	-0.050000518
	5	0	0	0.050000518
	15	0	0	0.150001554

V. OBSERVATIONS

1. From table (1) it is observed that as the rate of backlogging (θ) decreases, the unit time profit of the system increases.
2. From table (2) it is observed that as discount (d) increases the unit time profit of the system increases.
3. From table (3) it is observed that as (n) increases the unit time profit of the system increases.
4. Table (4) reveals that as the demand factor (a) increases, the unit time profit of the system also.
5. From Above sensitive table it has been observed (a) is insensitive to T, t_1 and moderate sensitive to $F(t_1,T)$.d and θ are negligible sensitive to $t_{1,}$T and moderate sensitive to $F(t_1,T)$.(n) is moderate sensitive to $t_{1,}$T and $F(t_1,T)$.

NUMERICAL ILLUSTRATION FOR CASE-2

M= 0.75, θ = 0.009, I_p = 0.12 units, I_e = 0.10 units, p= 550, n = 5, p_r = 200, s= 15 rs/unit, l=13 rs/unit, a=600, A = 100rs/ order

h= 1.5 rs/unit, d = 0.15, C = 120, δ = 2.2.

Applying the solution procedure described above the optimal values obtained is as follows:

t_1*= 0.650123, T^* = 1.13911, $F^*(t_1,T)$ = 589849rs, Q^* = 11325.19 units.

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

Effects of various parameters on Total Profit per Unit Time For Case-II

Effects of parameter"θ" on Total Profit per Unit Time

%change in θ	θ	t1	T	F
-15%	0.00765	0.650248	1.13921	590785
-10%	0.0081	0.650201	1.13917	590414
-5%	0.00855	0.650162	1.13914	590132
0	0.009	0.650123	1.13911	589849
5%	0.0135	0.649773	1.13883	589562
10%	0.0099	0.650051	1.13906	589336
15%	0.01035	0.650015	1.13903	589089

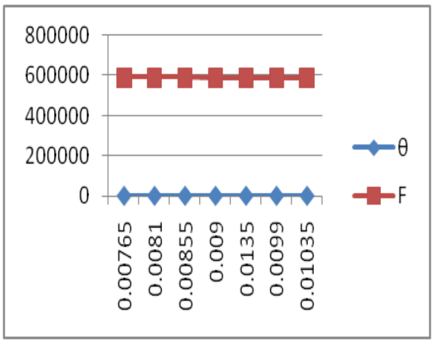


TABLE-5& FIGURE-6

Effects of parameter"d" on Total Profit per Unit Time

%change in d	d	t1	T	F
-15%	0.1275	0.638989	1.13142	349821
-10%	0.135	0.643069	1.13423	430529
-5%	0.1425	0.646688	1.13677	510206
0	0.15	0.650123	1.13911	589849
5%	0.1575	0.653306	1.14124	669426
10%	0.165	0.656215	1.14314	748852
15%	0.1725	0.658892	1.14464	828551

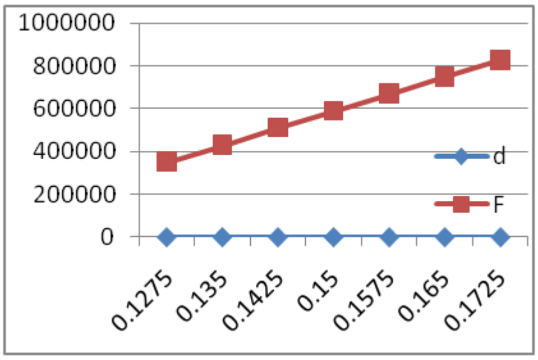
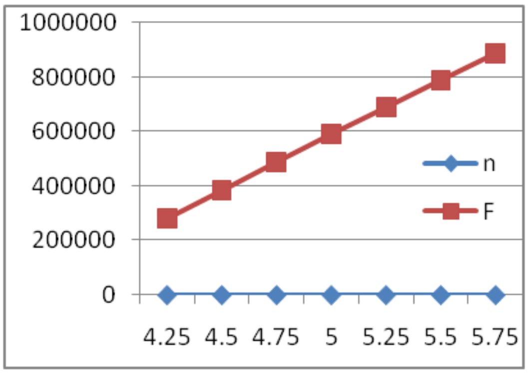


TABLE-6 & FIGURE-7

Effects of parameter"n" on Total Profit per Unit Time

%change in n	n	t1	T	F
-15%	4.25	0.635379	1.12888	281947
-10%	4.5	0.640847	1.13272	386612
-5%	4.75	0.645699	1.13611	488940
0	5	0.650123	1.13911	589849
5%	5.25	0.654081	1.14174	689485
10%	5.5	0.657608	1.14401	788197
15%	5.75	0.66072	1.14591	885979



International Journal of Advance Research in Science and Engineering
Vol. No.6, Issue No. 09 , September 2017
www.ijarse.com
IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

TABLE-7& FIGURE-8

Effects of parameter"a" on Total Profit per Unit Time

%change in a	a	t1	T	F
-15%	34000	0.650123	1.13911	501371
-10%	36000	0.650123	1.13911	530866
-5%	38000	0.650123	1.13911	560358
0	40000	0.650123	1.13911	589849
5%	42000	0.650123	1.13911	619344
10%	44000	0.650123	1.13911	648836
15%	46000	0.650123	1.13911	678326

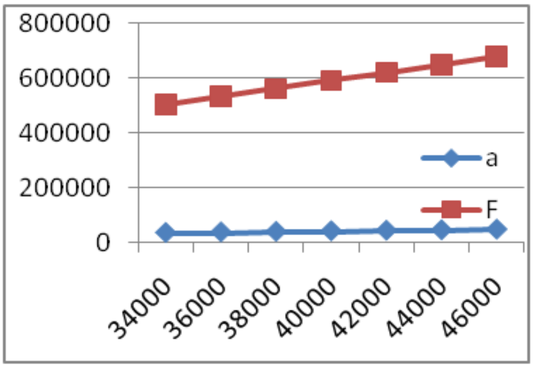


TABLE-8& FIGURE-9

Sensitivity Analysis

parameter	% change	%change t_1	%change T	%change $F(t_1)$
θ	-15	0.00019227	0.00008778	0.001586847
	-5	0.0000598	0.00002633	0.000479784
	5	-0.0005384	-0.000245806	-0.000486565
	15	-0.0001661	-0.00007023	-0.001288465
d	-15	-0.017126	-0.006750884	-0.406931265
	-5	-0.0052836	-0.002054235	-0.135022692
	5	0.004896	0.001869881	0.134910799
	15	0.01348822	0.004854667	0.404683232
n	-15	-0.0226788	-0.008980695	-0.522001394
	-5	-0.0068049	-0.002633635	-0.171075987
	5	0.00608808	0.00230882	0.168917808
	15	0.01629999	0.005969573	0.502043743
a	-15	0	0	-0.150001102
	-5	0	0	-0.049997542
	5	0	0	0.050004323
	15	0	0	0.149999407

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

VI. OBSERVATIONS

1. From table (5) it is observed that as the rate of backlogging (θ) decreases, the unit time profit of the system increases.
2. From table (6) it is observed that as discount (d) increases the unit time profit of the system increases.
3. From table (7) it is observed that as (n) increases the unit time profit of the system increases.
4. Table (8) reveals that as the demand factor (a) increases, the unit time profit of the system also.
5. From Above sensitive table it has been observed (a) is insensitive to T, t_1 and moderate sensitive to $F(t_1,T)$.(d) and (θ) are negligible sensitive to t_1,T and (d) is moderate sensitive to $F(t_1,T)$ and (θ) is negligible sensitive to $F(t_1,T)$. (n) is negligible sensitive to t_1, T and moderate sensitive to $F(t_1,T)$.

VII. CONCLUSION

In the present paper an inventory model is developed with realistic features of seasonal quadratic demand time dependent deterioration, partial backlogging. The impressive feature of the modal is back order price discount and permissible delay in payment. Because of credit period the retailers earn more interest on the accumulated sales revenue and pays interest on the amount not sold till credit period. The present paper is prepared considering the above mentioned realistic features. Two possible cases for credit period are discussed and the numerical illustrations, and sensitivity analysis represents the reality of the model to practical situations of the market. The present study can be further studied for some other situations of demand and partial backlogging useful for inventory problems.

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