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DEPENDABILITY ANALYSIS OF SYSTEMS WITH SEMI-MARKOV DEGRADATION

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ABSTRACT

Markov Models and evaluation techniques play a vital role in the dependability analysis. The main disadvantage of the use of Markov processes is that they do not allow any other distribution for the sojourn times beyond the exponential one as per the condition that future state depends only on the present state. The memoryless property of exponential distribution greatly helps satisfying this condition. But in most real world applications the life times and/or the repair times are not exponentially distributed. This paper deals with the dependability analysis of real life systems wherein the degradation process follows general statistical distributions. While modeling and evaluating single unit system and ideal two non-identical unit systems are considered. Expressions for availability and reliability are derived and applied to a real life case.

Keywords: Semi-Markov models, Markov models, Exponential distribution, Erlang distribution, Method of stages

I INTRODUCTION

Markov Models and evaluation techniques play a vital role in the evaluation of system reliability and associated reliability measures. Normally the dependability studies of many systems are based on Markov processes. The main disadvantage of the use of Markov processes is that they do not allow any other distribution for the sojourn times beyond the exponential one. But in most real world applications the life times and/or the repair times are not exponentially distributed [4]. In Markov models the holding times are exponentially distributed which is often too restrictive and might not fit the actual operating data well. Considering more general holding times leads to semi-Markov processes which are less amenable to analytic treatment but provide more flexible models [5].

In this paper our main goal is to study the reliability and availability measures of degradable and repairable systems. Thus a combination of general distribution for life time and repair time will be the underlying phenomenon for modeling and analysis. While modeling and evaluating single unit system and ideal two non-identical unit systems are considered. The analysis was extended to steady and transient state availability studies too. It was applied to a case and discussed.

II LITERATURE REVIEW

The main disadvantage of the use of Markov processes is that they do not allow any other distribution for the sojourn times beyond the exponential one. But in most real world applications the life times and/or the repair times are not exponentially distributed. The authors compared the dependability measured obtained using semi-Markov with that of Markov. Further in their analysis they have considered the life time to follow exponential and repair time general and only numerical examples with varying parameters were studied [4].

Average availability for holding time following general distributions are usually not available. To this effect the authors developed recursion procedures to apply to actual power plant data in the form of Weibull distribution. Authors compared the results with that of Markov models and commented that semi-Markov model using Weibull holding times fits the actual power plant data remarkably well [5].

Kharoufeh et al presented a hybrid, degradation based reliability model for a single unit system which is driven by a semi-Markov environment. By employing phase-type distributions the authors analyzed Markovian techniques to obtain reliability estimates. The viability of the approach they proposed was demonstrated by two numerical experiments [7].

Field units are subjected to a gradual deterioration process that progressively degrades their characteristics until a failure occurs. According to the authors only a few models are available with dependent increments processes. They proposed a degradation model in which the transition probabilities between unit states depend on both the current age and the current degradation level and analyzed with two real time case studies [6].

Limmnios presented a general model for dependability measures for continuous and discrete time semi-Markov processes. Author has proposed various theorems and Lemma to derive the dependability measures. The paper is limited to the theoretical and analytical treatment only [8].

Markovian models and their analysis techniques accomplish an important function in evaluating reliability/availability in complex engineering systems. Markovian modeling is essentially associated with constant rate of transition from one state to another state of the system and is strictly applicable to exponential distributions. When a more general form for the distribution of a system component failure or repair is considered, a non Markovian process takes place. In their study the approach of the method of stages is analyzed, provided that, offers a great potential applicability to solve industrial problems. An application, for the Chilean copper mining sector is presented, since these modeling methodologies can optimize the activities of maintenance and production in this sector [1].

As an extension to the popular hidden Markov model (HMM), a hidden semi-Markov model (HSMM) allows the underlying stochastic process to be a semi-Markov chain. Each state has variable duration and a number of observations being produced while in the state. This makes it suitable for use in a wider range of applications. Its forward–backward algorithms can be used to estimate/update the model parameters, determine the predicted, filtered and smoothed probabilities, evaluate goodness of an observation sequence fitting to the model, and find the best state sequence of the underlying stochastic process. It first provides a unified description of various HSMMs and discusses the general issues behind them. The boundary conditions of HSMM are extended. Then the conventional models, including the explicit duration, variable transition, and residential time of HSMM, are

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discussed. Various duration distributions and observation models are presented. Finally, the paper draws an outline of the applications [2].

A proactive handling of faults requires that the risk of upcoming failures is continuously assessed. One of the promising approaches is online failure prediction, which means that the current state of the system is evaluated in order to predict the occurrence of failures in the near future. More specifically, it focuses on methods that use event-driven sources such as errors. It uses Hidden Semi-Markov Models (HSMMs) for this purpose and demonstrates effectiveness based on field data of a commercial telecommunication system [3].

III RESEARCH METHODOLOGY

Markov Models and evaluation techniques play a vital role in the evaluation of system reliability. However following limitations apply to Markov Process:

- In the case of continuous process, the models are governed by sets of differential equations which can make them difficult to apply to complex systems.
- Markov modeling techniques are essentially associated with constant hazard rates and are therefore strictly applicable only to exponential distributions. If exponential distributions are not valid for the components, this technique cannot be used to evaluate transition probabilities.

The first limitation can be overcome by applying State transition Matrix for developing the solution. The second limitation has to be overcome by using Semi-Markov Process. In the below sections, cases with mathematical vigor have been analyzed with Semi-Markov Process, in particular Method of stages has been used in the following section as a solution technique to solve Semi-Markov models. In case of Weibull Distribution with $\beta \geq 1$, this distribution can be represented by a set of stages connected in series. When Weibull Distribution with $\beta < 1$, this distribution can be represented by a set of stages connected in parallel.

In general case, it is possible to corroborate that even though, the original components obey exponential laws, once combining the indexes of two or more components in parallel, series-parallel, stand-by, etc., the resulting terms are not exponential, necessarily. Strictly speaking the resulting distribution depends on the number of states that are combined and the way in which they are organized. It follows that the reverse processes is also true, that is, if a state is not exponentially distributed, then it can be divided into a number of stages which each one are exponentially distributed. The suggested methodology turns to finding:

- the number of sub states
- the way in which they are connected and
- their numerical parameters, in order to represent the state that is being considered.

This process to divide the system state in sub states denoting each sub state as a stage is known as method of stages.

3.1 Generic Algorithm to solve Semi-Markov Model using Method of stages

i) Carryout Weibull analysis

ii) Draw State transition diagram.

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iii) Calculate first and second moments of Weibull distribution.

iv) Using the given formulae, calculate no. of stages (α) and parameter of exponential distribution (λ).

v) Draw revised state transition diagram

vi) Determine LSTM or STM depending on whether Reliability or Availability model.

vii) Solve the matrix to get the desired solution.

IV MODELING AND ANALYSIS

Using the general algorithm presented in the previous section, the dependability analysis are formulated and analyzed for the single unit and two-unit ideal systems. The detailed derivation and analysis are presented in this section.

4.1 Case-1: Single unit System with Weibull failure

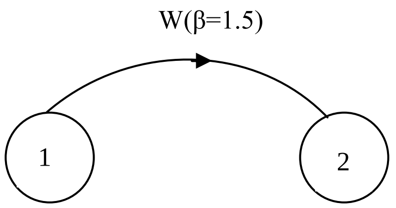

Fig.1: STD for single unit system

From the data analysis, MTBF = 346 hrs and Weibull with shape parameter, β=1

for an estimated mean life,

$$M_1 = \mathrm{MTBF} = 346 = \theta\,\Gamma\left(1+\frac{1}{\beta}\right)$$

$$\theta = \frac{346}{0.9033} = 383.04$$

$$M_2 = \theta^2\,\Gamma\left(1+\frac{2}{1.5}\right) = 174302.934$$

$$\rho = \lambda = \frac{M_1}{M_2 - M_1^2} = \frac{346}{174302.934 - 346^2} = 0.006339$$

$$\alpha = M_1\rho = 0.006339 \text{ x } 346 = 2.19 \sim 2$$

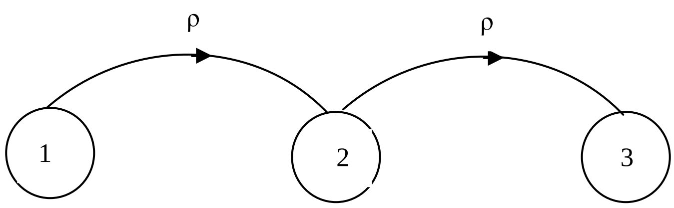

Fig.2: Revised STD using method of stages for single unit

In the revised STD, as of Markov with parameter ρ

The Laplacian State Transition Matrix is as given below

$$\begin{bmatrix} S+\lambda & 0 & 0 \\ -\lambda & S+\lambda & 0 \\ 0 & -\lambda & S \end{bmatrix} \begin{bmatrix} P_1(S) \\ P_2(S) \\ P_3(S) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

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Solving the Equation (1)

$P_1(t) = e^{-\lambda t}$ (2)

$P_2(t) = \lambda t\, e^{-\lambda t}$ (3)

$P_3(t) = 1 - e^{-\lambda t} - \lambda t . e^{-\lambda t}$ (4)

$\therefore R(t) = e^{-\lambda t} + \lambda t\, e^{-\lambda t}$ (5)

Assume t=100, then

$R(t) = e^{-0.006339 \text{ x } 100}[(100 \ x \ 0.006339) + 1]$

$= 0.8668$ (6)

By RBD method

$R(t) = e^{\left(t/_{\theta}\right)^{\beta}} = e^{\left(\frac{100}{383.04}\right)^{1.5}} = 0.8751$ (7)

The solutions of above model were also obtained by varying the parameters β and θ for different values of T.

4.2 Single Component System with Weibull Repair and exponential failure

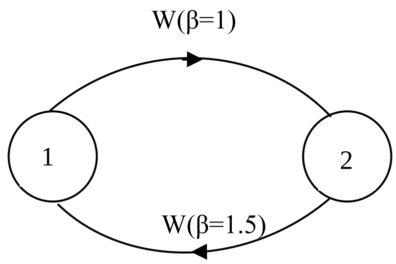

Fig.3: STD for single unit system with Weibull repair

<u>From data analysis</u>:

Avg. no. of repairs = 10 = M_1; exponential failure with λ = 0.003

Distribution is Weibull with β=2, θ = 11.28

Using method of stages to weibull repair, we get

$\rho = \lambda = 0.167$ (8)

$\alpha = \frac{10^2}{127.24 - 10^2} = 3.67 \sim 4$ (9)

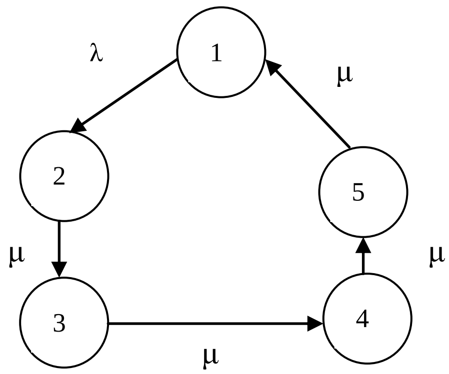

Fig.4: Revised STD using method of stages for single unit

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$$\begin{bmatrix} -\lambda & 0 & 0 & 0 & \mu \\ \lambda & -\mu & 0 & 0 & 0 \\ 0 & \mu & -\mu & 0 & 0 \\ 0 & 0 & \mu & -\mu & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (10)$$

$$P_5 = \frac{\lambda}{4\lambda+\mu} \qquad (11)$$

$$P_4 = P_3 = P_2 = \frac{\lambda}{4\lambda+\mu} \qquad (12)$$

$$P_1 = \frac{\mu}{4\lambda+\mu} \qquad (13)$$

$$\text{Availability of system } = A = P_1 = \frac{\mu}{4\lambda+\mu} = \frac{0.367}{0.367+0.003 \text{ x } 4} = 0.968 \qquad (14)$$

4.3 Two- unit Non-identical Active Parallel System with Weibull Repair

From data analysis for unit a, the failure follows exponential with parameter $\lambda_a = 0.1$

for unit b, the failure follows exponential with parameter $\lambda_b = 0.2$

Repair crew 1 for unit a follows Weibull with $\beta_1 = 1.5$, $\theta_1 = 22.14$hrs;

Repair crew 2 for unit a follows Weibull with $\beta_2 = 1.5$, $\theta_2 = 44.28$

For unit 1: applying method of stages will give $\rho_1 = \mu_1 = 0.1096$, $\alpha_1 \sim 2$

For unit 2: applying method of stages will give $\rho_1 = \mu_1 = 0.055$, $\alpha_1 \sim 2$

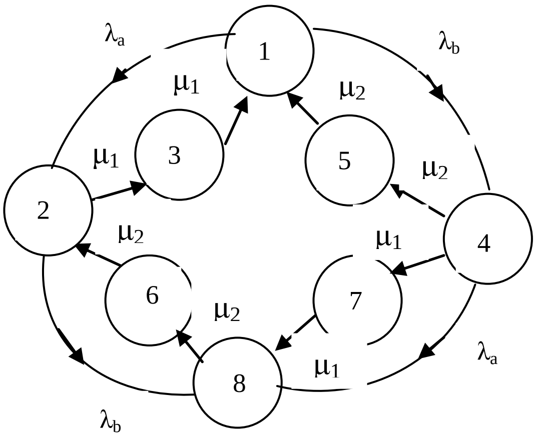

Fig.5: Revised STD using method of stages for 2 unit active

$$\begin{bmatrix} -\lambda_a - \lambda_b & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & 0 \\ \lambda_a & -\mu_1 - \lambda_b & 0 & 0 & 0 & \mu_2 & 0 & 0 \\ 0 & \mu_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_b & 0 & 0 & -\mu_2 - \lambda_a & 0 & 0 & \mu_1 & 0 \\ 0 & 0 & 0 & \mu_2 & -\mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & \mu_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & \mu_1 \\ 01 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Solving we get

$P_1 = 0.032$; $P_2 = 0.0605$; $P_3 = 0.0605$; $P_4 = 0.164$; $P_5 = 0.164$; $P_6 = 0.173$; $P_7 = 0.173$; $P_8 = 0.173$

$$A(\infty) = 1 - P_8 = 0.827 \qquad (15)$$

As the number of stages increase, the complexity of the solution increase. Hence this was coded in MATLAB.

V CASE STUDY- AVAILABILITY OF CENTRIFUGAL PUMP IN COPPER MINE

A case study was developed at a plant of the Copper Corporation of Chile (CODELCO), which is one of the greater underground mine of copper of the world, located to about 120 km to the south of the city of Santiago of Chile. The plant has modern facilities for processing copper minerals. Engineering department are permanently searching for new technologies and new decision making methods to support the optimization of the production operations [1].

The problem situation consisted of investigating the availability of the critical equipment that integrates the process of Humid Milling. This process is a subsystem from the concentrator and it is constituted by feeding transporters, the mill, the battery of hydrociclons and the pumps that send pulp from the ball mills to the hydrociclons. These pumps must resist the highly abrasive action of the pulp of the copper mineral, so they are made with special steel and require rigorous practices of maintenance. These equipments constitute the most critical units of the humid milling process and there is a great interest to improve the system reliability and to reduce its operational cost.

In the first step, applying Weibull analysis on the failure data revealed the failure phenomenon follows exponential distribution with the parameter failure rate as λ = 0.003/hr. For the repaired time an Erlang distribution was assumed considering a mean repairing time of 24 hours and a standard deviation of 12 hours as a first approximation. Applying the method of stages we get the required parameters no of stages α =4 and uniform repair rate $\mu=\rho$= 0.167. The diagram of the state space would be represented in the following way.

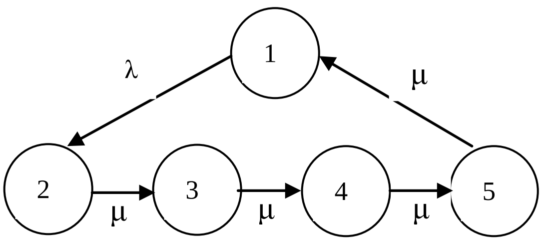

Fig.6: Revised STD using method of stages centrifugal

Forming LSTM and solving we get the availability of the centrifugal pump as given in Equation

$$\text{Availability of system } = A(\infty) = \frac{\mu}{4\lambda+\mu} = \frac{0.367}{0.367+0.003 \text{ x } 4} = 0.968$$

By conventional Markovian methods the steady state availability can be ascertained by using the formula given in Equation

$$A(\infty) = \frac{\mu}{\mu+\lambda} = \frac{1460}{1460+26.28} = 0.982 \qquad (16)$$

VI RESULTS AND DISCUSSION

For single unit system the parameters of the underlying distributions are perturbed and accordingly the reliability vales are obtained as shown in Table 1.

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Table 1 Comparison of Reliability results

β	θ	T	Conventional method	Semi-Markov model
1.5	400	50	0.9623	0.9568
	100	100	0.3020	0.3670
	10	30	0.0057	0.0055
2	400	50	0.9980	0.9845
	100	100	0.3983	0.368
	10	30	0.00017	0.00012
3	400	50	0.9998	0.9981
	100	100	0.3774	0.3679
	10	30	0.0000002	1.88E-11

As seen from above table, the values obtained from semi-Markov model and conventional reliability calculation method are comparable. Minor difference exists which can be attributed to below factor: If number of stages turns out to be non-integer value, it is being rounded off to nearest integer. This rounding off induces error and will be primary source of above minor difference. In the same way the analysis was applied to the real life case and compared with the conventional method and it is found results are comparable.

VII CONCLUSION

In Markov models the holding times are exponentially distributed which is often too restrictive and might not fit the actual operating data well. Considering more general holding times leads to semi-Markov processes which are less amenable to analytic treatment but provide more flexible models. To this effect a generalized algorithm was developed and applied to single two unit ideal systems. By varying various parameters the sensitiveness of dependability analysis carried out and found satisfactory. The technique can be extended to multi-unit systems for transient availability analysis. The limitation of integer constrained arisen during the assessment of number of stages can be further worked out to have an effective dependability analysis. As method of stages are specifically applicable to Weibull case with shape parameter greater than unity, for less than one appropriate algorithm is to be developed which can be considered as extension of this study.

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