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A New Technique for Solving the Assignment Problems Using Vogel Method

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ABSTRACT

In this paper, I propose a new technique for solving the assignment problems based on the Vogel Method which is well known in solving the Transportation problems. The assignment problem (AP) is a special case of transportation problems and both come under the type of linear programming problems (LP) [1]. *The assignment problems in which the objective function is to assign number of resources to number of activities with the minimum cost. From here I can define two types of the assignments problems:*

***1-** The Balanced assignment problems in which we assign number of resources (suppose equals N) to the same number of activities also equals N.*

***2-** The Unbalanced Assignment problems is achieved when the number of resourses differs or not equal to the number of activities or tasks.*

As we know, four methods are being used in solving the assignment problems: The Simplex Method, The Numeration Method, The Transportaion Method and The Hungarian Method [2]. *Among them all the Hungarian Method turns out to be the best, and many researches have already been published with this method. But here, in this research I propose nearly a new method which is a couple of the two well known methods the Hungarian method which is often used for solving the assignment problems and the other one is Vogel's Method which is used for solving the transportation problems but with a little new modifications*. *To show the efficiency of this method I'll consider some numerical examples*.

Keywords: Assignment Problems, Balanced and Unbalanced assignment Problems, Constraints, Objective Function, Optimal Solution,.

I. INTRODUCTION

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization and operations research in mathematics. The problem

instance has a number of *agents* and a number of *tasks*. Any agent can be assigned to perform any task, It is required to perform all tasks by assigning exactly one agent to each task and exactly one task to each agent in such a way that the *total cost* of the assignment is minimized. Different methods have been presented for assignment problem and various articles have been published on the subject. See **(references).** Recently, I introduced a quick new method for solving the Balanced Assignment Problem which is a combination between the Hungarian method and Vogel method**.**

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II. FORMULATION OF THE PROBLEM

Suppose that an assignment problem has n machines and n jobs so as to minimize the total cost or time in such a way that each machine can be assigned to one and only one job [3]. The cost matrix C_{ij} is given as:

$$\begin{array}{cc} & Activities \\ \text{Re}\,sources & \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \end{array}$$

Let X_{ij} denotes the assignment of i^{th} resource to j^{th} activity such that:

$$X_{ij} = \begin{cases} 1; if \ \ \text{resource i is assigned to activity j} \\ \text{0; otherwise} \end{cases}$$

Thus, the mathematical formulation of the assignment problem is:

$$Minimize \ \ Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to the constraints:

$$\sum_{i=1}^{n} X_{ij} = 1, \ for \ \text{all } i = 1, 2, \ldots.n$$

$$\sum_{j=1}^{n} X_{ij} = 1, \ for \ \text{all j} = 1, 2, \ldots.n$$

$$X_{ij} = 0 \ or \ 1$$

II. ALGORITHM OF THE PROBLEM

Let $X_1, X_2, \ldots, X_n$ denote the resources . And let A,B,.. ..denote the activities. Now perform the following steps:

1- Construct the cost matrix. Consider rows for resources and columns for activities.

2- Make row reduction and then column reduction as in Hungarian Method.

3- check for the zeros you obtain, the pivotal zero is the unique zero I mean the only zero in intersection between the row and column where this zero lies. If you obtain such a zero do the assignment here, and then delete this row and column. And repeat the procedure (as in example 2).

4- If you don't have such a zero do the following:

1- Add one row and one column

2- fill the new column by finding the difference between the two minimum cost in that row.

3- fill the new row by finding the difference between the two minimum cost in that column.

4- Then find the maximum difference among them all.

5- Now if the maximum lies in a row, make the assignment in that row with the minimum cost in row, then delete that row and column.

6- If the maximum lies in a column make the assignment in that column with the minimum cost in that column, then delete that column and row.

7- If there are more than maximum then choose the one in which the sum of its column and row is the maximum sum so as to get rid of that column and row. (As in example 1).

8- Repeat the procedure till you do all the assignments. After deleting the last column and row.

III. EXAMPLES

Example 1.

Consider a minimization assignment problem with the following 4×4 cost matrix which represents 4 employees and 4 jobs.

$$\begin{bmatrix} 4 & 5 & 2 & 5 \\ 3 & 1 & 1 & 4 \\ 13 & 1 & 7 & 4 \\ 12 & 6 & 5 & 9 \end{bmatrix}$$

First of all we apply Hungarian method by making row reduction and column reduction to obtain pivotal zero as follows:

$$\begin{bmatrix} 4 & 5 & 2 & 5 \\ 3 & 1 & 1 & 4 \\ 13 & 1 & 7 & 4 \\ 12 & 6 & 5 & 9 \end{bmatrix} \xrightarrow{\text{row reduction}} \begin{bmatrix} 2 & 3 & 0 & 3 \\ 2 & 0 & 0 & 3 \\ 12 & 0 & 6 & 3 \\ 7 & 1 & 0 & 4 \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 10 & 0 & 6 & 0 \\ 5 & 1 & 0 & 1 \end{bmatrix}$$

No pivotal zero is achieved then we go to apply vogel's method as folows:

$$\left[\begin{array}{cccc|c} 4 & 5 & 2 & 5 & \mathbf{2} \\ 3 & 1 & 1 & 4 & \mathbf{2} \\ 13 & 1 & 7 & 4 & \mathbf{3} \\ 12 & 6 & 5 & 9 & \mathbf{1} \\ \mathbf{1} & \mathbf{4} & \mathbf{1} & \mathbf{1} & \end{array}\right]$$

The maximum in the additional row and column is **4** so we begin making the assignment from that column with the minimum cost in the column.but as we have tow minimum (1,1) we begin with the assignment in which the sum of its column and row is the max value. This means we will assign employee **3** to job **2** with the cost **1.** Then delete that column and row. To obtain the submatrix:

$$\left[\begin{array}{ccc|c} 4 & 2 & 5 & \mathbf{2} \\ 3 & 1 & 4 & \mathbf{2} \\ 12 & 5 & 9 & \mathbf{4} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \end{array}\right]$$

The maximum in the additional row and column is **4** so do the assignment with the minimum cost in that row which is **5**.employee **4** to job **2** with the cost **5**. Then delete that row and column to get the submatrix:

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$$\left[\begin{array}{cc|c} 4 & 5 & \mathbf{1} \\ 3 & 4 & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \end{array}\right]$$

The minimum is 1, we begin with the one which has the maximum sum in row and column, I mean the one in this position $\left[\begin{array}{cc|c} 4 & 5 & \mathbf{1}\leftarrow \\ 3 & 4 & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \end{array}\right]$,

so we assign employee **1** to job **1** with the cost **4**. Then delete that row and column to have the last assignment which is employee **2** to job **4** with the cost **4**.

So we obtain the optimal solution with the minimum cost **Z** = 1+5+4+4=14

Example 2.

Suppose we want to assign 4 machines to 4 workers with the minimum cost . And here is the cost matrix:

$$\begin{array}{cc} & \textit{workers} \\ \textit{Machines} & \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix} \end{array}$$

If we apply row reduction and column reduction we arrive with the this matrix :

$$\begin{array}{cc} & \textit{workers} \\ \textit{Machines} & \begin{bmatrix} 4 & 0 & 5 & 6 \\ 0 & 1 & 0 & 4 \\ 1 & 7 & 0 & 7 \\ 0 & 2 & 5 & 0 \end{bmatrix} \end{array}$$

In which we have a pivotal zero C_{12} so do the assignment here with the machine **1** to worker **2** with the cost **2** then delete that row and column. If we make the row reduction and column reduction for the new submatrix we will obtain no pivotal zero. So we apply the Vogel's as follows:

$\left[\begin{array}{ccc|c} 6 & 3 & 7 & \mathbf{3} \\ 5 & 1 & 8 & \mathbf{4} \\ 7 & 9 & 4 & \mathbf{3} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} & \end{array}\right]$ the maximum is **4** do the assignment in that row with the minimum cost, I mean machine **3** to worker 3 with the cost **1**. Delete that row and column to obtain the submatrix :

$\begin{bmatrix} 6 & 7 \\ 7 & 4 \end{bmatrix} \xrightarrow{row\ \ reduction} \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} \xrightarrow{column\ \ reduction} \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ we obtain two pivotal zeros, I mean one zero in each column and row, so do the assignments in the zero positions to obtain machine **2** for worker **1** with the cost **6** and machine **4** for worker **4** with the cost **4**.

So we arrive with the optimal solution min **Z** = 2+1+6+4= 13

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Example 3.

Consider the following assignment problem [1]. Assign five workers to five jobs so as to minimize the total distance .

$$\begin{bmatrix} 12 & 8 & 7 & 15 & 4 \\ 7 & 9 & 1 & 14 & 10 \\ 9 & 6 & 12 & 6 & 7 \\ 7 & 6 & 14 & 6 & 10 \\ 9 & 6 & 12 & 10 & 6 \end{bmatrix}$$

Apply row and column reduction to obtain:

$$\begin{bmatrix} 7 & 4 & 3 & 11 & 0 \\ 5 & 8 & 0 & 13 & 9 \\ 2 & 0 & 6 & 0 & 1 \\ 0 & 0 & 8 & 0 & 4 \\ 2 & 0 & 6 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 & 3 & 11 & 0 \\ 5 & 8 & 0 & 13 & 9 \\ 2 & 0 & 6 & 0 & 1 \\ 0 & 0 & 8 & 0 & 4 \\ 2 & 0 & 6 & 4 & 0 \end{bmatrix}$$ the pivotal zero in position $_{C23}$ do the assignment here, I mean worker **2** for job **3** with the cost **1** then delete the row and column to arrive with the submatrix:

$$\begin{bmatrix} 12 & 8 & 15 & 4 \\ 9 & 6 & 6 & 7 \\ 7 & 6 & 6 & 10 \\ 9 & 6 & 10 & 6 \end{bmatrix}$$ if we apply row and column reduction we will get no pivotal zero. So we apply the vogel's to the submatrix to get:

$$\left[\begin{array}{cccc|c} 12 & 8 & 15 & 4 & \mathbf{4} \\ 9 & 6 & 6 & 7 & \mathbf{1} \\ 7 & 6 & 6 & 10 & \mathbf{1} \\ 9 & 6 & 10 & 6 & \mathbf{3} \\ \mathbf{2} & \mathbf{2} & \mathbf{4} & \mathbf{2} & \end{array}\right]$$ the maximum is **4** in the additional row and column. We do the assignment in the position with the maximum sum in column and row which is C_{14} , this means worker **1** for job **5** with the cost **4**, then delete this row and column to arrive with the submatrix :

$$\begin{bmatrix} 9 & 6 & 6 \\ 7 & 6 & 6 \\ 9 & 6 & 19 \end{bmatrix}$$ if we apply row and column reduction we will have no pivotal zero. So we apply the Vogel's to get :

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$\left[\begin{array}{ccc|c} 9 & 6 & 6 & \mathbf{3} \\ 7 & 6 & 6 & \mathbf{1} \\ 9 & 6 & 10 & \mathbf{3} \\ \mathbf{2} & \mathbf{0} & \mathbf{4} & \end{array}\right]$ we have the maximum 4, we do the assignment in that column with the minimum cost, as we have two minimum we choose the one with the maximum sum in row and column, this means we assign worker **3** to job **4** with the cost **6**, then delete that row and column to get the new submatrix:

$\begin{bmatrix} 7 & 6 \\ 9 & 6 \end{bmatrix}$ if we apply row and column reduction we will have no pivotal zero. So we apply the Vogel's to get :

$\left[\begin{array}{cc|c} 7 & 6 & \mathbf{1} \\ 9 & 6 & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \end{array}\right]$ the maximum is 3, so do the assignment in that row with the minimum cost, I mean we assign worker **5** to job **2** with the cost **6** then delete that row and column. And the last assignment is worker **4** to job **1** with the cost **7**.

So we obtain the optimal solution with the minimum distance (cost) min **Z** = 4+1+6+6+7= **24**.

IV. CONCLUSION

In this paper a new method was introduced to assign zeros directly for solving assignment problem. This new method is a combination between Hungarian method and Vogel's method with some new modifications. This new method is based on creating some zeros in the assignment matrix, and find an assignment in terms of the zeros.

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