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FIXED POINT THEOREMS FOR *F*-EXPANDING MAPPINGS OF G-METRIC SPACE

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ABSTRACT

Introduced Fixed point theorem is define new concept of F*-contraction mapping and which generalizes the Banach Space contraction principle, we present some new fixed point results for* F*-expanding mappings, especially on a complete* G*-metric space.*

Keywords: Fixed Point* F*-Contraction Map,* F*-Expanding Map,* G*-Metric Space

1 INTRODUCTION

Let (X,d)(X,d) be a metric space. A mapping T:X→X is said to be expanding if

$\forall x,y\in X \quad d(Tx,Ty) \geq \lambda d(x,y)$,where$\lambda>1$.

The condition $\lambda>1$ is important, the function T:R→R defined by $Tx=x+e^x$ satisfies the condition $|Tx-Ty| \geq |x-y|$ for all x,y∈R, and *T* has no fixed point.

For an expanding map, the following result is well known.

Theorem 1.1

Let (X,d)(X,d) *be a complete metric space, and let* T:X→X *be surjective and expanding. Then T is bijective and has a unique fixed point.*

It follows from the Banach contraction principle and the following very simple observation.

Lemma 1.2

If T:X→X *is surjective, then there exists a mapping* $T^*:X\to X$ *such that* $T\circ T^*$ *is the identity map on X.*

<u>Proof</u>

For any point x∈X, let yx∈X be any point such that Tyx= x. Let $T^*x=y_x$ for all x∈X Then $(T\circ T^*)(x)=T(T^*x)$ for all x∈X .

In the present paper, we introduce a new type of expanding mappings.

Definition 1.3

Let F be the family of all function F:(0,+∞)→R such that

(F1): F is strictly increasing, i.e., for all α,β∈(0,+∞), if α< β, then F(α)<F(β);

*(F2): f*or each sequence $\{\alpha_n\} \subset (0,+\infty)$, the following holds:

$\lim_{n\to\infty} \alpha n = 0$ if and only if $\lim_{n\to\infty} F(\alpha n) = -\infty$

(F3): there exists k∈(0,1) such that $\lim_{n\to 0+} \alpha k F(\alpha) = 0$

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Definition 1.4

Let (X,d) be a metric space. A mapping $T:X\to X$ is called *F*-expanding if there exist F∈F and $t > 0$ such that for all $x,y\in X$,

$d(x,y)>0 \Rightarrow F(d(Tx,Ty)) \geq F(d(x,y)) + t.$ (2)

When we consider in (2) the different types of the mapping F∈FF∈F, then we obtain a variety of expanding mappings.

Example 1.5

Let $F1(\alpha)=\ln\alpha$ .It is clear that F1F1 satisfies (*F*1), (*F*2), (*F*3) for any k∈(0,1). Each mapping $T:X\to X$ satisfying (2) is an F1-expanding map such that

$d(Tx,Ty) \geq e^t d(x,y)$ for all $x,y \in X$, .

It is clear that for $x,y \in X$ such that $x=y$, the inequality $d(Tx,Ty) \geq e^t d(x,y)$ also holds.

Example 1.6

If $F2(\alpha)=\ln\alpha+\alpha$, $\alpha>0$, then F1 satisfies (*F*1), (*F*2) and (*F*3), and condition (2) is of the form

$d(Tx,Ty)ed^{(Tx,Ty)-d(x,y)} \geq e^t d(x,y)$ for all $x,y\in X$..

Example 1.7

Consider $F3(\alpha)=\ln(\alpha^2+\alpha)$, $\alpha>0$. F3 satisfies (*F*1), (*F*2) and (*F*3), and for F3-expanding *T*, the following condition holds:

$d(Tx,Ty). \dfrac{d(Tx,Ty)+1}{d(x,y)+1} \geq e^t d(x,y)$ for all $x,y \in X$.

Example 1.8

Consider $F4(\alpha)=\arctan(-\frac{1}{(\alpha)})$, α>0α>0. F4 satisfies (*F*1), (*F*2) and (*F*3), and for F4-expanding *T*, the following condition holds:

$$d(Tx,Ty) \geq \left(\frac{1+\frac{\tan t}{d(x,y)}}{1-\tan t \,.d(x,y)}\right) d(x,y) \text{ for some } 0 < t < \frac{\pi}{2}$$

Here, we have obtained a special type of nonlinear expanding map

$$d(Tx,Ty) \geq \varphi(d(x,y))d(x,y).$$

Other functions belonging to F are, for example, $F(\alpha)=\ln(\alpha^n)$, $n\in N$, , $\alpha > 0$;

$F(\alpha)=\ln(\arctan\alpha)$, $\alpha > 0$.

Now we recall the following.

Definition 1.9

Let (X,d) be a metric space. A mapping $T:X\to X$ is an *F*-contraction on *X* if there exist F∈F and $t > 0$ such that for all $x, y \in X$,

$d(Tx,Ty)>0 \Rightarrow t + F(d(Tx,Ty)) \leq F(d(x,y))$.. (3)

For such mappings, Wardowski [1] proved the following theorem.

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Theorem 1.10

Let (X,d) *be a complete metric space and* $T:X\to X$ *be an F-contraction. Then T has a unique fixed point* $u \in X$ *and for every* $x\in X$, *a sequence* $\{x_n = T^n x\}$ *is convergent to u.*

II THE RESULT

In this section, we give some fixed point theorem for *F*-expanding maps.

Theorem 2.1

Let (X,d) *be a complete metric space and* $T:X\to X$ *be surjective and F-expanding. Then T has a unique fixed point.*

Proof

From Lemma 1.2, there exists a mapping $T^*:X\to X$ such that $T\circ T$ is the identity mapping on *X*.

Let $x,y \in X$ be arbitrary points such that $x\neq y$, and let $z=T^*x$ and $w=T^*y$ (obviously, $z\neq w$).

By using (2) applied to *z* and *w*, we have

$F(d(Tz,Tw)) \geq F(d(z,w)) + t.$

Since $Tz=T(T^*x) = x$ and $Tw=T(T^*y)=y$, then

$F(d(x,y)) \geq F(d(T^*x,T^*y)) + t,$

so $T^*:X\to X$ is an *F*-contraction. By Theorem 1.10, T^* has a unique fixed point $u\in X$. In particular, *u* is also a fixed point of *T* because $T^*u = u$ implies that $Tu=T(T^*u) = u$.

Let us observe that *T* has at most one fixed point. If $u,v \in X$ and $Tu=u\neq v$, then we would get the contradiction

$F(d(Tu,Tv)) \geq F(d(u,v)) + t,$

$0=F(d(Tu,Tv)) - F(d(u,v)) \geq t > 0,$

so the fixed point of *T* is unique.

Remark 2.2

If *T* is not surjective, the previous result is false. For example, let $X=[0,\infty)$ endowed with the metric $d(x,y) = |x-y|$ for all $x,y \in X$, and let $T:X\to X$ be defined by $Tx=2x + 1$ for all $x \in X$ Then *T* satisfies the condition $d(Tx,Ty) \geq 2d(x,y)$ for all $x,y \in X$ and *T* is fixed point free.

III APPLICATIONS TO *G*-METRIC SPACES

In 2006 Mustafa and Sims (see [2] and the references therein) introduced the notion of a *G*-metric space and investigated the topology of such spaces. The *G*-metric space is as follows.

Definition 3.1

Let *X* be a nonempty set. A function $G:X\times X\times X\to[0,\infty)$ satisfying the following axioms:

(*G1*) $G(x,y,z) = 0$ if $x=y=z$,

(G2) $G(x,x,y) > 0$ for all $x, y \in X$ with $x \neq y$,

(*G3*) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z\in X$ with $z \neq y$,

(G4) $G(x,y,z) \leq G(x,z,y) = G(y,z,x)=$ (symmetry in all three variables),

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(G5) $G(x,y,z) \le G(x,a,a) + G(a,y,z)$ for all $x, y, z, a \in X$,

is called a *G*-metric on *X*, and the pair (X,G) is called a *G*-metric space.

Recently, Samet et al. [3] observed that some fixed point theorems in the context of *G*-metric spaces can be concluded from existence results in the setting of quasi-metric spaces. Especially, the following theorem is a simple consequence of Theorem 1.10.

Theorem 3.2

Let (X,G) *be a complete G-metric space, and let* $T:X\to X$ *satisfy one of the following conditions*:

(a) T is an F-contraction of type I on a G-metric space X, i.e., there exist

$F\in \mathcal{F}$ *and* t>0t>0 *such that for all* $x,y \in X$,

$G(Tx,Ty,Ty) > 0 \Rightarrow t + F(G(Tx,Ty,Ty)) \le F(G(x,y,y));$ (4)

(b) T is an F-contraction of type II on a G-metric space X, i.e., ther exist $F\in\mathcal{F}$

and $t>0$ *such that for all* $x, y, z \in X$,

$G(Tx,Ty,Tz) > 0 \Rightarrow t + F(G(Tx,Ty,Tz)) \le F(G(x,y,z)).$ (5)

Then T has a unique fixed point $u\in X$, *and for any* $x\in X$, *a sequence* $\{ x_n=T^n x \}$ *is G-convergent to u*.

The previous ideas lead also to analogous fixed point theorems for *F*-expanding mappings on *G*-metric spaces.

Definition 3.3

A mapping $T:X\to X$ from a *G*-metric space (X,G) into itself is said to be

1. *(a)* *F*-expanding of type I on a *G*-metric space *X* if there exist $F\in \mathcal{F}$ and $t > 0$ such that for all $x, y \in X$,

$G(x,y,y)>0 \Rightarrow F(G(Tx,Ty,Ty)) \ge F(G(x,y,y)) + t$; (6)

(b) *F*-expanding of type II on a *G*-metric space *X* if there exist $F \in \mathcal{F}$ and $t > 0$ such that for all $x,y,z \in X$,

$G(x,y,z)>0 \Rightarrow F(G(Tx,Ty,Tz)) \ge F(G(x,y,z)) + t$.. (7)

Theorem 3.4

Let (X,G) *be a complete G-metric space and* $T:X\to X$ *be a surjective and F-expanding mapping of type I(or type II). Then T has a unique fixed point.*

Proof

Let *T* be an *F*-expanding mapping of type I. From Lemma 1.2, there exists a mapping $T^*: X\to X$ such that ToT^* is the identity mapping on *X*. Let $x,y\in X$ be arbitrary points such that $x\neq y$ and let $\xi=T^*x$ and $\eta=T^*y$.

Obviously, $\xi \neq \eta$, and $G(\xi,\eta,\eta) > 0$. By using (6) applied to ξ and η, we have

$F(G(T\xi,T\eta,T\eta)) \ge F(G(\xi,\eta,\eta)) + t.$

Since $T\xi=T(T^*x) = x$ and $T\eta=T(T^*y) = y$, then

$F(G(x,y,y)) \ge F(G(T^*x,T^*y,T^*y)) + t$,,

so T^* is an *F*-contraction of type I on a *G*-metric space (X,G). Theorem 3.2 guarantees that T^* has a unique fixed point $u \in X$. The point *u* is also a fixed point of *T* because $Tu=T(T^*u)=u$.

Now, we prove the uniqueness of the fixed point. Assume that *v* is another fixed point of *T* different from *u*: $Tu = u\neq v = Tv$. This means $G(u,v,v) > 0$, so by (6)

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$0 < t \leq F(G(Tu,Tv,Tv)) - F(G(u,v,v))=0$,,

which is a contradiction, and hence u=v.

For *F*-expanding mappings of type II, it is necessary to take z = y and apply the proof for *F*-expanding mappings of type I.

As a corollary of Theorem 3.4, taking F1∈ F, see Examples 1.5, we obtain the following.

Corollary 3.5[2], Corollary 9.1.4 *Let* (X,G) *be a complete G-metric space and* $T:X\to X$ *be surjective, and let there exist* $\lambda > 1$ *such that*

$G(Tx,Ty,Ty) \geq \lambda G(x,y,y)$ for all x, y ∈ X,

or

$G(Tx,Ty,Tz) \geq \lambda G(x,y,z)$ for all x ,y, z ∈ X.

Then T has a unique fixed point.

Remark 3.6 If *T* is not surjective, the previous results are false.Consider $X=(-\infty,-1]\cup[1,\infty)$ endowed with the *G* metric $G(x,y,z)= |x-y| +| x-z| + |y-z|$ for all x, y ,z ∈ X and the mapping $T: X\to X$ defined by $Tx= -2x$.

Then $G(Tx, Ty, Tz) \geq 2G(x,y,z)$ for all x, y , z ∈ X and *T* has no fixed point.

Now, we will improve some results contained in the book [2]. We will use the following observation: if $T:X\to X$ is a subjective mapping, based on each x0∈ X, there exists a sequence $\{x_n\}$ such that $Tx_{n+1}=x_n$ for all $n \geq 0$

Generally, a sequence {xn} verifying the above condition is not necessarily unique.

Theorem 3.7

Let (X,G) *be a complete G-metric space, and let* $T:X\to X$ *be a surjective mapping. Suppose that there exist* F∈F *and* $t > 0$ *such that for all* x,y ∈ X,

$G(x,Tx,y)>0 \Rightarrow F(G(Tx,T2x,Ty)) \geq F(G(x,Tx,y)) + t.$ (8)

Then T has a unique fixed point.

Proof

Let x0 ∈ X be arbitrary. Since *T* is surjective, there exists x1∈ X such that Tx1 = x0 . By continuing this process, we can find a sequence $\{x_n=Tx_{n+1}\}$ for all n=0,1,2,……. If there exists n0 ∈N U{0} such that $x_{n0}=x_{n0+1}$, then xn0+1 is a fixed point of *T*.

Now assume that xn≠xn+1 for all $n \geq 1$. Then $G(x_{n+1},x_n,x_n) > 0$ for all $n \geq 1$, and from (8) with x = x_{n+1} and $y=x_n$, we have, for all $n \geq 1$,

$$F(G(x_n,x_{n-1},x_{n-1})) = F(G(Tx_{n+1},T^2x_{n+1},Tx_n))$$
$$\geq F(G(x_{n+1},Tx_{n+1},x_n)) + t = F(G(x_{n+1},x_n,x_n)) + t,$$

and hence

$$t + F(G(x_{n+1},x_n,x_n)) \leq F(G(x_n, x_{n-1}, x_{n-1})). \quad (9).$$

Using (9), the following holds for every $n \geq 1$:

$$F(G(x_{n+1},x_n,x_n)) \leq F(G(x_n,x_{n-1},x_{n-1})) - t$$
$$\leq F(G(x_{n-1},x_{n-2},x_{n-2})) - 2t \leq ---- \leq F(G(x_1,x_0,x_0)) - nt. \quad (10)$$

From (10) we obtain

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$\lim_{n\to\infty} F(G(xn+1, xn, xn)) = -\infty,$

which together with (*F*2) gives

$\lim_{n\to\infty} F(G(xn+1, xn, xn)) = 0,$ (11)

From (*F*3) there exists k∈ (0,1) such that

$\lim_{n\to\infty} F(G(xn+1,xn,xn))^k F(G(xn+1, xn, xn)) = 0$ (12)

By (10), the following holds for all $n \geq 1$:

$[G(x_{n+1},x_n,x_n)]^k F(G(x_{n+1},x_n,x_n)) - [G(x_{n+1},x_n,x_n)]^k F(G(x_1,x_0,x_0))$

$\leq [G(x_{n+1},x_n,x_n)]^k(F(G(x_1,x_0,x_0)) - nt)$

$- [G(x_{n+1},x_n,x_n)]^k F(G(x1,x0,x0)) = - [G(x_{n+1},x_n,x_n)]^k . nt$ (13)

Letting $n\to\infty$ in (13) and using (11), (12), we obtain

$$\lim_{n\to\infty} [G(x_{n+1},x_n,x_n)]^k .n=0 \quad . (14)$$

Now, let us observe that from (14) there exists $n1 \geq 1$ such that

$$[G(x_{n+1},x_n,x_n)]^k .n \leq 1 \text{ for all } n \geq n_1.$$

Consequently, we have

$$G(x_{n+1},x_n,x_n) \leq \frac{1}{n1/k} \quad \text{all } n \geq n_1..$$

Since the series $\sum_{i=0}^{\infty} \frac{1}{n1/k}$ converges, for any ε > 0, there exists $n2 \geq 1$ such that

$\sum_{i=0}^{\infty} \frac{1}{n1/k} < \varepsilon$ In order to show that {xn} is a Cauchy sequence, we consider m > n > max{n1,n2}. From [2],

Lemma 3.1.2(4), we get

$G(x_{n+1},x_n,x_n) \leq \sum_{j=n}^{m-1} G(x\,j+1, xj, xj) \leq \sum_{j=n}^{\infty} G(x\,j+1, xj, xj)$

$$\leq \sum_{j=n}^{\infty} \frac{1}{J1/k} \leq \sum_{j=n2}^{\infty} \frac{1}{J1/k} \leq \varepsilon.$$

Therefore by [2], Lemma 3.2.2 and axiom (G4) {xn} is a Cauchy in a *G*-metric space (X,G).From the completeness of (X,G), there exists u ∈ X such that {xn}→u As *T* is surjective, there exists w ∈ X such that u=Tw. From (8) with $x = x_{n+1}$ and y=w, we have, for all $n \leq 1$,

$F(G(x_{n+1},x_n,u)) = F(G(Tx_{n+1},T^2x_{n+1},T_w))$

$\geq F(G(x_{n+1},Tx_{n+1},w)) + t = F(G(x_{n+1},x_n,w)) + t,$

and hence

$F(G(x_{n+1},x_n,u)) > F(G(x_{n+1},x_n,w))$ (15)

By (*F*1) from (15), we have

$$G(x_{n+1},x_n,u) > G(x_{n+1},x_n,w) \quad \text{for all } n \geq 1 \quad (16)$$

Using the fact that the function *G* is continuous on each variable ([2], Theorem 3.2.2), taking the limit as $n\to\infty$ in the above inequality, we get

$G(u,u,w) = F(G(x_n,x_{n+1},u)) =0,,$

that is, u= w. Then *u* is a fixed point of *T* because $u = T_w = T_u$.

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To prove uniqueness, suppose that $u,v \in X$ are two fixed points. If $Tu=u\neq v=Tv$, then $G(u,u,v) > 0$ So, by (8),

$F(G(u,u,v)) = F(G(Tu,T^2u,Tv))$

$\geq F(G(u,Tu,v)) + t = F(G(u,u,v)) + t$,

which is a contradiction, because $t > 0$. Hence, $u=v$.

Taking $F1 \in F$, see Example 1.5, we obtain the following.

***Corollary 3.8* [2], Theorem 9.1.2** *Let* (X,G)(X,G) *be a complete G-metric space and* $T:X\to X$ *be a surjective mapping*. *Suppose that there exists* $\lambda>1$ *such that*

$G(Tx,T^2x,Ty) \geq \lambda G(x,Tx,y)$f or all$x,y \in X$..

Then T has a unique fixed point.

Next result does not guarantee the uniqueness of the fixed point.

Theorem 3.9

Let (X,G)(X,G) *be a complete G-metric space, and let* $T:X\to X$ *be a surjective mapping*. *Suppose that there exist*$F \in f$ *and* $t > 0$ *such that for all* $x,y \in X$,

$$G(x,Tx,T^2x) > 0 \Rightarrow F(G(Tx,Ty,T^2y)) \geq F(G(x,Tx,T^2x)) + t. \qquad (17)$$

Then T has a fixed point.

Proof

Let $x0 \in X$ be arbitrary. Since T is surjective, there exists $x1 \in X$ such that $x0=Tx1$. By continuing this process, we can find a sequence $\{xn=Tx_{n+1}\}$ for all $n \geq 0$. If there exists $n0 \geq 0$ such that $xn_0=xn_0+1$, then $xn0+1$ is a fixed point of T.

Now, assume that $xn\neq xn+1$ for all $n \geq 0$. From (17) with $x=x_{n+1}$ and $y=xn$,

we have $G(xn+1,Tx_{n+1},T^2x_{n+1}) = G(xn+1,xn,xn-1) > 0$ and

$F(G(x_n,x_{n-1},x_{n-2}))=F(G(Tx_{n+1},Tx_n,T^2x_n))$

$\geq F(G(x_{n+1},Tx_{n+1},T^2x_{n+1})) + t = F(G(x_{n+1},x_n,x_{n-1})) + t$,

and hence

$$\begin{aligned} F(G(x_{n+1},x_n,x_{n-1})) &\leq F(G(x_n,x_{n-1},x_{n-2})) - t \\ &\leq F(G(x_{n-1},x_{n-2},x_{n-3})) - 2t \\ &\leq F(G(x_2,x_1,x_0)) - (n-1)t. \end{aligned} \qquad (18)$$

From (18), we obtain

$$\lim_{n\to\infty} F\big(G(xn+1, xn, xn-1)\big) = -\infty,$$

which together with (F2) gives

$$\lim_{n\to\infty} F\big(G(xn+1, xn, xn-1)\big) = 0,$$

Mimicking the proof of Theorem 3.7, we obtain

$$\lim_{n\to\infty} F\big(G(xn+1, xn, xn-1)\big)]k\,.(n-1) = 0;$$

and consequently, there exists $n_1 \geq 1$ such that

$$G(x_{n+1},x_n,x_{n-1}) \leq \frac{1}{(n-1)1/k} \quad \text{for all } n > n_1$$

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Since the series $\sum_{i=1}^{\infty} \frac{1}{i1/k}$ converges, for any ε > 0, there exists n2 ≥ 1 such that $\sum_{i=n2}^{\infty} \frac{1}{i1/k} < \varepsilon$. In order to show that {xn} is a Cauchy sequence, we consider m>n>max{n1,n2}. From [2], Lemma 3.1.2(4) and axioms (G3), (G4), we get

$$G(x_m,x_n,x_n) \le \sum_{j=n}^{m-1} G(x\,j+1, xj, xj) \le \sum_{j=n}^{\infty} G(x\,j+1, xj, xj)$$

$$\le \sum_{j=n}^{\infty} G(x\,j+1, xj, xj-1) \le \sum_{j=n}^{\infty} \frac{1}{j1/k} \le \sum_{j=n2}^{\infty} \frac{1}{j1/k} < \varepsilon$$

Therefore, by [2], Lemma 3.2.2, {xn}{xn} is a Cauchy in a *G*-metric space (X,G). From the completeness of (X,G), there exists u ∈ X such that {xn}→u. As *T* is surjective, there exists w ∈ X such that u=Tw. From (17) with x=w and y=xn+1, we have

$$F(G(u,x_n,x_{n-1})) = F(G(Tw,Tx_{n+1},T^2x_{n+1})) \ge F(G(w,Tw,T^2w)) + t ,$$

so

$$F(G(w,Tw,T^2w)) \le F(G(u,x_n,x_{n-1})) - t < F(G(u,x_n,x_{n-1}))$$

Using (*F*1), we have

$$G(w,Tw,T^2w) < G(u,x_n,x_{n-1}) \text{for all } n \ge 1.$$

Using the fact that the function *G* is continuous on each variable ([2], Theorem 3.2.2), taking the limit as n→∞n→∞ in the above inequality, we get

$$G(w,Tw,T^2w)= \lim_{n\to\infty} G(u,x_n,x_{n-1}) = 0$$

that is, $w=Tw=T^2w$. Hence, u=Tu.

Taking F_1∈ F, see Examples 1.5, we obtain the following.

IV CONCLUSION

Theorme:- *Let* (X,G)(X,G) *be a complete G-metric space and* T:X→X *be a surjective mapping*. *Suppose that there exists* λ > 1 *such that*

$G(Tx,Ty,T^2y) \ge \lambda G(x,Tx,T^2x)$ for all x, y ∈ X..

Then T has, a fixed point.with F –Expanding Mapping .

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