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ONE MODULO THREE GEOMETRIC MEAN LABELING OF SOME FAMILIES OF GRAPHS

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ABSTRACT

A graph G is said to be one modulo three geometric mean graph if there is an injective function ϕ from the vertex set of G to the set $\{ a \mid 1 \le a \le 3q - 2 \text{ and either } a \equiv 0 (\text{mod } 3) \text{ or } a \equiv 1 (\text{mod } 3) \}$ where q is the number of edges of G and ϕ induces a bijection ϕ^* from the edge set of G to $\{ a \mid 1 \le a \le 3q - 2 \text{ and } a \equiv 1 (\text{mod} 3) \}$ given by $\phi^*(uv)= \left\lceil \sqrt{\phi(u)\phi(v)} \right\rceil$ or $\left\lfloor \sqrt{\phi(u)\phi(v)} \right\rfloor$ and the function ϕ is called one modulo three geometric mean labeling of G. In this paper , we establish that some families of graphs are one modulo three geometric mean graphs

Keywords: Mean labeling, one modulo three mean labeling, geometric mean labeling, one modulo three geometric mean labeling, one modulo three geometric mean graph.

AMS Classification (2010): 05C78

I. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected. The vertex set and the edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively. We follow the basic notations and terminologies of graph theory as in [1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions and a detailed survey of graph labeling can be found in [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [3]. A graph $G = (p, q)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0,1,2,3,\dots,q\}$ such that for each edge uv, is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Jeyanthi andMaheswari introduced the concept of one modulo three mean labeling in [4]. A graph G is called one modulo three mean graph if there is an injective function ϕ from the vertex set of G to the set $\{a \mid 0 \le a \le 3q - 2 \text{ and either } a \equiv 0 (\text{mod } 3) \text{ or } a \equiv 1 (\text{mod } 3)\}$ where q is the number of edges of G and ϕ induces a bijection ϕ^* from the edge set of G to $\{ a \mid 1 \le a \le 3q - 2 \text{ and either } a \equiv 1 (\text{mod} 3) \}$ given by $\phi^*(uv)= \left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil$ and the function ϕ is called one modulo three mean labeling of *G*. The concept of geometric mean labeling was due to Somasundram et al.[5], A graph G=(V,E) with *p* vertices and *q* edges is said to be geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels *f*(*x*) from 1,2,…,*q*+1 in such a way that when each edge *e*=*uv* is labeled with *f* (*e*=uv)= $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$, then the resulting edge labels are all distinct. In this case, the function *f* is called geometric mean labeling of *G*.

Motivated by the concepts in [5] we define a new type of labeling called one modulo three geometric mean labeling as follows: A graph *G* is said to be one modulo three geometric mean graph if there is an injective function ϕ from the vertex set of G to the set $\{a \mid 1 \leq a \leq 3q-2 \text{ and either } a \equiv 0 (\text{mod } 3) \ \ or\ a \equiv 1 (\text{mod } 3)\}$ where q is the number of edges of G and ϕ induces a bijection ϕ * from the edge set of G to $\{\ a \mid 1 \leq a \leq 3q-2 \text{ and either } a \equiv 1 (\text{mod} 3)\ \}$ given by ϕ *(uv)= $\left\lceil \sqrt{\phi(u)\phi(v)} \right\rceil$ or $\left\lfloor \sqrt{\phi(u)\phi(v)} \right\rfloor$ and the function ϕ is called one modulo three geometric mean labeling of *G*. In [6] we proved that P_n , $k_{1,n} (n > 2)$, comb, $P_n \odot \overline{k_2}$, $s(P_n \odot k_1)$, $s(P_n \odot \overline{k_2})$,$c_n (n \geq 5)$, $L_n = P_n\ X\ P_2$, are one modulo three geometric mean graphs and also we proved that if *G* is a graph in which every edge lies on a triangle, then *G* is not a one modulo three geometric mean graph.

We begin with a brief summary of definitions which are necessary for the present study.

Definition1.1: Duplication of an edge e_k=$v_k v_{k+1}$ of a graph G produces a new graph G' such that $N(v_k{}') = N(v_k) \cup \{v_{k+1}{}'\} - \{v_{k+1}\}$and $N(v_{k+1}{}') = N(v_{k+1}) \cup \{v_k{}'\} - \{v_k\}$.

Definition1.2: The tadpole graph is formed by joining the end point of a path P_m to a cycle C_n. It is denoted by C_n @ P_m.

Definition1.3:A key graph is a graph obtained from K_2 by appending one vertex of $C_m$to one end point and comb graph $P_n \odot K_1$ to the other end of K_2.It is denoted as *KY*(*m*,*n*).

Theorem 1.4[6]: The comb graph is aone modulo three geometric mean graph.

Theorem 1.5[6]: The cycle C_n is a one modulo three geometric mean graph for *n*≥5.

Theorem 1.6[6]: The path $P_n$is a one modulo three geometric mean graph

II. ONE MODULO THREE GEOMETRIC MEAN LABELING OF FAMILIES OF GRAPHS

Theorem 2.1: Let $G_1(p_1, q_1), G_2(p_2, q_2), \ldots, G_n(p_n, q_n)$ be one modulo three geometric mean cycles or path with $q_i (1 \leq i \leq n)$ and u_i , v_i be the vertices of $G_i (1 \leq i \leq n)$ labeled with $3q_i - 2$ and 1. Then the graph G obtained by joining u_1 with v_2 and u_2 with v_3 and u_3 with v_4 and so on until we join u_{n-1} with v_n by an edge is an one modulo three geometric mean graph.

Proof:

The graph G has $p_1 + p_2 + \cdots + p_n$ vertices and $\sum_{i=1}^{n} q_i + (n-1)$ edges.

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Let ϕ_i be one modulo three geometric mean labeling of $G_i (1 \leq i \leq n)$.

Define a vertex labeling $\phi: V(G) \to \{1, 3, 4, \ldots, 3(\sum_{i=1}^{n} q_i + (n-1)) - 2\}$ as

If$x \in V(G_1)$, $\phi(x) = \phi_1(x)$

If $x \in V(G_i)$, $2 \leq i \leq n$

$$\phi(x) = \begin{cases} 3\left(\sum_{k=1}^{i-1}(q_k+1)\right) & for\ the\ lowest\ vertex\ label\ of\ G_i \\ \phi_i(x) + 3\left(\sum_{k=1}^{i-1}(q_k+1)\right) & for\ all\ other\ remaining\ vertices\ of\ G_i \end{cases}$$

Then the induced edge labels of G are $1, 4, \ldots, 3(\sum_{i=1}^{n} q_i + (n-1)) - 2$. Hence ϕ is one modulo three geometric mean labeling of G.

Theorem 2.2: Let $G_1(p_1, q_1), G_2(p_2, q_2), \ldots, G_n(p_n, q_n)$ be one modulo three geometric mean cycles with $q_i (1 \leq i \leq n)$ and u_i , v_i be the vertices of $G_i (1 \leq i \leq n)$ labeled with $3q_i - 2$ and 1. Then the graph G obtained by identifying u_1 with v_2 and u_2 with v_3 and u_3 with v_4 and so on until u_{n-1} identified with v_n is an one modulo three geometric mean graph.

Proof:

The graph G has $p_1 + (p_2 - 1) + \cdots + (p_n - 1) = (p_1 + p_2 + \cdots + p_n - (n-1))$ vertices and $\sum_{i=1}^{n} q_i$ edges.

Let ϕ_i be one modulo three geometric mean labeling of $G_i (1 \leq i \leq n)$.

Define a vertex labeling $\phi: V(G) \to \{1, 3, 4, \ldots, 3(\sum_{i=1}^{n} q_i) - 2\}$ as

If$x \in V(G_1)$, $\phi(x) = \phi_1(x)$

If $x \in V(G_i)$, $2 \leq i \leq n$ $\quad \phi(x) = \begin{cases} 3\left(\sum_{k=1}^{i-1} q_k\right) - 2 & for\ the\ lowest\ vertex\ label\ of\ G_i \\ \phi_i(x) + 3\left(\sum_{k=1}^{i-1} q_k\right) & for\ all\ other\ remaining\ vertices\ of\ G_i \end{cases}$.

Then the induced edge labels of G are $1, 4, \ldots, 3(\sum_{i=1}^{n} q_i) - 2$. Hence ϕ is one modulo three geometric mean labeling of G.

Theorem 2.3: Let $G_1(p_1, q_1), G_2(p_2, q_2), \ldots, G_n(p_n, q_n)$ be one modulo three geometric mean cycles with $q_i (1 \leq i \leq n)$ and e_i , $e_i{}'$ be the edges of $G_i (1 \leq i \leq n)$ labeled with $3q_i - 2$ and 1. Then the graph G obtained by identifying e_1 with $e_2{}'$ and e_2 with $e_3{}'$ and e_3 with $e_4{}'$ and so on until e_{n-1} identified with $e_n{}'$ is an one modulo three geometric mean graph.

Proof:

The graph G has $p_1 + (p_2 - 2) + \cdots + (p_n - 2) = (p_1 + p_2 + \cdots + p_n - 2(n-1))$ vertices and $\sum_{i=1}^{n} q_i - (n-1)$ edges.

Let ϕ_i be one modulo three geometric mean labeling of $G_i (1 \leq i \leq n)$.

Define a vertex labeling $\phi: V(G) \to \{1, 3, 4, \ldots, 3(\sum_{i=1}^{n} q_i - (n-1)) - 2\}$ as

If $x \in V(G_1)$, $\phi(x) = \phi_1(x)$ and If$x \in V(G_i)$, $2 \leq i \leq n$ $\phi(x) = \phi_i(x) + 3(\sum_{k=1}^{i-1}(q_k - 1))$

Then the induced edge labels of G are $1, 4, \ldots, 3(\sum_{i=1}^{n} q_i) - 2$. Hence ϕ is one modulo three geometric mean labeling of G.

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Theorem 2.4: The Key graph $KY(m,n)$ is a vertex equitable graph if $m \geq 5$.

Proof:

Let $G_1 = P_n \odot K_1$ and $G_2 = C_m$. Since G_1 has 2n-1 edges and G_2 has m edges, By Theorem 1.4, 1.5 $P_n \odot K_1, C_m$ are one modulo three geometric mean graphs.

Let ϕ_i be one modulo three geometric mean labeling of $G_i (1 \leq i \leq 2)$.

If $x \in V(G_1)$, $\phi(x) = \phi_1(x)$

If $x \in V(G_2)$ $\phi(x) = \begin{cases} 3\left(\sum_{k=1}^{i-1}(q_k+1)\right) & for\ the\ lowest\ vertex\ label\ of\ G_2 \\ \phi_i(x) + 3\left(\sum_{k=1}^{i-1}(q_k+1)\right) & for\ all\ other\ remaining\ vertices\ of\ G_2 \end{cases}$

Then the induced edge labels of G are $1, 4, \ldots, 3(2n-1+m)-2$. Hence ϕ is one modulo three geometric mean labeling of G.

Theorem 2.5: The tadpole graph $C_n @ P_m$ is a one modulo three geometric mean graph for $n \geq 5$.

Proof:

Let $G_1 = C_m$ and $G_2 = P_n$. Clearly G_1 has m edges and G_2 has n-1 edges, By Theorem 1.5, 1.6 C_m and P_n are one modulo three geometric mean graphs.

Let G = $C_n @ P_m$, then G has $n+m$ vertices and $n+m$ edges. By Theorem 2.1 the tadpole graph $C_n @ P_m$ is a one modulo three geometric mean graph.

Theorem 2.6:

The graph obtained by duplication of an arbitrary edge in $c_n (n \geq 4)$ admits an one modulo three geometric mean labelling.

Proof:

Let C_n be the cycle $u_1, u_2, \ldots . u_n, u_1$.

Let G be the graph obtained by duplicating an arbitrary edge of C_n .

With out of loss of generality let this edge be $e = u_1 u_2$ and the newly added edge be $e' = u_1' u_2'$.

Then $V(G) = \{u_1, u_2, \ldots, u_n, u_1' u_2'\}$, $E(G) = \{E(C_n), e_1', e', e''\}$ where $e_2' = u_2' u_3$ and $e_1' = u_n u_1'$.

Then |V(G)| = n+2 and |E(G)| = n+3.

Define $\phi: V(G) \rightarrow \{1, 3, 4, \ldots, 3q-2\}$ by

Case i: If n is odd, $n \geq 9$

$\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 21, , \phi(u_1') = 10, \phi(u_2') = 16 . \phi(u_i) = 28 + 6(i-4), 4 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$.

$\phi\left(u_{\left\lceil \frac{n}{2} \right\rceil + 1}\right) = 3n + 6, \phi(u_n) = 12 \phi(u_{n-i}) = 19 + 6(i-1),\ 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 3.$

Case ii : If n is even, $n \geq 10$

$\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 21, \phi(u_1{}') = 10, \phi(u_2{}') = 16$, $\phi(u_i) = 25 + 6(i-4), 4 \le i \le \frac{n}{2} + 1$.

$\phi\left(u_{\frac{n}{2}+2}\right) = 3n + 6, \phi(u_n) = 12, \phi(u_{n-1}) = 24$, $\phi(u_{n-i}) = 28 + 6(i-2)$, $2 \le i \le \frac{n}{2} - 3$.

Then the induced edge labels of G are $1, 4, \dots, 3(n+3) - 2$. Hence ϕ is one modulo three geometric mean labeling of G.

If $n = 4$, we define the labeling as $\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 13, \phi(u_4) = 18, \phi(u_1{}') = 9, \phi(u_2{}') = 19$.

If $n = 5$, we define the labeling as $\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 15$, $\phi(u_4) = 10$, $\phi(u_5) = 12$, $\phi(u_1{}') = 21, \phi(u_2{}') = 22$.

If $n = 6$, we define the labeling as $\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 21$, $\phi f(u_4) = 16$, $\phi(u_5) = 12$, $\phi(u_6) = 10, \phi(u_1{}') = 24, \phi(u_2{}') = 25$.

If $n = 7$, we define the labeling as $\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 21$, $\phi(u_4) = 28$, $\phi(u_5) = 27$, $\phi(u_6) = 19, \phi(u_7) = 12, \phi(u_1{}') = 10, \phi(u_2{}') = 16$.

If $n = 8$, we define the labeling as $\phi(u_1) = 1, \phi(u_2) = 3, \phi(u_3) = 21$, $\phi(u_4) = 25$, $\phi(u_5) = 31$, $\phi(u_6) = 30, \phi(u_7) = 22, \phi(u_8) = 12, \phi(u_1{}') = 10, \phi(u_2{}') = 16$.

Theorem 2.7: The graph $c_n \odot k_1$ is a one modulo three geometric mean graph for $n > 3$..

Proof: Let $u_1, u_2, \dots, u_n$ be the vertices of C_n. Let v_i be the pendant vertices attached at each $u_i$for $1 \le i \le n$.

Then $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$, $E(G) = E(C_n) \cup \{e_i = u_i v_i : 1 \le i \le n\}$Then |V(G)| = 2n and |E(G)| = 2n. Define $\phi: V(G) \to \{1, 3, 4, \dots, 3(2n) - 2\}$ by

When $n = 4$, we define the labeling as $\phi(u_1) = 3, \phi(u_2) = 7, \phi(u_3) = 22, \phi(u_4) = 21, \phi(v_1) = 1, \phi(v_2) = 12, \phi(v_3) = 18, \phi(v_4) = 13$.

When $n = 5$, we define the labeling as $\phi(u_1) = 3, \phi(u_2) = 6, \phi(u_3) = 28, \phi(u_4) = 27$, $\phi(u_5) = 13, \phi(v_1) = 1, \phi(v_2) = 15, \phi(v_3) = 21, \phi(v_4) = 19, \phi(v_5) = 18$,

Case i : If n is odd, $n \ge 7$

$\phi(u_1) = 3, \phi(u_2) = 13, \phi(u_i) = 28 + 12(i-3), 3 \le i \le \left\lceil \frac{n}{2} \right\rceil$. $\phi\left(u_{\left\lceil \frac{n}{2} \right\rceil + 1}\right) = 6n - 3, \phi(u_n) = 7, \phi(u_{n-1}) = 21, \phi(u_{n-i}) = 37 + 12(i-2)$, $2 \le i \le \frac{n-5}{2}$.

$\phi(v_1) = 1, \phi(v_2) = 18, \phi(v_i) = 24 + 12(i-3)$, $3 \le i \le \left\lceil \frac{n}{2} \right\rceil$.$\phi\left(v_{\left\lceil \frac{n}{2} \right\rceil + 1}\right) = \begin{cases} 25 & n = 7 \\ 6n - 23 & n \ge 9 \end{cases}$, $\phi(v_n) = 12, \phi(v_{n-1}) = 22, \phi(v_{n-2}) = 25, \phi(v_{n-i}) = 33 + 12(i-3)$, $3 \le i \le \frac{n-3}{2}$.

Case ii : If n is even, $n \ge 6$

$\phi(u_1) = 3, \phi(u_2) = 7, \phi(u_3) = 24$, $\phi(u_i) = 34 + 12(i-4), 4 \le i \le \frac{n}{2} + 1$. $\phi\left(u_{\frac{n}{2}+2}\right) = 6n - 3, \phi(u_n) = 30$, $\phi(u_{n-i}) = 31 + 12(i-1)$, $1 \le i \le \frac{n-6}{2}$.

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$\phi(v_1) = 1, \phi(v_2) = 6, \phi(v_3) = 10, \phi(v_4) = 18, \phi(v_i) = 42 + 12(i-5),\quad 5 \leq i \leq \frac{n}{2} + 1,$

$$\phi\left(v_{\frac{n}{2}+2}\right) = \begin{cases} 15 & n = 6 \\ 27 & n = 8 \\ 6n - 24 & n \geq 10 \end{cases}, \phi(v_n) = 12, \phi(v_{n-i}) = 15 + 12(i-1),\ 1 \leq i \leq \frac{n-4}{2}.$$

Then the induced edge labels of G are $1, 4, \dots, 2n-2$ .Hence ϕ is one modulo three geometric mean labeling of G.

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