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SOME NEW RESULTS USING COMPATIBLE MAPPINGS OF TYPE B IN FUZZY METRIC SPACES

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ABSTRACT

Here we have taken the concept compatibility type B in fuzzy metric space to solve the theorems of common fixed point.

***Index Terms*— Mappings,Compatible mapping of type B.**

I. INTRODUCTION

Fuzzy sets were given by Zadeh[1]. Then some new definations given by byKramosil ,Michalek[2] andGeorge and Veeramani[3]. Grabiec[4] gave some new results with different mappings on fuzzy metric space.Many results were given by Singh and Chauhan[5] using the concept of compatible mappings in Fuzzy metric space. The concept of compatible mapping of type (A) and type(B) was given by Jungck[6] .Cho ,Pathak and [9] also proved some theorems on mapping of compatible type B in fuzzy metric space.

II. FUZZY METRIC SPACE

Definition[3]:A 3-tuple (X,M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty]$ satisfying the following conditions

(f1) $M(x, y, t) > 0$

(f2) $M(x, y, t) = 1$ if and only if $x = y$

(f3) $M(x, y, t) = M(y, x, t)$;

(f4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(f5) $M(x, y, .) : (0, \infty) \to (0, 1]$ is continuous.

$t,s > 0$ and $x,y,z \in X$

Then M is called a fuzzy metric on X.

Compatible mappings and Non compatible mappings[4]: Let A and B be mapping from a fuzzy metric space (X,M, *) into itself.Then these mappings are said to be compatible if

$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1, \forall\, t > 0,$

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where $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} x \in X$$

from the above condition it is referred that A and S are non compatible maps from a fuzzy metric space (X,M,*) into itself if

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$$

but either $\lim_{n\to\infty} M(ABx_n, BAx_n, t) \neq 1$, or not existence of limit.

Defination[6]: Let S and T be maps from an FM-space (X,M,*)into itself. The maps S and T are said to be type B compatible ,if

$\lim n \to \infty M(SSx_n, TTx_n, t) = 1$

For all t>0 where $\{x_n\}$ is a sequence in X such that

$\lim n \to \infty Sx_n = \lim n \to \infty Tx_n = p$ for some $p \in X$

III. MAIN RESULTS

Theorem:-Let A,S,B,Qbe self maps of complete fuzzy metric space (X,M,*) such that

(1)$AB(X) \cup QS(X), A(X) \subset SB(X)$

(2)There is a constant $c \in (0,1)$ such that

$M(Ax,Qy,ct)*[M(Sx,Ax,ct)M(By,Qy.ct)]*M(By,Qy,ct)+am(By,Qy,ct)M(Ax,Qy,2ct) \geq [pM(Sx,Px,t)+ qM(Sx,By,t)]M(Sx,Qy,2ct)M(Sx,By,t)$

For all x,y in X ,t> 0 where 0<p,q<1,

(3)[A,S],[Q,B] are compatible of type B

(4)S,B are continuous and SB=BS

Then A,Q,S,B have common fixed point in X.

Proof: Suppose we are taking x_0 is an arbitrary point of X.we can make a sequence $\{x_n\}$ in X as follows

$ABx_{2n}=SBx_{2n+1}$, $QSx_{2n+1}= SBx_{2n+2}$, n=0,1,2.......

Now let $z_n=SBx_n$then put $x=Bx_{2n}$ and $y=Sx_{2n+1}$

$M(ABx_{2n},QSx_{2n+1},ct)*[M(SBx_{2n},ABx_{2n},ct)M(BSx_{2n+1},QSx_{2n+1},ct)]*M(BSx_{2n+1},QSx_{2n+1},ct)]*M(BSx_{2n+1},QSx_{2n+1},ct)+am(BSx_{2n+1},QSx_{2n+1},ct)M(SBx_{2n},QSx_{2n+1},2ct)$

$\geq [pM(SBx_{2n},ABx_{2n},t) +qm(SBx_{2n},BSx_{2n+1},t)] M(SBx_{2n},QSx_{2n+1},2ct)+M(SBx_{2n},BSx_{2n+1},t)$

And

$M(SBx_{2n+1},SBx_{2n+2},ct)*[M(z_{2n},SBx_{2n+1},ct)M(z_{2n+1},SBx_{2n+1},ct)]*M(z_{2n+1},SBx_{2n+2},ct)]*aM(z_{2n+1},SBx_{2n+2},ct)+M(z_{2n},SBx_{2n+2},2ct)$

$\geq [pM(z_{2n},SBx_{2n+1},t)+qm(z_{2n},z_{2n+1},t)] M(z_{2n},SBx_{2n+1},2ct)$

Then

$M(z_{2n+1},z_{2n+2},ct)*[M(z_{2n},z_{2n+1},ct)M(z_{2n+1},z_{2n+2},ct)]*M(z_{2n+1},z_{2n+2},ct)]*aM(z_{2n+1},z_{2n+2},ct)+M(z_{2n},z_{2n+2},2ct)$

$\geq [pM(z_{2n},z_{2n+1},t)+qm(z_{2n},z_{2n+1},t)] M(z_{2n},z_{2n+2},2ct)+M(z_{2n},z_{2n+1},t)$

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So

$M(z_{2n+1},z_{2n+2},ct)*[M(z_{2n},z_{2n+1},ct)M(z_{2n+1},z_{2n+2},ct)]*M(z_{2n+1},z_{2n+2},ct)]*aM(z_{2n+1},z_{2n+2},ct)+M(z_{2n},z_{2n+2},2ct)$

$\geq [p+q+1]M(z_{2n},z_{2n+1},t)+qm(z_{2n},z_{2n+1},t)\ M(z_{2n},z_{2n+2},2ct)$

and

$M(z_{2n+1},z_{2n+2},ct)*M(z_{2n},z_{2n+2},2ct)*aM(z_{2n+1},z_{2n+2},ct)+M(z_{2n},z_{2n+2},2ct)$

$\geq [p+q+1]M(z_{2n},z_{2n+1},t)\ M(z_{2n},z_{2n+1},t)\ M(z_{2n},z_{2n+2},2ct)$

Thus it follows that

$M(z_{2n+1},z_{2n+2},ct) \geq M(z_{2n},z_{2n+1},t)$

$0<k<1$ and for all $t>0$

Similarly we also have

$M(z_{2n+2},z_{2n+3},ct) \geq M(z_{2n+1},z_{2n+2},t)$

$0<k<1$ and for each $t>0$

In general we have for m= 1,2,3......

$M(z_{m+1},z_{m+2},ct) \geq M(z_m,z_{m+1},t)$

$0<k<1$ and for all $t>0$ aan we know that $\{z_n\}$ is a Cauchy sequence in Xand (X,M,*) is complete,it converges to a point z in X.Since $\{ABx_{2n}\}$ and $\{QSx_{2n+1}\}$ are subsequences of $\{z_n\}$,$\{ABx_{2n}\}\to z$ and $\{QSx_{2n+1}\}\to z$ as $n\to\infty$

Takingy$_n$=Bx$_n$ and w$_n$=Sx$_n$ for n=1,2,..... then we have $Ay_{2n}\to z$,$Sy_{2n}\to z$,$Bw_{2n+1}\to z$

$M(AAy_{2n},SSy_{2n},t)\to 1$ and $M(QQw_{2n+1},BBw_{2n+1},t)\to 1$ as $n\to\infty$.Moreover by the continuity of B,we have $BQw_{2n+1}\to Tz$ and $QQw_{2n+1}\to Bz$

As $n\to\infty$.,now taking $x=y_{2n}$ and $y=Qw_{2n+1}$ in main equation,we have

$M(Ay_{2n},QQw_{2n+1},ct)*[M(Sy_{2n},Ay_{2n},ct)M(BQw_{2n+1},QQw_{2n+1},ct)]*M(BQw_{2n+1},QQw_{n+1},ct)+am(BQw_{2n+1},QQw_{2n+1},ct)M(Sy_{2n},QQw_{2n+1},2ct)$

$\geq[pM(z,z,t)+qM(z,Bz,t)]M(z,Bz,2ct),(z,Bz,t)$

Then it follows that

$M(z,Bz,ct)+aM(z,Bz,2ct)\geq[p+qM(z,Bz,t)]M(z,Bz,2ct)$

And since M(x,y,.) is non decreasing for all x,y in X,we have

$M(z,Bz,2ct)M(z,Bz,t)+aM(z,Bz,2ct) \geq[p+qM(z,Bz,t)]M(z,Bz,2ct)$

Thus

$M(z,Bz,t) +a\geq p+qM(z,Bz,t)$ And

$M(z,Bz,t)\geq(P-a)/(1-q) =1$ As each $t>0$ so z=Bz.Same as we have z=Sz

Now taking $x=y_{2n}$ and y =z in main equation

$M(Ay_{2n},Qz,ct)*[M(Sy_{2n},Ay_{2n},ct)M(Bz,Qz,ct)]*M(Bz,Qz,ct)]*M(Bz,Qz,ct)+aM(Bz,Qz,ct)M(Sy_{2n},Qz,2ct)$

$\geq[pM(Sy_{2n},Ay_{2n},t)+qM(Sy_{2n},Bz,t)]M(Sy_{2n},Qz,t)]M(Sy_{2n},Qz,t)M(Sy_{2n},Bz,t)$

And

$M(z,Qz,ct)*[M(z,z,ct)M(z,Qz,kt)]*M(z,Qz,ct)]+aM(z,Qz,ct)+M(z,Qz_{2n+2},2ct)$

$\geq [pM(z,z,t) +qM(z,z,t)\ M(z,Qz,2ct),(z,z,t)$

Then

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M(z,Qz,ct)*M(z,Qz,ct)+aM(z,Qz,ct)M(z,Qz,2ct)

$\geq$ (p+q)M(z,Qz,2ct)

Thus it follows that

M(z,Qz,ct)+aM(Z,Qz,ct) ≥ p+q And

M(z,Qz,ct)≥ (p+q/1+a) =1 0<c<1 and for each t>0

so that z=Qz similarly we have z=Az.

IV. CONCLUSION

Here we proved some new results using the concept of compatibility of type B without exploiting the condition of t-norm.

REFERENCES

[1] L. A. Zadeh, Fuzzy sets, Infor. and Control, 8(1965), 338-353

[2] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika,11(1975), 336-344.

[3] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets andSystems, 64(1994), 395-399.

[4] M. Grabiec, Fixed points in fuzzy metric space, Fuzzy Sets and Systems, 27(1988), 385-389.

[5] B. Singh and M.Chauhan“ Common fixed points of compatible maps in fuzzy metric space”,Fuzzy sets and systems,115(2000),PP.471-475.

[6] G. Jungck “Compatible mappings and common fixed points” .IntrnationalJ.Math.Math.Sci,(1988).PP- 285-288.

[7] Y.J.Cho,H.K. Pathak,S.M. Kang and J.S. Jung “Common fixed point theorems of compatible type B on fuzzy metric space, Fuzzy sets and systems, 93(1),113(1998),pp 99-111.