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Flow of a Dusty Non-Newtonian Viscous Fluid Through an

Inclined Equilateral Triangular Channel

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ABSTRACT

In this paper the motion of a Non-Newtonian dusty fluid through an inclined equilateral channel placed under a transverse magnetic field has been studied by using the concern equation of a straight channel after their modification for inclined triangular channel with suitable conditions and then through for a series process, an systematic explanation for the velocity division has been got for the both fluid and particle phase. The result of various parameters connected with the flow problem such as magnetic field, frequency of oscillation and inclination are analyst by plotting the graph.

Keywords: Dusty fluid, Non- Newtonian MHD flows, Triangular -Channel, Magneticfield, Twophase flow.

I. INTRODUCTION

The studies of the flow of dusty incompressible and electrically-conducting fluid throughout an leaning pipe of different cross-sections in company of magnetic field has various important practical applications in industries and engineering sciences as the effectiveness and presentation of many devices are affected by the company of rejected solid particles covered by the fluid in magnetic field. A big effort has been prepared to recognize relations between dusty fluid flow and magnetic field time to time by number of researchers. Saffman¹ considered the consequence of stability of laminar flow of dusty gas. Dube and Srivastava² studied the equation for the unsteady flow of a Dusty viscous fluid by letting consistent distribution of dust particles in a channel bounded by 2 parallel flat plates. Pateriya³ examined the unsteady viscous fluids flow throughout electrical ducts. Rukmangadachari⁴ has found the solution of dusty viscous flow through a cylinder of triangular crosssection. Das and Gupta⁵ studied the unsteady viscous flow of an incompressible fluid viscous liquid through an equilateral triangular channel in presence of magnetic- field. Malekzadeh⁶ investigated the magnetic field effect on laminar heat transfer in a pipe for thermal entry region. Chernyshov⁷ obtained the exact solution for unsteady two dimensional problem of the motion of an incompressible viscous fluid in rigid tube of triangular crosssection.Attia⁸ explained MHD Hartman flow of a dusty fluid with exponential decaying pressure gradient. Sandsoo Lim⁹ examined the MHD micro pump with side-walled electrodes. Lee¹⁰ has discovered arithmetic solution of study on electro hydrodynamic induction pumps by CFD modelling. Khare& Avinash¹¹ considered Magnetoydrodynamic flow of a Dusty Fluid through an Equilateral Triangular Channel.

In the current learning, the inclined channel through equilateral triangular cross-section has believed for the motion of fluid. The fluid is supposed to be dusty, incompressible and electrically conducting while the particle

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phase is assumed be incompressible and electrically non-conducting Dust particle are assumed to be sphereshaped and of equal size and mass. The flow is persuaded by a decaying pressure gradient & other force of interactions has been ignored. The inclined channel is located under an applied transverse magnetic field while no electric field is applied and the persuaded magnetic field is neglected by assuming a very little magnetic Reynolds number.

The motion of system has been observed for the fluid and particle phase separately. The differential equations so formed have been solved analytically with the established boundary conditions using different mathematical techniques and related expressions have been derived by considering parameters viz. Magnetic Field, Frequency-Parameter, Dust Relaxation Parameter and Dust Concentration Parameter. Choosing the numerical values for these parameters, the derived relations have been used to find the numerical values for the steady and unsteady part of velocity. Then the graphs have been drawn to analyze the results which are also examined on theoretical basis.

II. FORMULATION OF THE PROBLEM

Consider the flow of a dusty viscous incompressible fluid through an inclined equilateral triangular channel placed under transversely applied magnetic field taking the flow along the axis of the channel.



Fig.Shape of Channel

The governing equations of the motion of a dusty incompressible electricity conducting fluid in a straight channel under the influence of applied external transverse uniform magnetic field are given by

$$\frac{\partial u}{\partial t} + (u\nabla)u = -\frac{\nabla p}{\rho} + v\nabla^2 u + \frac{KN}{\rho}(u-v) + \frac{\mu_e(J \times H)}{1+m^2} + g\sin\beta$$
(1.1)

$$\left[\frac{\partial v}{\partial t} + (v \cdot \nabla)\nabla\right] = \frac{K}{m}(u - v) + g\sin\beta$$
(1.2)

$$\nabla \boldsymbol{\mu} = \nabla \boldsymbol{.} \boldsymbol{\nu} = \boldsymbol{0} \tag{1.3}$$

$$\frac{\partial N}{\partial t} + \nabla . \left(N v \right) = 0 \tag{1.4}$$

The equations (1.1) and (1.2) may be written as

$$\frac{\partial u}{\partial t} = \frac{1}{\rho} \left(-\frac{\partial p}{\partial z} \right) + \upsilon \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{KN_0}{\rho} (v - u) - \frac{\sigma}{\rho} \left(\frac{B_0^2 u}{1 + m^2} \right) + g \sin \beta$$
(1.5)

$$\frac{\partial v}{\partial t} = \frac{1}{\tau} (u - v) + g \sin \beta$$
(1.6)

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ISSN (O) 2319 - 8354 ISSN (P) 2319 - 8346 Where u: Axial velocity of fluid, v: Axial velocity of dust particles, P: Pressure, μ : Viscosity of the fluid, K: Stokes's resistance coefficient, M: Mass of the each particle, NO: Number density of particles assumed to be constant, v: Kinematic viscosity, ρ : Density of fluid, μ e: Permeability, σ : Electrical conductivity, B0:Magnetic induction $\tau = \frac{m}{K}$: Relaxation time for dust particle, $f = \frac{mN}{2}$: Mass concentration parameter of

the dust particle.

The boundary conditions are
$$u = 0$$
 and $v = 0$ on the boundary of the channel (1.7)

The flow is induced by a pressure gradient of the form

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = A\left(1 + \varepsilon e^{i\omega t}\right)$$
(1.8) Where ω is the frequency of the

oscillation, A being a constant and ε is a dimensionless small quantity.

III. SOLUTION OF THE PROBLEM

Now transforming the equation (1.5), (1.6) and (1.8) by using the following non-dimensional variables

$$u = u_{1}u^{*}, \quad v = u_{1}v^{*}, \quad x = ax^{*}, \quad y = ay^{*}, \quad z = az^{*}, \quad \tau = \frac{a}{u_{1}}\tau^{*}, \quad p = \rho u_{1}^{2}p^{*}$$

$$R = \frac{au_{1}}{v}, \quad t = \frac{a}{u_{1}}t^{*}, \quad M = B_{0}a\sqrt{\frac{\sigma}{\rho v}}, \quad f = \frac{mN_{0}}{\rho}, \quad \tau = \frac{m}{K}, \quad \omega^{*} = \frac{\omega a}{u_{1}}, \quad A^{*} = \frac{Aa}{u_{1}^{2}}$$
(1.9)

Where M is the Hartman Number.

Putting them in equation (1.5), (1.6) and (1.8) we have

$$\frac{\partial u^*}{\partial t^*} = \left(-\frac{\partial P}{\partial z}\right) + \frac{1}{R} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) + \frac{f}{\tau^*} (v^* - u^*) - \frac{M^2 u^*}{R(1 + m^2)} + g \sin \beta$$
(1.10)
$$\tau^* \frac{\partial v^*}{\partial t^*} = (u^* - v^*) + g \sin \beta$$
(1.11)

$$-\frac{\partial p^*}{\partial z^*} = A^* \left(1 + \varepsilon e^{i\omega^* t^*}\right) \tag{1.12}$$

On dropping the star and corresponding non dimensional initial and boundary conditions are

(1)
$$t \le 0$$
, $u(x, y, t) = 0$ everywherein the channel (1.13)
 $v(x, y, t) = 0$ on the boundary of the channel (1.14)
 $v(x, y, t) = 0$ on the boundary of the channel (1.14)

Now we transform the equations (1.10) to trilinear co-ordinate system. Let *PQR* be the equilateral triangular tube and O is its centroid taken as origin. The lines perpendicular and parallel to QRare taken as x-axis and y-axis. Let 2a be the length of each side of the triangle and r be radius of in-circle.Let p_1 , p_2 and p_3 are the perpendicular from any point within the triangle on the sides QR, RP and PQ respectively.

Therefore $p_{1} = r - x$ $p_{2} = r + \frac{x}{2} - \frac{\sqrt{3}}{2} y$ $p_{3} = r + \frac{x}{2} + \frac{\sqrt{3}}{2} y$ (1.16) $p_{1} + p_{2} + p_{3} = \sqrt{3} a$

Now from equations (1.15) and (1.16), we have

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial p_1^2} + \frac{\partial^2}{\partial p_2^2} + \frac{\partial^2}{\partial p_3^2} - \frac{\partial^2}{\partial p_1 \partial p_2} - \frac{\partial^2}{\partial p_2 \partial p_3} - \frac{\partial^2}{\partial p_3 \partial p_1}$$
(1.17)

Under the transformation of equation (1.17), the equation (1.10) be

$$\frac{\partial u}{\partial t} = \left(-\frac{\partial P}{\partial z}\right) + \frac{1}{R} \left(\frac{\partial^2 u}{\partial p_1^2} + \frac{\partial^2 u}{\partial p_2^2} + \frac{\partial^2 u}{\partial p_3^2} - \frac{\partial^2 u}{\partial p_1 \partial p_2} - \frac{\partial^2 u}{\partial p_2 \partial p_3} - \frac{\partial^2 u}{\partial p_3 \partial p_1}\right) + \frac{f}{\tau} (v - u) - \frac{M^2 u}{R(1 + m^2)} + g \sin \beta.$$
(1.18)

and the boundary conditions are

(1)
$$t \le 0$$
, $u = v = 0$, everywhere in channel
(2) $t > 0$, $u = v = 0$ at $p_1 = 0$, $p_2 = 0$ and $p_3 = 0$ (1.19)

For the given conditions, we can let the solutions for u and v are of the form

$$u(p_1, p_2, p_3, t) = A\{u_0(p_1, p_2, p_3) + \varepsilon u_1(p_1, p_2, p_3) e^{i\omega t}\}$$
(1.20)

And

$$v(p_1, p_2, p_3, t) = A\{v_0(p_1, p_2, p_3) + \varepsilon v_1(p_1, p_2, p_3)e^{i\omega t}\}$$
(1.21)

And the boundary conditions change to

$$u_0 = 0, u_1 = 0, v_0 = 0 \text{ and } v_1 = 0 \text{ at } p_1 = 0, p_2 = 0 \text{ and } p_3 = 0$$
 (1.22) Now

putting the value of different derivatives, u and v in equation (1.18) and separately the terms free from $e^{i\omega t}$ and the coefficients of $e^{i\omega t}$, we get

$$\frac{\partial^2 u_0}{\partial p_1^2} + \frac{\partial^2 u_0}{\partial p_2^2} + \frac{\partial^2 u_0}{\partial p_3^2} - \frac{\partial^2 u_0}{\partial p_1 \partial p_2} - \frac{\partial^2 u_0}{\partial p_2 \partial p_3} - \frac{\partial^2 u_0}{\partial p_3 \partial p_1} + \frac{R f}{\tau} \Big(v_0 - u_0 \Big) - \frac{M^2 u_0}{(1 + m^2)} + \frac{gR}{A} \sin \beta = -R$$
(1.23)

$$\frac{\partial^2 u_1}{\partial p_1^2} + \frac{\partial^2 u_1}{\partial p_2^2} + \frac{\partial^2 u_1}{\partial p_3^2} - \frac{\partial^2 u_1}{\partial p_1 \partial p_2} - \frac{\partial^2 u_1}{\partial p_2 \partial p_3} - \frac{\partial^2 u_1}{\partial p_3 \partial p_1} + \frac{Rf}{\tau} \Big(v_1 - u_1 \Big) - \frac{M^2 u_1}{(1 + m^2)} + R - Ru_1 i\omega + \frac{gR}{A} \sin \beta = 0$$
(1.24)

Again putting the value of different derivatives, u and v in equation (1.11), we get

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$$\frac{1}{\tau} \left(u_0 - v_0 + \frac{g\tau}{A} \sin\beta \right) = 0 \Longrightarrow u_0 = v_0 - \frac{g\tau}{A} \sin\beta \quad (1.25)$$
$$v_1 = \frac{1}{(1 + i\omega\tau)} u_1 \qquad (1.26)$$

Now from equation (1.25) in (1.23), we get

$$\frac{\partial^2 u_0}{\partial p_1^2} + \frac{\partial^2 u_0}{\partial p_2^2} + \frac{\partial^2 u_0}{\partial p_3^2} - \frac{\partial^2 u_0}{\partial p_1 \partial p_2} - \frac{\partial^2 u_0}{\partial p_2 \partial p_3} - \frac{\partial^2 u_0}{\partial p_3 \partial p_1} - \frac{M^2 u_0}{(1+m^2)} + \frac{gR}{A} \sin\beta(1+f) = -R$$
(1.27)

$$\frac{\partial^2 u_1}{\partial p_1^2} + \frac{\partial^2 u_1}{\partial p_2^2} + \frac{\partial^2 u_1}{\partial p_3^2} - \frac{\partial^2 u_1}{\partial p_1 \partial p_2} - \frac{\partial^2 u_1}{\partial p_2 \partial p_3} - \frac{\partial^2 u_1}{\partial p_3 \partial p_1} - R\omega(C_1 + iC_2)u_1 + \frac{gR}{A}\sin\beta = -R^{(1.28)}$$

Where
$$C_1 = \left(\frac{M^2}{(1+m^2)\omega R} + \frac{f\omega\tau}{1+\omega^2\tau^2}\right)$$
, and $C_2 = \left\{1 + \frac{f}{1+\omega^2\tau^2}\right\}$ (1.29)

Now we let the solution of the equation (1.27) satisfying the boundary conditions of equation (1.22) as

$$u_{0} = \sum_{n=1}^{\infty} \alpha_{n} \left(\sin \frac{2n\pi}{\sqrt{3}a} p_{1} + \sin \frac{2n\pi}{\sqrt{3}a} p_{2} + \sin \frac{2n\pi}{\sqrt{3}a} p_{3} \right)$$
(1.30)
$$u_{1} = \sum_{n=1}^{\infty} \beta_{n} \left(\sin \frac{2n\pi}{\sqrt{3}a} p_{1} + \sin \frac{2n\pi}{\sqrt{3}a} p_{2} + \sin \frac{2n\pi}{\sqrt{3}a} p_{3} \right)$$
(1.31)

Substituting the value of different derivatives of $u_0 \& u_1$ and $u_0 \& u_1$ from equation (1.30) and (1.31) in equation (1.27) and (1.28), we get

$$\sum_{n=1}^{\infty} \alpha_n \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) \frac{M^2}{(1+m^2)} = R + \frac{gR}{A} \sin \beta (1+f)^{(1.32)}$$
$$\sum_{n=1}^{\infty} \beta_n \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) \omega(C_1 + iC_2) + \frac{g}{A} \sin \beta = 1^{(1.33)}$$

Since $p_1 + p_2 + p_3 = 3^{1/2}$, we can write

$$R = \frac{R}{\sqrt{3}a} \left(\sqrt{3}a - 2p_1 \right) + \left(\sqrt{3}a - 2p_2 \right) + \left(\sqrt{3}a - 2p_3 \right) \right)$$
(1.34)

Now expressing $(\sqrt{3}a - 2p_1)$ etc. as Fourier's sine series, we get

$$\sum_{n=1}^{\infty} \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) \frac{2R}{\pi} = R$$
(1.35)

Now from equations (1.32) and (1.33), we have

$$\alpha_n = \frac{2R(1+m^2)}{\pi n M^2} + g \frac{(1+m^2)R}{AM^2} \sin\beta(1+f)$$
 (1.36)

$$\beta_n = \frac{2}{\pi n \omega (C_1 + iC_2)} + \frac{g}{A} \sin \beta$$
(1.37)

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Hence the solution of equations (1.27) and (1.28) is

$$u_{0} = \sum_{n=1}^{\infty} \left(\frac{2R(1+m^{2})}{\pi nM^{2}} + g \frac{(1+m^{2})R}{AM^{2}} \sin\beta(1+f) \right) \left(\sin\frac{2n\pi}{\sqrt{3}a} p_{1} + \sin\frac{2n\pi}{\sqrt{3}a} p_{2} + \sin\frac{2n\pi}{\sqrt{3}a} p_{3} \right)$$
(1.38)

$$v_0 = \sum_{n=1}^{\infty} \left(\frac{2R(1+m^2)}{\pi n M^2} + g \frac{(1+m^2)R}{AM^2} \sin\beta(1+f) \right) \left(\sin\frac{2n\pi}{\sqrt{3}a} p_1 + \sin\frac{2n\pi}{\sqrt{3}a} p_2 + \sin\frac{2n\pi}{\sqrt{3}a} p_3 \right) + \frac{g\tau}{A} \sin\beta$$
(1.39)

and also

$$u_{1} = \sum_{n=1}^{\infty} \left(\frac{2}{\pi n \omega (C_{1} + iC_{2})} + g \sin \beta \right) \left(\sin \frac{2n\pi}{\sqrt{3}a} p_{1} + \sin \frac{2n\pi}{\sqrt{3}a} p_{2} + \sin \frac{2n\pi}{\sqrt{3}a} p_{3} \right)^{(1.40)}$$
$$v_{1} = \sum_{n=1}^{\infty} \frac{2}{\pi n (1 + i\omega\tau) \omega (C_{1} + iC_{2})} \left(\sin \frac{2n\pi}{\sqrt{3}a} p_{1} + \sin \frac{2n\pi}{\sqrt{3}a} p_{2} + \sin \frac{2n\pi}{\sqrt{3}a} p_{3} \right)^{(1.40)}$$

Now from equations (1.38) and (1.39), we get

$$u = A \begin{cases} \left(\sum_{n=1}^{\infty} \left(\frac{2R(1+m^2)}{\pi n M^2} + g \frac{(1+m^2)R}{AM^2} \sin \beta (1+f) \right) \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) \right) \\ + \varepsilon \left(\sum_{n=1}^{\infty} \left(\frac{2}{\pi n \omega (C_1 + iC_2)} + g \sin \beta \right) \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) e^{i\omega t} \right) \end{cases}$$
(1.42)

Again from equations (1.40) and (1.41), we get

$$v = A \begin{cases} \left(\sum_{n=1}^{\infty} \left(\frac{2R(1+m^2)}{\pi n M^2} + g \frac{(1+m^2)R}{AM^2} \sin \beta (1+f) \right) \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) + \frac{g\tau}{A} \sin \beta \\ + \varepsilon \left(\sum_{n=1}^{\infty} \frac{2}{\pi n (1+i\omega\tau)\omega(C_1+iC_2)} \left(\sin \frac{2n\pi}{\sqrt{3}a} p_1 + \sin \frac{2n\pi}{\sqrt{3}a} p_2 + \sin \frac{2n\pi}{\sqrt{3}a} p_3 \right) \right) e^{i\omega\tau} \end{cases}$$
(1.43)

Graph between Magnetic field parameter & Velocity of Fluid. Graph between Magnetic field parameter & Velocity of Particles.



IV. CONCLUSION

The above equations are used to find numerical computation for various values of the parameters to analyze their effect on the steady and unsteady part of the velocity distributions for both fluid and particle phase.

(i) The study indicates that for every inclination both velocities show a resonances character which occurs nearly magnetic field parameter, the amplitude of unsteady velocity of both phases' increases and take maximum value then decreases with increase of f, the fluid density ρ_{i} decreases with increase in mass concentration parameter f, would increase of the velocity.

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- (ii) The assessment of the velocity in case of particle is very very high in comparison to that of fluid. The cause is obvious that magnetic field is more efficient on magnetic sensitive particle. Both graphs are shows similar in nature except the magnitudes of the changes in parameter are different. It is observed that the steady part of flow of the fluid and particle phase is the identical with increase in Hartman number and cross-section area of the channel. It is also originate that the assessment of Reynolds number has no effects on the fully developed velocity profile. Also the maximum/minimum velocities for both phases are going on at the same inclinations.
- (iii) Their further studies shows that as mass concentration of dust particle increases, the amplitude of unsteady part of velocities of both the phase increases and take the maximum value and then decreased due to increment of magnetic field. The amplitude of the unsteady part of the velocities of fluid and particle phases increase with decrease of frequency of oscillation. It is also observed that as relaxation time of dust particles increases, the amplitude of unsteady velocity of both the phases' increases but when τ decreases and tends to zero, the amplitudes of velocities of fluid and particle phase become the same.

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