



# NUMERICAL ANALYSIS OF CO-CURRENT PARALLEL FLOW THREE FLUID HEAT EXCHANGER

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## ABSTRACT

*This paper contains numerical analysis of a three fluid heat exchanger. The heat exchanger is of co-current parallel flow type. Results are obtained by varying the number of transfer units (NTU) and also varying the two dimensional longitudinal conduction parameter and the axial dispersion parameter. The performance analysis has been carried out numerically. Finite difference method has been used to discretize the non-dimensional form of energy equations and the iterations are obtained by Gauss-Siedel iterative technique. Results are obtained to optimize the losses due to the two dimensional longitudinal conduction and there is a major impact on the performance by varying the number of transfer units (NTU) of the three fluid heat exchanger.*

**Keywords-** Axial dispersion, Co-current, Heat exchanger, Longitudinal conduction, Three fluid.

## I. INTRODUCTION

A heat exchanger is a device which is used to transfer the thermal energy i.e. heat between a solid object and a fluid or between two or more fluids. The fluid can be separated by a solid wall to prevent mixing (as in tube in tube type heat exchanger) or may be in direct physical contact as in cooling tower. Three fluid heat exchanger is a type of heat exchanger which involves the thermal interaction between three fluids. Three fluid heat exchanger are widely used in cryogenics technology and many other chemical processes such as purification and liquefaction of hydrogen, ammonia gas synthesis, air purification and separation systems, helium- air separation system. In cryogenic technology we need to achieve a very very low temperature and this can be achieved by a compact multifluid heat exchanger by setting up of desired flow conditions and exchanging heat with the other streams. Apart from this the three fluid heat exchanger has its applications in aerospace industries where we need to achieve a certain fixed level of temperature.

On the basis of number of thermal energy interaction a three fluid heat exchanger can be classified into two types namely two thermal communication and three thermal communication. In two thermal communication type of heat exchanger only one fluid exchanges heat with the other two fluids and the other two fluids do not have any thermal interaction with each other. While in three thermal communications all the three streams have thermal energy interaction with each other. One can also classify the three fluid heat exchangers based on the flow arrangements such as parallel flow and cross flow arrangements. Parallel flow heat exchangers are further

classified as cocurrent, countercurrent and cocurrent- countercurrent type. Cross flow arrangements can also be further divided into single pass arrangements and multipass flow arrangements.

In past many researchers have tried to explain various aspects such as thermal performance, temperature distribution, effectiveness, size estimation of the three fluid heat exchanger. Morley [1] was the first to perform the analysis of three fluid heat exchanger. Based on the energy balances and rate equations, he got the temperature distributions of all the three fluid streams within the heat exchanger. Although he gave a formula but it was in dimensional form. Instead of giving the general solution he gave unknown coefficient of integration, which he decided to be evaluated in terms of boundary conditions. He neither presented the final explicit temperature nor does he define the design procedure. He gave a complicated heat exchanger sizing procedure which is a trial and error method and is very laborious too. Paschkis and Heisler [2] used an electric analogue method for the design of a parallel stream extended surface three fluid heat exchanger with three thermal communications. In this case he worked on the problems in which some of the inlet and outlet temperatures were specified and some parameters like length of heat exchanger and some unknown temperatures need to be found. The whole analysis was verified using a small heat exchanger model using fluids namely nitrogen, oxygen and air at low temperatures. Sorlie [3] developed a design theory He solved the three first order linear differential equations and derived some closed form formulas for the temperatures of the streams. He followed the effectiveness-NTU approach and

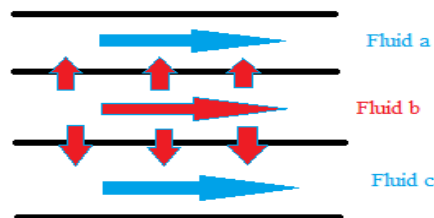


Figure 1(a) Co-current type three fluid heat exchanger with two thermal communication.

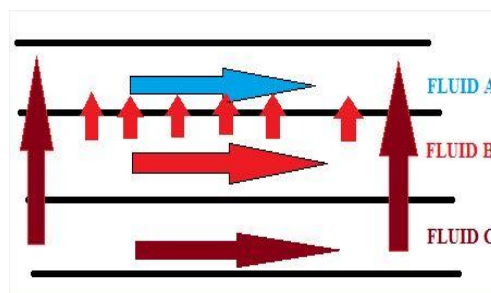


Figure 1(b) co-current type three fluid heat exchanger with three thermal communication.

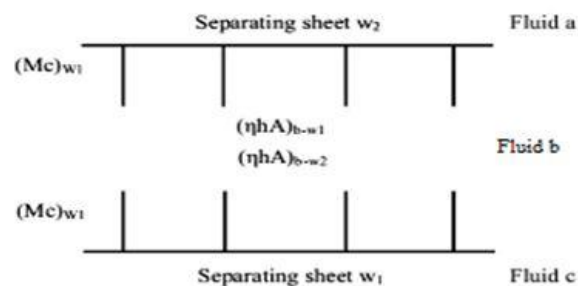
formulated his results in two temperatures effectiveness. Krishnamurthy and form formulas for the temperatures of the streams. He followed the effectiveness-NTU approach and formulated his results in two temperatures effectiveness. Krishnamurthy and Rao [4] analyzed the single pass four streams and three steam heat exchanger



with any of the flow arrangements By integrating the energy balance and rate equations for each fluid, second order linear differential equations were formulated for a three fluid parallel stream heat exchanger. From this he derived the expressions for the heat exchanger length. Rao[5] analyzed concurrent and counter concurrent parallel stream three fluid heat exchangers in terms of temperature differences between fluids as dependent variables. The heat exchanger has three simultaneous thermal communications. The general solutions for the temperature difference along the heat exchanger with steady state operations and constant properties were formulated. No qualitative or quantitative evaluation of the obtained solutions was performed to compare other existing solutions. Aulds and Barron [6] extended the Sorlie's work by taking the case of three thermal interactions. They presented analytical relationship between the effectiveness – NTU for a countercurrent parallel stream three fluid heat exchanger with three thermal communications.. Skeulic and shah [7] has tried to explain the thermal design theory of three fluid and multifluid heat exchanger. He felt that there is a need of unified approach for thermal design even for a single class of three fluid heat exchangers. So he presented a unified theory for all flow arrangements having two thermal communications. He also presented a step-by-step procedure for rating and sizing problems. He also gave a single analytical expression to determine temperature across all combinations of fluids and for all fluid arrangements. Barron and Yeh [8] obtained a numerical solution for the temperature distribution and overall heat exchanger effectiveness for a three fluid heat exchanger with the effect of longitudinal conduction included in the separating sheets. Das and Murugesan [9] predicted the transient simulation of multipass plate heat exchanger with axial thermal dispersion in fluid. They have chosen examples to show the effects of dispersive pecllet number, NTU and number of plates. It has been found that the effect of dispersion is to primarily increase the initial delay period and effectiveness of heat exchanger. Bielski and Malinowski [10] presented semi analytical as well as analytical solutions for a parallel flow arrangements involving step change in inlet temperature of central fluid. Later on, they solved a set of partial differential equations involving three thermal communications and considering longitudinal conduction and its thermal capacity. Mishra et al. [11] also presented the transient temperature response of the cross flow heat exchanger with finite wall capacitance in his technical note. The transient response has been obtained for a combined effect of finite core capacity, two dimensional longitudinal conduction in wall and the axial dispersion in fluids. He analysed in detail the effects of two dimensional longitudinal conduction and axial dispersion. He further stated that te axial dispersion in both the fluid stream neutralizes the effect of conductive heat transfer. Also presence of axial dispersion of high order reduces the effect of two dimensional longitudinal conduction to some extent. It has been observed that the longitudinal conduction plays an important role with increase in NTU and the role of axial dispersion is dominant when the values of pecllet number are small. Singh et al [12] presented the transient behaviour of three fluid heat exchanger which is of concurrent parallel flow type in his technical note. He has used the numerical analysis technique using the Finite difference method. He further stated that the two dimensional longitudinal conduction plays an important role in heat transfer at values of higher NTU and the values of axial dispersion plays an important role at the values of lower pecllet number. It is observed that at lower NTU , the heat exchanger reaches the steady state condition within a very short time, whereas at values of higher NTU the difference in temperature of three fluid increases.

A lot of paper has dealt with the temperature distribution of three fluid heat exchanger. This paper deals with the ways to optimize the three fluid heat exchanger for increased thermal performance and effectiveness. It also considers losses in the form of two dimensional longitudinal conduction and axial dispersion due to which the effectiveness of three fluid heat exchanger is largely reduced. We are also going to observe the effects on the performance of the three fluid heat exchanger by varying the number of transfer units (NTU) of the heat exchanger. In this paper numerical analysis of three fluid heat exchanger has been done with the help of a C program. Finite difference method has been adopted to discretize the non-dimensional form of energy equations and the iterations are obtained using Gauss Siedal iterative technique.

## II. INDENTATIONS AND EQUATIONS



**Figure 2 Distribution of convective resistance of central fluid 'b' and the heat capacity of separating sheet with fins**

A co-current type parallel flow three fluid heat exchanger with two thermal communication is one in which all the three fluid flows in the same direction and the third fluid exchanges the thermal energy with the other two. In Fig. 2, it is shown that the one fluid is flowing in between two separating sheets and having two thermal communications with the other two fluid. A similar type mathematical formulation has been given by Singh et al[15] in their technical note. Following are the assumptions made in this regard -

1. There is no phase change of any fluid in a three fluid system.
2. There is no source of heat generation within the fluids.
3. The fluid in centre is assumed to be hottest or the coldest.
4. No heat transfer to the surrounding.
5. The heat transfer area is uniformly distributed for each fluid.
6. Two thermal communication exists between the fluid.
7. Thermophysical properties of fluids streams and walls are assumed to be independent of space, time and temperature.
8. There is no variation in velocity and temperature of fluid streams in perpendicular direction of flow.
9. The primary and secondary area of the separating sheets have been lumped together, so variation in wall temperature is two dimensional.

10. The fouling resistance and the film heat transfer coefficient are assumed to be constant and uniform.
11. Transverse conduction through fins between adjacent separating sheets is neglected. This implies there will be temperature extremum in the fin temperature profile.
12. Transverse thermal resistance of the separating sheets in a direction normal to it is considered negligible.

As the properties of the top and bottom fluid may be different in a general case therefore we need to consider two separating sheet walls in which the top separating sheet is depicted as (w1) whereas the bottom separating sheet wall is depicted as (w2).

The conservation of energy for the three fluid streams and for the two separating sheets can be expressed in non dimensional form for an infinitesimal small control volume.

For fluid streams a, b, c one gets Eqs. (5), (6), (7) respectively

$$\frac{Va}{Rab} \frac{\partial Ta}{\partial \theta} = Tw1 - Ta - \frac{Eab}{Rab} \frac{\partial Ta}{\partial X} \tag{1}$$

$$Vb \frac{\partial Tb}{\partial \theta} = \frac{1}{\phi} (Tw1 - Tb) + \frac{1}{(1-\phi)} (Tw2 - Tb) - \frac{\partial Tb}{\partial X} \tag{2}$$

$$\frac{Vc}{Rcb} \frac{\partial Tc}{\partial \theta} = Tw2 - Tc - \frac{Ecb}{Rcb} \frac{\partial Tc}{\partial X} \tag{3}$$

Similarly for separating sheets w1 and w2, the energy equations are given as Eqs. (4) and (5) respectively

$$\psi \frac{\partial Tw1}{\partial \theta} = Rab(Ta - Tw1) + \frac{1}{\phi} (Tb - Tw1) + \lambda x Na \frac{\partial^2 Tw1}{\partial X^2} + \lambda y Na \frac{\partial^2 Tw1}{\partial Y^2} \tag{4}$$

$$(1 - \psi) \frac{\partial Tw2}{\partial \theta} = \frac{1}{(1-\phi)} (Tb - Tw2) + Rcb(Tc - Tw2) + \lambda x Na \frac{\partial^2 Tw2}{\partial X^2} + \lambda y Na \frac{\partial^2 Tw2}{\partial Y^2} \tag{5}$$

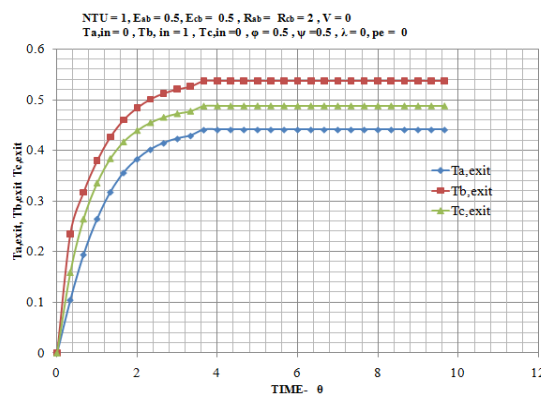


Figure 3(a) Graph showing NTU=1 and no longitudinal conduction and axial dispersion

One need to replace the partial derivatives in the governing equations with discrete numbers. Discretization of partial differential equation is known as finite difference and discretization of the integral form of the equation is called finite volume. It is also needed a different type of coordinate system for transformation and grid generation. The term discrete can be defined as unique, unconnected or a separate thing but the term discretization is a new and esoteric word. Analytic solutions of partial differential equations involves some closed form expression which give the variation of the dependent variables continuously throughout the domain. In constant numerical solutions can give answers at only discrete points in the domain, called Grid points. Uniform spacing greatly simplifies the programming of the solution, saves storage space and usually results in greater accuracy. The partial difference equations are totally replaced by the system of algebraic equation which can be solved for the values of the flow field variables at discrete points only. This method is Finite difference method.

The Gauss-Seidel method is an iterative technique for solving a square system of  $n$  linear equations with unknown  $x$ . Gauss-Seidel iterative technique is also known as method of successive displacement

### III. METHOD OF SOLUTION-

The above equations from Eqs.1-5, are discretized with the help of finite difference method and the solutions are obtained by the Gauss-Seidel iterative techniques. The results are obtained in the tabular form by developing a C program. Graphs are drawn from the data obtained and the results are analysed to optimize the performance of the three fluid heat exchanger.

### IV. RESULTS AND DISCUSSIONS

(a) Effects by varying the number of transfer unit(NTU) with no longitudinal conduction and axial dispersion are shown in Fig.3(a)-3(g).

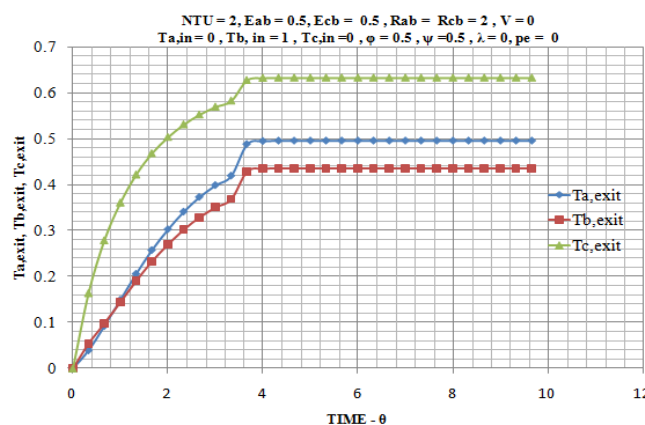


Figure 3(b) Graph showing NTU=2 and no longitudinal conduction and axial dispersion

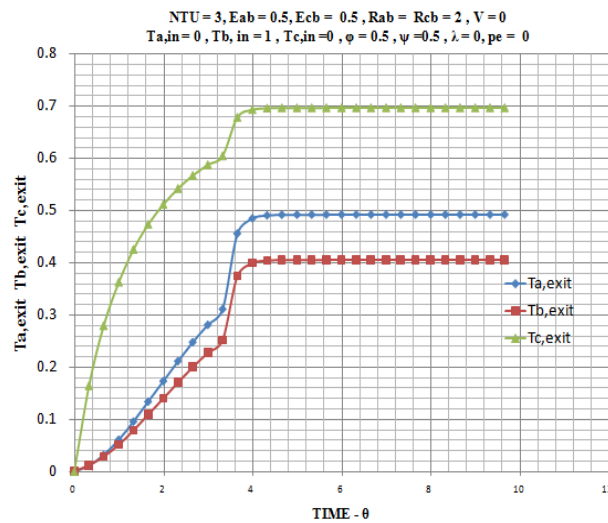


Figure 3(c) Graph showing NTU=3 and no longitudinal conduction and axial dispersion

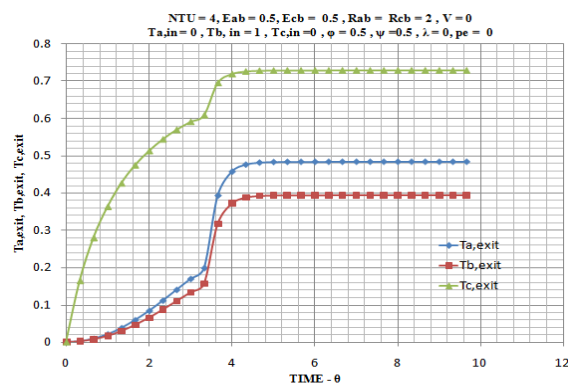


Fig.3 (d) Graph showing NTU=4 and no longitudinal conduction and axial dispersion

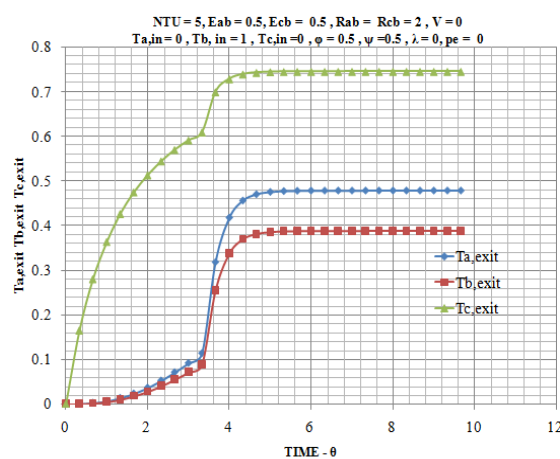


Fig.3 (e) Graph showing NTU=5 and no longitudinal conduction and axial dispersion



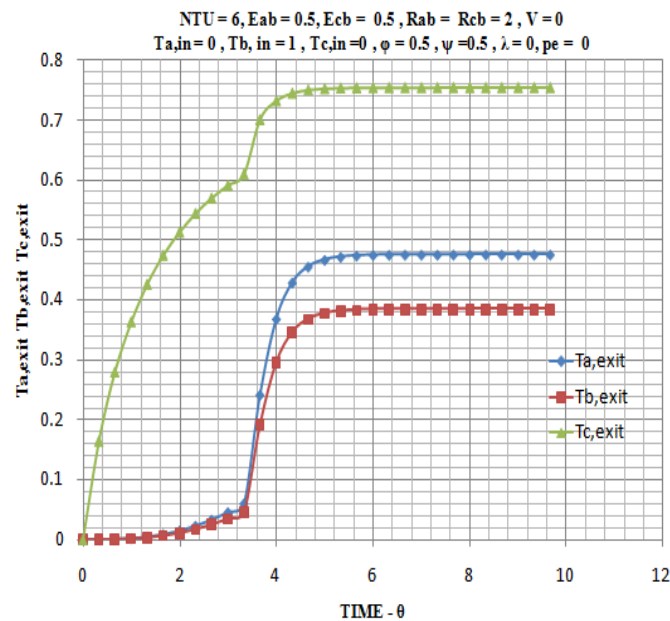


Fig.3 (e) Graph showing NTU=6 and no longitudinal conduction and axial dispersion

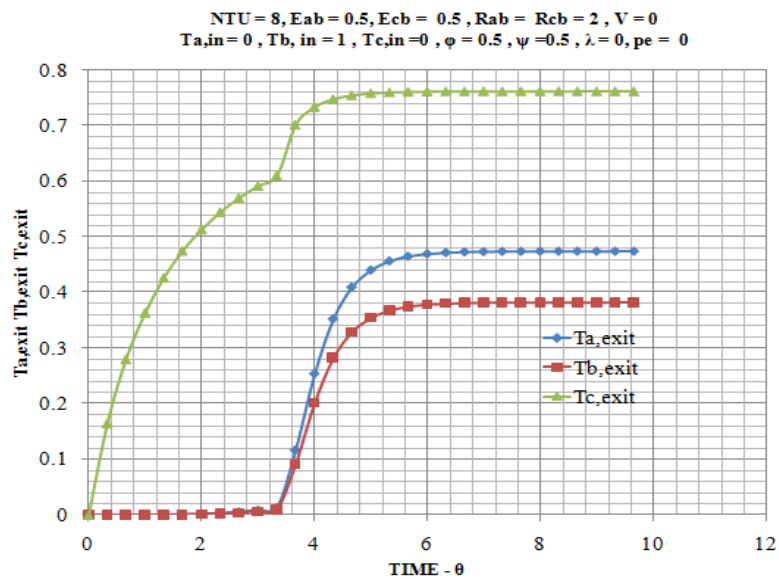


Fig.3 (g) Graph showing NTU=8 and no longitudinal conduction and axial dispersion

Performance analysis of co-current type parallel flow three fluid heat exchanger reveals that there is severe impact on performance by varying the NTU. On varying the NTU one by one several results are obtained. Graphs are plotted for each case. It can be clearly observed from the graph that at lower values of NTU say 1-3, the mean temperature of the three fluids attains a steady state condition very quickly. On further increasing the NTU from 4-8, it can be observed that it requires some time to attain a steady state condition of the three fluids. So it can be stated that the performance of the heat exchanger is lowered at higher values of NTU. The results are verified when compared with that of Singh et al [15]. The results hold very close to their analysis



(b) Effects of two dimensional longitudinal conduction-

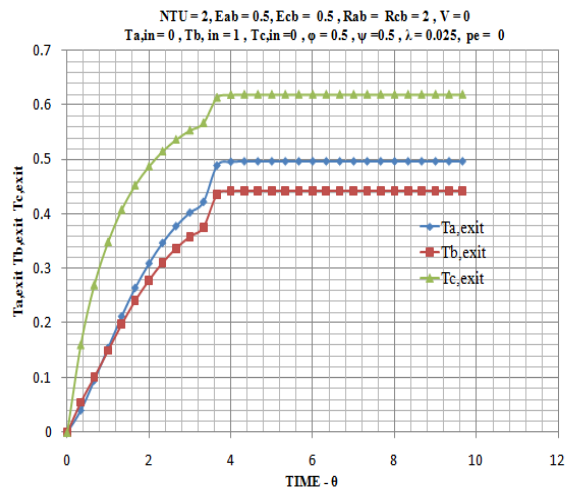


Fig.4 a Effect of varying longitudinal conduction parameter  $\lambda=0.025$

Results are further obtained by varying the values of longitudinal conduction and keeping other parameters such as NTU, thermal heat capacity ratio and conductance ratio of the fluids as constant. Several graphs are plotted as shown in Fig.3(a) to Fig.3(d).

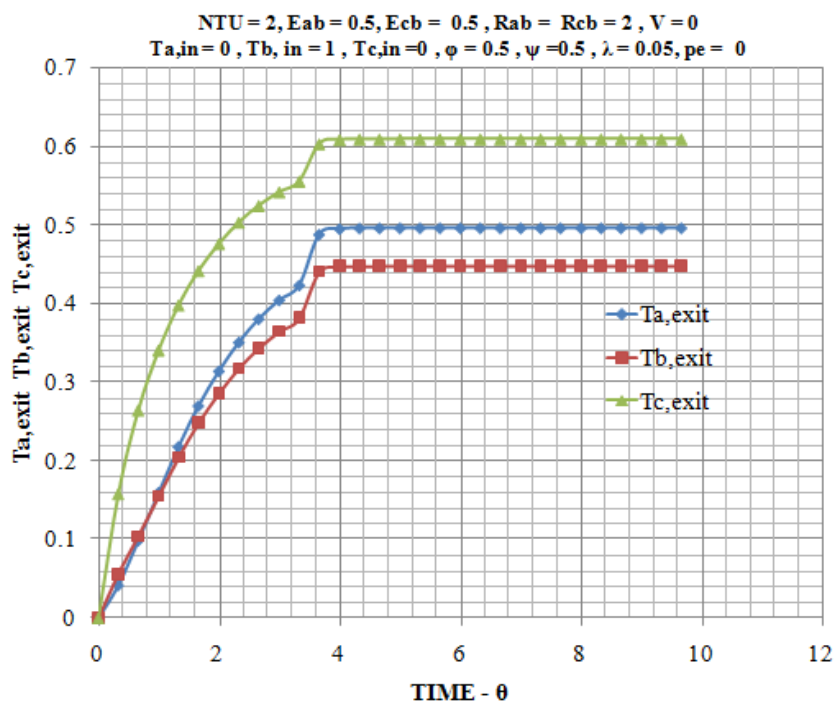


Fig.4 b Effect of varying longitudinal conduction parameter  $\lambda=0.05$

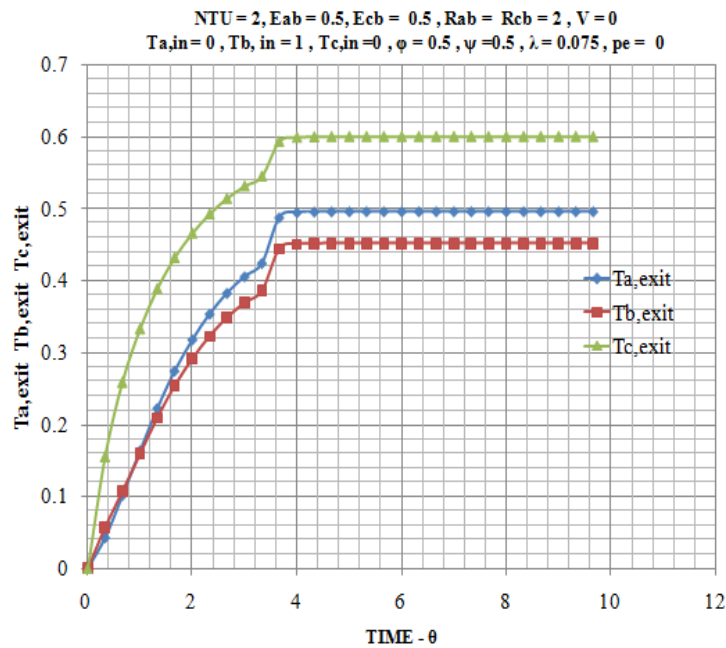


Fig.4 c Effect of varying longitudinal conduction parameter  $\lambda=0.075$

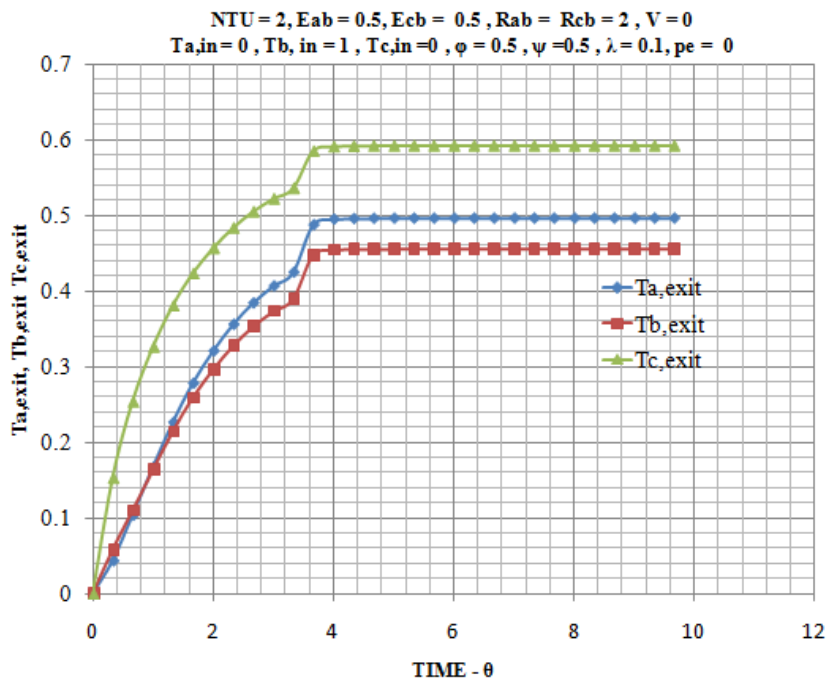


Fig.4 d Effect of varying longitudinal conduction parameter  $\lambda=0.1$

With the help of graph one can depict that the longitudinal conduction has great influence on the performance of the heat exchanger. As the longitudinal conduction increases the mean temperature of each fluid increases. Then after it attain a steady state where the effects of longitudinal conduction has no influence.

(c) Effects of axial dispersion-

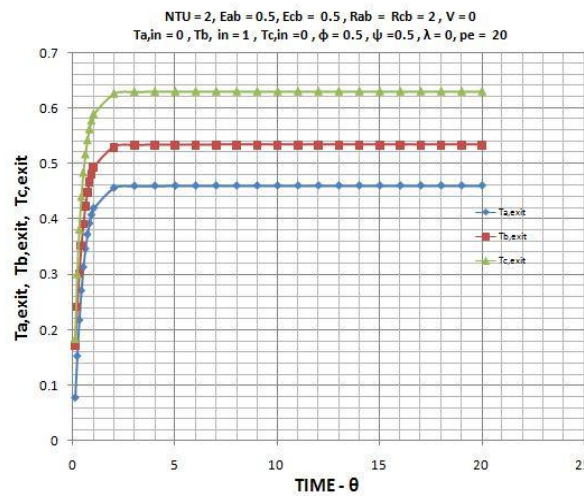


Figure 5 Graph for variation of axial dispersion parameter i.e. pecllet number(Pe) = 20

We can observe that when we increase the pecllet number let say from 5 to 20 or even more, the transient phase reduces and the graph reaches the steady state very quickly. Theoretically we can say that if the pecllet number is infinite the performance of the three fluid heat exchanger is the highest. But losses due to eddies formation and surface irregularities deviates the flow condition and the axial dispersion is induced in the fluid. So we can see that the performance of three fluid heat exchanger decreases when we lower the pecllet number from 20. The effect of axial dispersion dominates at lower values of pecllet number.

V. CONCLUSION

Numerical analysis of co-current type parallel flow three fluid heat exchanger has been carried out. The above analysis was investigated numerically with the help of a computer program. It can be concluded that at lower values of NTU say 1-3 the performance is higher. And if we further increase the NTU from 4-8 the performance is decreased. At higher values of longitudinal conduction parameter the performance is lower. At lower values of Peclet number say upto 10 the performance is lowered. Results obtained will surely help the design engineer in optimization of performance and design the three fluid heat exchanger.

VI. NOMENCLATURE

- A area of heat transfer,  $m^2$
- C heat capacity rate (mc), W/K
- Ea,c-b capacity rate ratio =  $(mc)_{a,c}/(mc)_b$
- h heat transfer coefficient,  $W/m^2K$
- M mass flow rate of fluid, kg/s
- M mass of separating sheet, kg
- Na  $(\eta hA/mc)_b$



NTU	number of transfer unit
Pe	Peclet number
Ra,c-b	conductance ratio = $(\eta hA)_{a,c}/(\eta hA)_b$
T	dimensionless temperature = $(t-t_{a,in})/(t_{b,in}-t_{a,in})$
T	dimensionless mean temperature
U	velocity of flow, m/s
V	heat capacity ratio = $LA_c\rho c/M_c w$
X	= $(\eta hA/mc)_{bx}/L_x = Na(x/L_x)$
Y	= $(\eta hA/mc)_{by}/L_y = Na(y/L_y)$

## Greek letters

$\theta$	= $(\eta hA)_b \tau / (M_c) w$ , dimensionless time
$\phi$	= $(\eta hA)_b^{-w} / (\eta hA)_b$
$\eta$	overall surface efficiency
$\lambda$	longitudinal heat conduction parameter
$\psi$	= $(M_c) w_1 / (M_c) w$

## Subscripts

a,b,c	fluid streams a,b and c
ex	exit
in	inlet
max	maximum
min	minimum
w	separating wall
x,y	x and y coordinate
1, 2	sheets 1 and 2

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