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A PROBLEM ON DEFORMATION IN MICROPOLAR GENERALIZED THERMOELASTIC MEDIUM WITH MASS DIFFUSION SUBJECTED TO THERMO MECHANICAL LOADING DUE TO THERMAL LASER PULSE BY USING INTEGRAL TRANSFORMS TECHNIQUE

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ABSTRACT

The present investigation deals with the deformation in micropolar generalized thermoelastic medium with mass diffusion subjected to thermo mechanical loading due to thermal laser pulse. Laplace and Fourier transform technique is used to solve the problem. Concentrated normal force and thermal source are taken to illustrate the utility of approach. The closed form expressions of normal stress, tangential stress, tangential couple stress, mass concentration and temperature distribution are obtained in the transformed domain.

Keywords: Laser Pulse, Micropolar, Mass Diffusion, Uniformly And Linearly Distributed Source.

I. INTRODUCTION

Micropolar theory of elasticity was introduced by Eringen [1]. This theory incorporates the local deformation and rotation of the material points of the composite. This theory provides a model that can support body couples and surface couples and exhibits a high frequency optical wave spectrum. Eringen [2, 3], Maugin and Mild [4], Nowacki [5] developed the linear theory of micropolar thermoelasticity by excluding the micropolar theory of elasticity to include the thermal effects. Touchert et al. [6] derived the basic equations of linear theory of micropolar coupled thermoelasticity. However, there is a certain degree of coupling with temperature and temperature gradients as temperature speeds up the diffusion process. Nowacki [7, 10] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [11] and Olesiak and Pyryev [12], respectively, discussed the theory of thermo diffusion and coupled quasi stationary problems of thermal diffusion for an elastic layer. Rapidly oscillating contraction and expansion generates temperature changes in materials susceptible to diffusion of heat by conduction [13]. This mechanism has attracted considerable attention due to the extensive use of pulsed laser technologies in material processing and non-destructive testing and characterization [14, 15]. The so-called ultra-short lasers are those with pulse durations ranging from nanoseconds to femto seconds. In the case of ultra-short pulsed laser heating, the high intensity energy flux and ultra-short duration lead to a very large thermal gradients or ultra-high heating may exist at the boundaries. In such cases, as pointed out by many investigators, the classical Fourier model, which leads to an

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infinite propagation speed of the thermal energy, is no longer valid [16]. Researchers have proposed several models to describe the mechanism of heat conduction during short-pulse laser heating, such as the parabolic one-step model [17], the hyperbolic one-step model [18], and the parabolic two-step and hyperbolic two-step models [19, 20].Dubois [21] experimentally demonstrated that penetration depth play a very important role in the laser-ultrasound generation process. Ezzat et al. [22] discussed the thermo-elastic behavior in metal films by fractional ultrafast laser. Al-Huniti and Al-Nimr [23] investigated the thermoelastic behavior of a composite slab under a rapid dual-phase lag heating. The comparison of one-dimensional and two-dimensional axisymmetric approaches to the thermo mechanical response caused by ultrashort laser heating was studied by Chen et al. [24].Kim et al.[25] studied thermoelastic stresses in a bonded layer due to pulsed laser radiation. Thermoelastic material response due to laser pulse heating in context of four theorems of thermoelasticity was discussed by Youssef and Al-Bary [26],Theoretical study of the effect of enamel parameters on laser induced surface acoustic waves in human incisor was studied by Yuan et al [27].A two- dimensional generalized thermoelastic diffusion problem for a thick plate under the effect of laser pulse thermal heating was studied by Elhagary [28].Othman et al. [29]studied the influence of thermal loading due to laser pulse on generalized micropolar thermoelastic solid with comparison of different theories.

II. PROBLEM FORMULATION

Following Eringen [3] and Al-Qahtani and Datta [30] the basic equations for homogeneous, isotropic micropolar generalized thermoelastic solid with mass diffusion in the absence of body forces and body couples are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\ddot{\boldsymbol{u}}$$
,(1)

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}},(2)$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)(\beta_1T_0\nabla.\dot{\boldsymbol{u}}-Q)+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C$$
, (3)

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0$$
, (4)

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}C$$
, (5)

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}, \qquad (6)$$

The plate surface is illuminated by laser pulse given by the heat input

$$Q=I_0f(t)g(x_1)h(x_3) \qquad (7)$$

where I_0 is the energy absorbed. The temporal profile $f(t)$ is represented as,

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$f(t) = \frac{t}{t_0^2} e^{-\left(\frac{t}{t_0}\right)}$(8)

Here t_0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in x_1

$$g(x) = \frac{1}{2\pi r^2} e^{-\left(\frac{x_1^2}{r^2}\right)} \quad (9)$$

where r is the beam radius, and as a function of the depth x_3 the heat deposition dueto the laser pulse is assumed to decay exponentially within the solid,

$h(x_3) = \gamma^* e^{-\gamma^* x_3}$(10)

Equation (7) with the aid of (8,9 and 10) takes the form

$$Q = \frac{I_0 \gamma^*}{2\pi r^2 t_0^2} t e^{-\left(\frac{t}{t_0}\right)} e^{-\left(\frac{x_1^2}{r^2}\right)} e^{-\gamma^* x_3} \ , \quad (11)$$

We consider plane strain problem with all the field variables depending on x_1, x_3 and t. For two dimensional problems, we take

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \quad (12)$$

For further consideration, it is convenient to introduce in equations (1.1)-(1.4) the dimensionless quantities defined as:

$$\varepsilon = \frac{\gamma^2 T_0}{\rho^2 c^* c_1}, m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij} \ , \quad C' = \frac{\beta_2}{\rho c_1^2} C \ , \ Q = \frac{K^* \omega^{*2}}{c^*} Q \quad (13)$$

Making use of (13) in (1)-(4) and with the aid of (12), we obtain:

$$a_1 \frac{\partial \varepsilon}{\partial x_1} + a_2 \nabla^2 u_1 - a_3 \frac{\partial \phi_2}{\partial x_3} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x_1} - a_4 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = \rho \frac{\partial^2 u_1}{\partial t^2} , \quad (14)$$

$$a_1 \frac{\partial \varepsilon}{\partial x_3} + a_2 \nabla^2 u_3 + a_3 \frac{\partial \phi_2}{\partial x_1} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x_3} - a_4 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = \rho \frac{\partial^2 u_3}{\partial t^2} , \quad (15)$$

$$\nabla^2 \phi_2 - 2a_6 \phi_2 + a_6 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = a_7 \ddot{\phi}_2 \quad (16)$$

$$-\nabla^2 T + \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + a_5 \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right) e + a_8 \left(\frac{\partial}{\partial t} + \gamma_1 \frac{\partial^2}{\partial t^2}\right) C = Q_0 f^*(x_1, t) e^{-\gamma^* x_3} \quad (17)$$

$$\nabla^2 e + a_9 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 T + a_{10} \left(1 + \varepsilon \tau^0 \frac{\partial}{\partial t}\right) \dot{C} - a_{11} \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla^2 C = 0 \ , \quad (18)$$

The displacement components u_1 and u_3 are related to the non- dimensional potential functions ϕ and ψ as:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3} \ , \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \quad (19)$$

Substituting the values of u_1 and u_3 from (19) in (14)-(18) and with the aid of (12), we obtain:

$$\nabla^2\phi - \ddot{\phi} - \left(1+\tau_1\frac{\partial}{\partial t}\right)T - a_4\left(1+\tau^1\frac{\partial}{\partial t}\right)C = 0, \quad (20)$$

$$\nabla^4\phi + a_9\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T + a_{10}\left(1+\epsilon\tau^0\frac{\partial}{\partial t}\right)\dot{C} - a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0 \quad (21)$$

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T} + a_5\left(\frac{\partial}{\partial t}+\epsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi + a_8\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C - \nabla^2 T = Q_0 f^*(x_1,t)e^{-\gamma^* x_3} \quad (22)$$

$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0, \quad (23)$$

$$\nabla^2\phi_2 - 2a_6\phi_2 - a_6\nabla^2\psi = a_7\ddot{\phi}_2, \quad (24)$$

III. SOLUTION OF THE PROBLEM

We define Laplace transform and Fourier transform respectively as:

$$\bar{f}(s,x_1,x_3) = \int_0^\infty f(t,x_1,x_3)e^{-st}dt\,, \quad (25)$$

$$\hat{f}(x_3,\xi,s) = \int_{-\infty}^{\infty}\bar{f}(s,x_1,x_3)e^{\iota\xi x_1}dx_1\,, \quad (26)$$

Applying Laplace transform defined by (25) on (20)-(24) and then applying Fourier transforms defined by (26) on the resulting quantities and eliminating $\hat{C}$&$\hat{T}$, $\hat{\phi}$&$\hat{T}$,$\hat{\phi}$&$\hat{C}$ and $\hat{\phi}_2$ respectively from the resulting equations, we obtain:

$$[D^6 + AD^4 + BD^2 + C]\hat{\phi} = f_1 e^{-\gamma^* x_3} \quad (27)$$

$$[D^6 + AD^4 + BD^2 + C]\hat{T} = f_2 e^{-\gamma^* x_3} \quad (28)$$

$$[D^6 + AD^4 + BD^2 + C]\hat{C} = f_3 e^{-\gamma^* x_3} \quad (29)$$

$$[D^4 + ED^2 + F]\hat{\psi} = 0\,, \quad (30)$$

The solutions of the equations (27)-(30) satisfying the radiation conditions that $(\hat{\phi},\hat{\phi}^*,\hat{T},\hat{\phi_2},\hat{\psi}) \to 0$ as $x_3 \to \infty$ are given by:

$$\hat{\phi} = B_1 e^{-m_1 x_3} + B_2 e^{-m_2 x_3} + B_3 e^{-m_3 x_3} + L_1 e^{-\gamma^* x_3} \quad (31)$$

$$\hat{T} = d_1 B_1 e^{-m_1 x_3} + d_2 B_2 e^{-m_2 x_3} + d_3 B_3 e^{-m_3 x_3} + L_2 e^{-\gamma^* x_3} \quad (32)$$

$$\hat{C} = e_1 B_1 e^{-m_1 x_3} + e_2 B_2 e^{-m_2 x_3} + e_3 B_3 e^{-m_3 x_3} + L_3 e^{-\gamma^* x_3} \quad (33)$$

$$\hat{\psi} = B_4 e^{-m_4 x_3} + B_5 e^{-m_5 x_3} \quad (34)$$

$$\hat{\phi_2} = h_4 B_4 e^{-m_4 x_3} + h_5 B_5 e^{-m_5 x_3} \quad (35)$$

IV. BOUNDARY CONDITIONS

We consider concentrated normal force and concentrated thermal source at the boundary surface $x_3 = 0$, mathematically, these can be written as:

$$t_{33} = -F_1\psi_1(x_1)\delta(t), t_{31} = 0, m_{32} = 0,$$
$$T = F_2\psi_1(x_1)\delta(t)\,,\ C = F_3\psi_1(x_1)\delta(t) \quad (36)$$

Also

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$t_{33} = \lambda e + (2\mu + K)u_{3,3} - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)T - \beta_2\left(1 + \tau^1\frac{\partial}{\partial t}\right)C t_{31} = (2\mu + K)u_{3,1} - K\phi_2$

$m_{32} = \beta\phi_{2,3}$

(37)

Substituting the values of $\hat{\phi}, \widehat{\phi^*}, \hat{T}, \hat{\psi}, \widehat{\phi_2}$ from the equations (31)-(35) in the boundary condition (36) and using (5)-(11), (12)-(13), (25)-(26) and solving the resulting equations, we obtain:

$\widehat{t_{33}} = \sum_{i=1}^{5} G_{1i} e^{-m_i x_3} - M_1 e^{-\gamma^* x_3}, i = 1,2,\ldots,5$ (38)

$\widehat{t_{31}} = \sum_{i=1}^{5} G_{2i} e^{-m_i x_3} - M_2 e^{-\gamma^* x_3}, i = 1,2,\ldots,5$ (39)

$\widehat{m_{32}} = \sum_{i=1}^{5} G_{3i} e^{-m_i x_3} - M_3 e^{-\gamma^* x_3}, i = 1,2,\ldots,5$ (40)

$\hat{T} = \sum_{i=1}^{5} G_{4i} e^{-m_i x_3} - M_4 e^{-\gamma^* x_3}, i = 1,2,\ldots,5$ (41)

$\sum_{i=1}^{5} G_{5i} e^{-m_i x_3} - M_5 e^{-\gamma^* x_3}, i = 1,2,\ldots,5$ (42)

V. SPECIAL CASE

Micropolar Thermoelastic Solid: In absence of mass diffusion effect in Equations (38) - (42), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar generalized thermoelastic half space.

5.1 Inversion of the transforms

The transformed displacements, stresses and temperature changes are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence these are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied .

VI. DISCUSSIONS

The analysis is conducted for a magnesium crystal-like material.

6.1 Linearly distributed normal force

The variation of normal stress t_{33} with the distance x_1. It is noticed that for MPMDT1 and MPMDT2, t_{33} show similar behavior. The value of normal stress monotonically increases as x_1 and then oscillates. The value of t_{33} increases near the application of the normal force due to the mass diffusion effect and then remain oscillating for all values of x_1.

The variation of tangential stress t_{31} with the distance x_1. It is noticed that initially the behavior of t_{31} for MPMDT1 and MPT1 show variable trend but for MPMDT1, MPMDT2 and MPT1, MPT2 exhibits similar behavior. t_{31} Initially decrease monotonically for all the cases. The variation in tangential stress in micropolar thermoelastic is more than that of micropolar thermoelastic with mass diffusion.

VII. CONCLUSIONS

The problem consists of investigating displacement components, scalar mass concentration, temperature distribution and stress components in a homogeneous isotropic micropolar mass diffusion thermoelastic half space due to various sources subjected to laser pulse. Integral transform technique is employed to express the results mathematically. Theoretically obtained field variables are also exemplified through a specific model to present the results in the transformed domain.

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