

SIDDHALIC 5/12 AND SIDDHALIC 6/14 RULE IN NUMERICAL INTEGRATION

Siddharth Mishra

Ex-Student, Department of Physics, Banaras Hindu University, UP, (India)

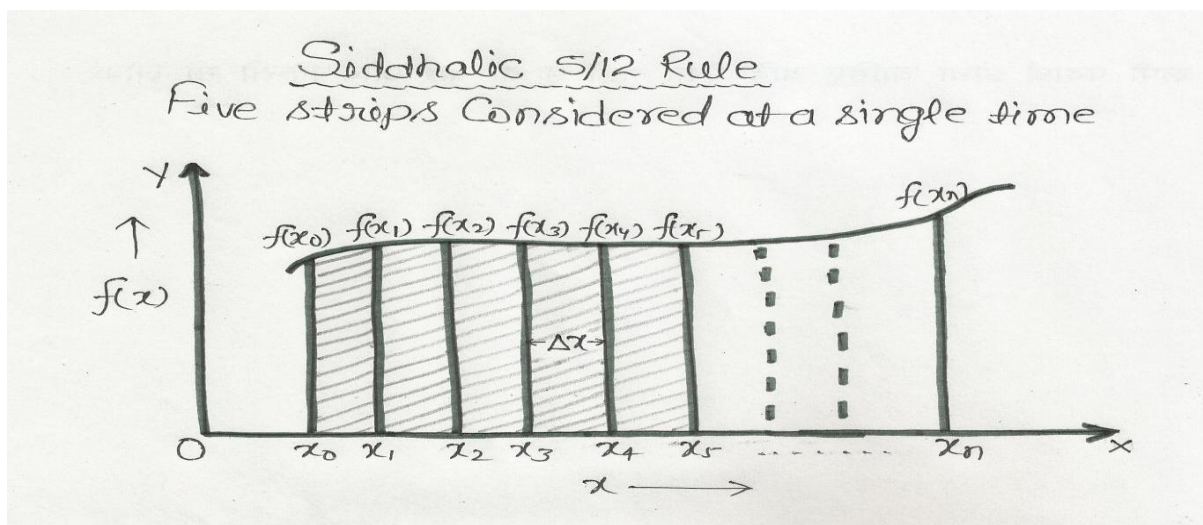
ABSTRACT

Numerical Integration technique involves finding out the value of the integration between the desired limits of any function $f(x)$ numerically. In this regard, there are most famous formula like Mid point rule, trapezoidal rule, Simpson's $1/3^{\text{rd}}$ rule & Simpson's $3/8$ rule. These formula are frequently used in calculating the desired integration of the chosen function between the limits. In this perspective, Siddhalic 5/12 & Siddhalic 6/14 rule are going to provide important contribution in evaluating the integrals of any function between the desired limits of integration. These formula are derived employing a new approach or concept which is presented in this paper in a detailed way of expression.

I INTRODUCTION

Here in this paper presented below the Siddhalic 5/12 & Siddhalic 6/14 rules are derived using the basic mathematical way of formulation. The presented formula are derived by employing the definition of arithmetic mean in a precise manner which can be seen thoroughly by insightful inspection of the paper. The two formula are listed in order with their formulation as below

(1). SIDDHALIC 5/12 RULE



Let us consider a function $f(x)$ as shown in the figure plotted against different values of x namely ; $x_0, x_1, x_2, x_3, x_4, x_5, \dots, x_n$. we have to calculate the value of the integral $\int_{x_0}^{x_n} f(x) dx$, in order to calculate this we divide the whole area into a number of thin strips of equal width Δx , as shown in the figure. Here what we are going to do is considering a combination of five strips at a single time & then calculating the corresponding area as under , writing $y_n = f(x_n)$ we have the area of the first such combination of strips

= Total width of strip combination \times average height of strip boundaries

= $(5\Delta x) \times \{ \{y_0+y_1+y_2+y_3+y_4+y_5\}/6 \}$

= $(5\Delta x/12) \times \{ 2y_0+2y_1+2y_2+2y_3+2y_4+2y_5 \}$

the above expression can be written as

= $(5\Delta x/12) \times [\{y_0+y_5\} + \{y_1+y_4\} + \{2y_2\} + \{2y_3\} + y_0+y_5+y_1+y_4]$

Now from the definition of arithmetic mean & from the figure above ,if we have

$\{y_0+y_5\}/2 = \{y_1+y_4\}/2 = \{y_2+y_3\}/2$

therefore, $y_0+y_5 = y_1+y_4 = y_2+y_3$, applying this the area of first combination of five strips becomes

= $(5\Delta x/12) \times [\{y_2+y_3\} + \{y_2+y_3\} + 2y_2+2y_3+y_0+y_5+y_1+y_4]$

= $(5\Delta x/12) \times [4y_2+4y_3+y_0+y_1+y_4+y_5]$

= $(5\Delta x/12) \times [y_0+y_1+4y_2+4y_3+y_4+y_5]$

similarly for other combination of strips we obtain the expression as

$(5\Delta x/12) \times [y_5+y_6+4y_7+4y_8+y_9+y_{10}]$

..... & so on

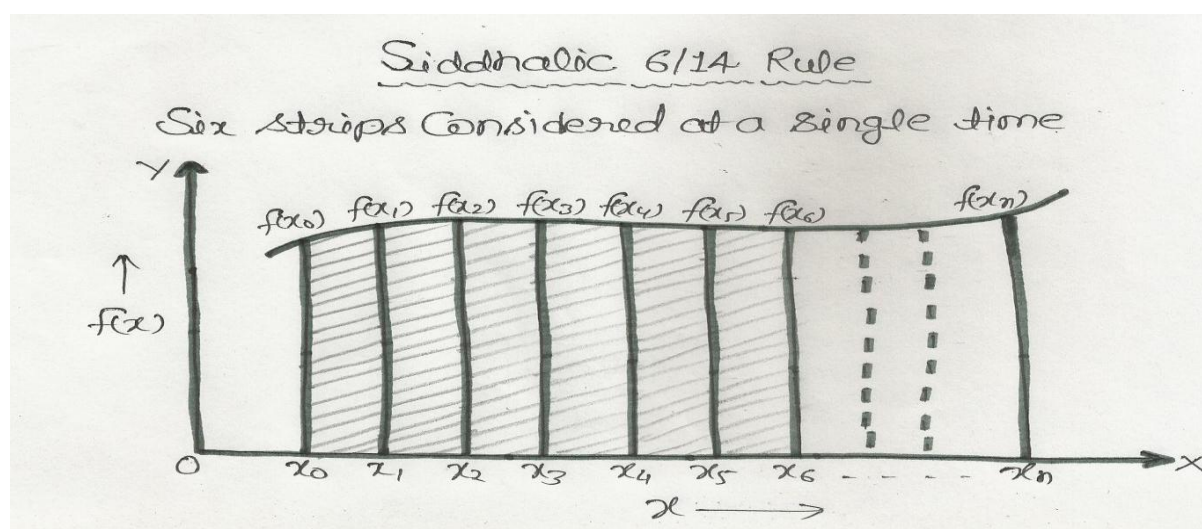
thus adding the combinations final area will be

$\int_{x_0}^{x_n} f(x) dx = (5\Delta x/12) \times [y_0+y_1+4y_2+4y_3+y_4+2y_5+\dots+y_n]$

$\int_{x_0}^{x_n} f(x) dx = (5\Delta x/12) \times [f(x_0)+f(x_1)+4f(x_2)+4f(x_3)+f(x_4)+2f(x_5)+\dots+f(x_n)]$

above formula denotes the Siddhalic 5/12 rule for numerical integration method & can be applied without hesitation

(2). SIDDHALIC 6/14 RULE





Let us consider a function $f(x)$ plotted as shown in figure against the x -values namely ; $x_0, x_1, x_2, x_3, x_4, x_5, x_6, \dots, x_n$. In order to evaluate the value of the integral $\int_{x_0}^{x_n} f(x) dx$ we divide the whole area under the curve into a number of thin strips of equal width Δx . Considering the combination of six such strips at a single time, the area of one such combination of strips

$$= \text{Total width of strip combination} \times \text{average height of strip boundaries}$$

$$= (6\Delta x) \times [\{y_0+y_1+y_2+y_3+y_4+y_5+y_6\}/7]$$

$$= (6\Delta x/14) \times [2y_0+2y_1+2y_2+2y_3+2y_4+2y_5+2y_6]$$

$$= (6\Delta x/14) \times [\{y_0+y_6\}+\{y_1+y_5\}+\{y_2+y_4\}+2y_3+y_0+y_6+y_1+y_5+y_2+y_4]$$

Now by the definition of arithmetic mean & from the figure, if we have

$$(y_0+y_6)/2 = (y_1+y_5)/2 = (y_2+y_4)/2 = y_3$$

or

$$y_0+y_6 = y_1+y_5 = y_2+y_4 = 2y_3$$

applying this the area of first combination of six strips becomes

$$= (6\Delta x/14) \times [2y_3+2y_3+2y_3+2y_3+y_0+y_6+y_1+y_5+y_2+y_4]$$

$$= (6\Delta x/14) \times [y_0+y_1+y_2+8y_3+y_4+y_5+y_6]$$

similarly for the other combination of strips we obtain the expression as follows

$$(6\Delta x/14) \times [y_6+y_7+y_8+8y_9+y_{10}+y_{11}+y_{12}]$$

..... & so on

Thus adding the combinations final area will be

$$\int_{x_0}^{x_n} f(x) dx = (6\Delta x/14) \times [y_0+y_1+y_2+8y_3+y_4+y_5+2y_6+\dots+y_n]$$

$$\int_{x_0}^{x_n} f(x) dx = (6\Delta x/14) \times [f(x_0)+f(x_1)+f(x_2)+8f(x_3)+f(x_4)+f(x_5)+2f(x_6)+\dots+f(x_n)]$$

the above expression denotes the Siddhalic 6/14 rule for numerical integration method & is applicable in numerical analysis undoubtedly.

II CONCLUSION

Thus by the employed method we have obtained the Siddhalic 5/12 & Siddhalic 6/14 rule for numerical integration method. These formula can be used with the ease of method without any formulation doubt because employing the above concept or approach , the famous simpson's 1/3rd rule & simpson's 3/8 rule can also be derived which ensures strong support to this method of derivation.