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RADICAL TRANSVERSAL LIGHTLIKE HYPERSURFACES OF KAEHLER NORDEN MANIFOLD WITH TOTALLY UMBILICAL SCREEN DISTRIBUTIONS

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ABSTRACT

In this paper we study the geometry of Radical transversal lightlike hypersurface of Kaehler Norden manifold with totally umbilical screen distributions. Our main result is a classi_cation theorem for radical transversal lightlike hypersurfaces of Kaehler Norden manifold with totally umbilical screen distribution.

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I. INTRODUCTION

The theory of Almost Complex manifolds with Norden metric was introduced by A. P. Norden [13]. Further, Ganchev et al.[6],[7], [8] studied Kaehler manifolds with Norden metric and developed the theory of their holomorphic hypersurfaces with constant totally real sectional curvatures. The geometry of an indefinite almost Hermitian manifold is completely different from the geometry of an almost complex manifold with Norden metric. The difference arises due to the behaviour of an almost complex structure $\bar{J}$ which is an isometry with respect to the semi-Riemannian metric $\bar{g}$ in first case and is an anti-isometry with respect to the metric $\bar{g}$ in the second case.

The general theory of lightlike submanifolds that of classical theory of non-degenerate submanifolds. Since in case of lightlike submanifold, the intersection of tangent bundle and the normal bundle called the radical distribution is non-trivial, there are more difficulties in studying lightlike submanifolds than in the non-degenerate case. The lightlike geometry has been developed by K. L. Duggal and A. Bejancu [1][4]. Many geometers have proved various important results for lightlike hypersurfaces using the lightlike theory introduced by Duggal and Bejancu. A classification theorem of lightlike hypersurface with totally umbilical screen distribution of a semi Riemannian space form has been proved by D. H. Jin [9]. Recenty, Nakova [10] clubbed together the theory of almost complex manifolds with Norden metric and the geometry of lightlike submanifolds. In particular, she studied submanifolds of an almost complex manifold with Norden metric which are non-degenerate with respect to the one Norden metric and lightlike with respect to the other Norden metric on the manifold. Further, Nakova [11] has proved that the induced Ricci tensor on (M, g) is symmetric for radical transversal lightlike hypersurfaces of Kaehler norden manifold.

In this paper, we study the geometry of radical transversal lightlike hypersurfaces of Kaehler norden manifold with totally umbilical screen distributions. We prove that for totally umbilical

IJARSE
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screen distribution, local fundamental form of M or $S(TM)$ vanishes. Finally, we have proved that every null plane of pointwise tangent space has zero null sectional curvature.

II. PRELIMINARIES

2.1 Almost complex manifolds with Norden metric

An almost complex manifold with Norden metric $(\bar{M}, \bar{J}, \bar{g})$ is defined to be an even dimensional differentiable manifold $\bar{M}$ endowed with an almost complex structure $\bar{J}$ and a pseudo-Riemannian metric $\bar{g}$ on $\bar{M}$ such that

$$\bar{J}^2 X = -X, \quad \bar{g}(\bar{J}X, \bar{J}Y) = -\bar{g}(X, Y), \tag{1}$$

for all differentiable vector fields X, Y on $\bar{M}$. The associated metric $\bar{\tilde{g}}$ of $\bar{g}$ on $\bar{M}$ defined by

$$\bar{\tilde{g}}(X, Y) = \bar{g}(\bar{J}X, Y) \tag{2}$$

is a Norden metric, too on $\bar{M}$. Both metrics $\bar{g}$ and $\bar{\tilde{g}}$ are necessarily of neutral signature. Let $\bar{\nabla}$ and $\bar{\tilde{\nabla}}$ be the Levi-Civita connection of $\bar{g}$ and $\bar{\tilde{\nabla}}$ respectively. An almost complex manifold with Norden metric is said to be *Kaehler Norden manifold* if $\bar{\nabla} \bar{J} = 0$. Let $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$ be a Kaehler Norden manifold. The curvature tensors $\bar{R}$ of type $(0, 4)$ is defined by

$$\bar{R}(X, Y, Z, W) = \bar{g}(\bar{R}(X, Y)Z, W)$$

for all $X, Y, Z, W \in TM$ and has the property

$$\bar{R}(X, Y, Z, W) = -\bar{R}(X, Y, JZ, JU)$$

The curvature tensor $\bar{\tilde{R}}$ is defined by

$$\bar{\tilde{R}}(X, Y, Z, W) = \bar{R}(X, Y, Z, JW)$$

In the geometry of Kaehler Norden manifolds the following tensors are essential :

$$\begin{aligned}
\bar{\pi}_1(\bar{X}, \bar{Y}, \bar{Z}, \bar{W}) &= \bar{g}(\bar{Y}, \bar{Z})\bar{g}(\bar{X}, \bar{W}) - \bar{g}(\bar{X}, \bar{Z})\bar{g}(\bar{Y}, \bar{W}), \\
\bar{\pi}_2(\bar{X}, \bar{Y}, \bar{Z}, \bar{W}) &= \bar{g}(\bar{Y}, \bar{J}\bar{Z})\bar{g}(\bar{X}, \bar{J}\bar{W}) - \bar{g}(\bar{X}, \bar{J}\bar{Z})\bar{g}(\bar{Y}, \bar{J}\bar{W}), \\
\bar{\pi}_3(\bar{X}, \bar{Y}, \bar{Z}, \bar{W}) &= -\bar{g}(\bar{Y}, \bar{Z})\bar{g}(\bar{X}, \bar{J}\bar{W}) + \bar{g}(\bar{X}, \bar{Z})\bar{g}(\bar{Y}, \bar{J}\bar{W}) \\
&\quad -\bar{g}(\bar{Y}, \bar{J}\bar{Z})\bar{g}(\bar{X}, \bar{W}) + \bar{g}(\bar{X}, \bar{J}\bar{Z})\bar{g}(\bar{Y}, \bar{W}),
\end{aligned} \tag{3}$$

$\bar{X}, \bar{Y}, \bar{Z}, \bar{W} \in T_p\bar{M}, p \in \bar{M}$. The following two sectional curvatures are defined for every non-degenerate section $\beta = span\{X, Y\}$ with respect to $\bar{g}$ in $T_p\bar{M}$, $p \in \bar{M}$,

$$K(\beta; p) = \frac{\bar{R}(\bar{X}, \bar{Y}, \bar{Y}, \bar{X})}{\bar{\pi}_1(\bar{X}, \bar{Y}, \bar{Y}, \bar{X})}, \qquad \tilde{K}(\beta; p) = \frac{\bar{R}(\bar{X}, \bar{Y}, \bar{Y}, \bar{J}\bar{X})}{\bar{\pi}_1(\bar{X}, \bar{Y}, \bar{Y}, \bar{X})},$$

A section β in T_pM is said to be *holomorphic* if $\bar{J}\beta = \beta$ and its sectional curvature is called a holomorphic sectional curvature. A section β is said to be *totally real* with respect to $\bar{g}$ if $\beta \neq \bar{J}\beta$ and $\beta \perp \bar{J}\beta$.

An indefinite Kaehler manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$ of constant holomorphic sectional curvature c is called an indefinite *complex space form* and is denoted by $\bar{M}(c)$. So, we are considering the corresponding totally real sectional curvatures $K(\beta; p)$ and $\tilde{K}(\beta; p)$ with respect to $\bar{g}$.

IJARSE
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Theorem 2.1. *[3] Let $(\bar{M}, \bar{J}, \bar{g}, \bar{\tilde{g}})$ $(dim\bar{M} = 2n \geq 4)$ be a Kaehler manifold with Norden metric. M is of pointwise constant totally real sectional curvatures $\bar{c}$ and $\bar{\tilde{c}}$ with respect to $\bar{g}$, i.e.*

$$K(\beta, p) = \bar{c}(p), \qquad \tilde{K}(\beta, p) = \bar{\tilde{c}}(p)$$

for an arbitrary non degenerate totally real 2-plane β in T_pM, $p \in M$, if and only if

$$\bar{R} = \bar{c}(\bar{\pi}_1 - \bar{\pi}_2) + \bar{\tilde{c}}\bar{\pi}_3.$$

Both functions $\bar{c}$ and $\bar{\tilde{c}}$ are constant if $\bar{M}$ is connected and $dimM \geq 6$.

2.2 Lightlike hypersurfaces of semi-Riemannian manifolds

In this section, we follow [4][5] for the notations and fundamental equations for lightlike hypersurfaces of semi-Riemannian manifolds. Let M be a hypersurface of an $(m+2)$-dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$. M is called a *lightlike hypersurface* of $\bar{M}$ if at any $x \in M$ the tangent space T_xM and the normal space $T_xM^{\perp}$ have a non-empty intersection denoted by $RadT_xM$. $RadTM$ is called the *radical distribution* on M. Since for a hypersurface $dim(T_xM^{\perp}) = 1$ it follows that $dim(RadT_xM) = 1$ and $RadT_xM = T_xM^{\perp}$. Thus, the induced metric g by $\bar{g}$ on a lightlike hypersurface M has a constant rank m. Thus, there exists a non-degenerate complementary vector bundle $S(TM)$ of the normal bundle $TM^{\perp}$ in TM, which is called the *screen distribution* on M. Thus we have the following decomposition of TM

$$TM = S(TM) \oplus TM^{\perp} \tag{4}$$

where $\oplus$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $(M, g, S(TM))$.

Theorem 2.2. *[4] Let $(M, g, S(TM))$ be a lightlike hypersurface of a semi-Riemannian manifold $(\bar{M}, \bar{g})$, Then there exists a unique lightlike vector bundle $tr(TM)$ of rank 1 over M, such that for any non-zero section ξ of $TM^{\perp}$ on a coordinate neighborhood $u \in M$, there exists a unique section N of $tr(TM)$ on u satisfying:*

$$\bar{g}(N, \xi) = 1, \qquad \bar{g}(N, N) = \bar{g}(N, W) = 0, \qquad \forall W \in (S(TM)|_u). \tag{5}$$

Hence for any screen distribution $S(TM)$ we have a unique bundle $tr(TM)$ which is the complementary vector bundle to TM in $T\bar{M}|_M$ and satisfies (5). Then we have the following decompositions:

$$T\bar{M}|_M = S(TM) \oplus (Rad(TM) \oplus tr(TM)) = TM \oplus tr(TM). \tag{6}$$

We call $tr(TM)$ and N the transversal vector bundle and the null transversal vector field of M with respect to $S(TM)$ respectively.

Let P be the projection morphism of TM on $S(TM)$ with respect to the decomposition (4) and $\bar{\nabla}$ be the Levi-Civita connection of $\bar{M}$. For any $X, Y \in \Gamma(TM), N \in \Gamma(tr(TM))$ and $\xi \in \Gamma(Rad(TM))$, the Gauss and Weingarten formulas of M and $S(TM)$ are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N, \tag{7}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N, \tag{8}$$

and

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi \tag{9}$$

$$\nabla_X \xi = -A^*_{\xi} X - \tau(X)\xi \tag{10}$$

IJARSE
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respectively, where ∇ and ∇^* are the induced connections on TM and $S(TM)$ respectively, B and C are called locally second fundamental forms of M and $S(TM)$ respectively. A_N and A^*_ξ are linear operators on TM and $S(TM)$ respectively and τ is a 1-form on TM defined by $\tau(X) = \bar{g}(\bar{\nabla}_X N, \xi)$. Since $\bar{\nabla}$ is a torsion-free and metric connection on $\bar{M}$, it is easy to see that B is symmetric and independent of the choice of a screen distribution and satisfies

$$B(X, \xi) = 0, \forall X \in \Gamma(TM) \tag{11}$$

Denote a local 1-form η by $\eta(X) = g(X, N)\ \forall X \in \Gamma(TM)$, then the induced metric g on M satisfies

$$(\nabla_X g)(Y, Z) = B(X, Y)\eta(Z) + B(X, Z)\eta(Y), \tag{12}$$

for all $X, Y, Z \in \Gamma(TM)$, which means that ∇ is not a metric connection on M. But a simple computation implies that ∇^* is a metric connection on $S(TM)$. The above local second fundamental forms B and C of M and $S(TM)$ are related to their shape operators by

$$B(X, Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{13}$$

$$C(X, PY) = g(A_N X, PY), \qquad \bar{g}(A_N X, N) = 0, \tag{14}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma(tr(TM))$. From the above equations we see that A^*_ξ and A_N are $\Gamma(S(TM))$-valued shape operators related to B and C respectively, and A^*_ξ is self-adjoint on TM such that

$$A^*_\xi \xi = 0. \tag{15}$$

Donate by $\bar{R}$, R and R^* the curvature tensor of semi-Riemannian connection $\bar{\nabla}$ of $\bar{M}$, the induced connection ∇ on M and the induced connection ∇^* on $S(TM)$ respectively, we obtain the following Gauss-Codazzi equations for M and $S(TM)$ such that, for any vector fields $X, Y, Z, W \in \Gamma(TM)$

$$\bar{g}(\bar{R}(X, Y)Z, PW) = g(R(X, Y)Z, PW) + B(X, Z)C(Y, PW) - B(Y, Z)C(X, PW), \tag{16}$$

$$\bar{g}(\bar{R}(X, Y)Z, \xi) = g(R(X, Y)Z, \xi) = (\nabla_X B)(Y, Z) - (\nabla_Y B)(X, Z) + B(Y, Z)\tau(X) - B(X, Z)\tau(Y), \tag{17}$$

$$\bar{g}(\bar{R}(X, Y)Z, N) = g(R(X, Y)Z, N), \tag{18}$$

$$g(R(X, Y)PZ, PW) = g(R^*(X, Y)PZ, PW) + C(X, PZ)B(Y, PW) - C(Y, PZ)B(X, PW) \tag{19}$$

$$g(R(X, Y)PZ, N) = (\nabla_X C)(Y, PZ) - (\nabla_Y C)(X, PZ) + C(X, PZ)\tau(Y) - C(Y, PZ)\tau(X), \tag{20}$$

for all $X, Y, Z, W \in \Gamma(TM)$, $\xi \in \Gamma(Rad(TM))$ and $N \in \Gamma(tr(TM))$.

2.3 Radical transversal lightlike hypersurfaces

Let $(M, g, S(TM), S(TM)^\perp)$ be a lightlike hypersurface of an almost complex manifold with Norden metric $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$. M is called *Radical transversal lightlike hypersurfaces* if

$$\bar{J}(RadTM) = ltr(TM) \tag{21}$$

$$\bar{J}(S(TM)) = S(TM) \tag{22}$$

IJARSE
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Further, we consider a radical transversal lightlike hypersurface $(M, g, S(TM))$ of a Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$as defined in [11]. Let $\{\xi, N\}$ be the the pair on (M, g) which satisfies condition (5) Using the definition of M, we have

$$\bar{J}\xi = bN$$

, where $b \in \Gamma(\bar{M})$. For an arbitrary $X \in \Gamma(TM)$ we have the following decomposition

$$X = PX + \eta(X)\xi$$

and we obtain

$$\bar{J}X = \bar{J}(PX) + \eta(X)bN$$

. Since $S(TM)$ is holomorphic with respect to $\bar{J}$, it follows that $\bar{J}(PX)$ belongs to $S(TM)$.

III. TOTALLY UMBILICAL SCREEN DISTRIBUTIONS

$S(TM)$ is said to be totally umbilical in M if on any co-ordinate neighborhood $u \in M$, there is a smooth function ρ such that

$$C(X, PY) = \rho g(X, Y), \forall X, Y \in \Gamma(TM). \tag{23}$$

In case $\rho = 0$ on u, we say that $S(TM)$ is totally geodesic. In general, $S(TM)$ is not necessarily integrable. In case of Kaehler Norden manifold, the following result give equivalent conditions for the integrability of $S(TM)$.

Theorem 3.1. *[11] Let $(M, g, S(TM))$ be a Radical transversal lightlike hypersurface of a Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$. Then the following are equivalent*

1. *$S(TM)$ is integrable*
2. *C is symmetric on $\Gamma(S(TM))$*
3. *A_N is self-conjugate on $\Gamma(S(TM))$ with respect to g.*

Let $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$ be a Kaehler Norden manifold having constant totally real sectional curvatures $\bar{c}$ and $\bar{\bar{c}}$ with respect to $\bar{g}$ and $S(TM)$ a totally umbilical screen distribution of M. then the equation (17) reduces to

$$(\nabla_X B)(Y, Z) - (\nabla_Y B)(X, Z) = B(X, Z)\tau(Y) + B(Y, Z)\tau(X) \tag{24}$$

using (12), (18), (20) and (23), for any $X, Y, Z \in \Gamma(TM)$, we get

$$\rho\gamma B(X, Y) = \{\zeta[\rho] + \rho\tau(\zeta) + \bar{\bar{c}}\}g(X, Y) + \bar{c}g(X, JY) \tag{25}$$

Theorem 3.2. *Let $(M, g, S(TM))$ be an $(m+1)(m > 2)$-dimensional Radical lighlike hypersurface of a Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$ of constant totally real sectional curvatures $\bar{c}$ and $\bar{\bar{c}}$ with respect to $\bar{g}$. Then the second fundamental form C or B vanishes if $S(TM)$ is totally umbilical. Moreover,*

1. *If $C = 0$ then $S(TM)$ is totally geodesic and $\bar{\bar{c}} = 0$*
2. *If $B = 0$ then $S(TM)$ is totally geodesic immersed in $\bar{M}$ and the induced connection $\tilde{\nabla}$ on M is a metric connection.*

IJARSE
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ISSN (P) 2319 - 8346

Proof. Assume that $C \neq 0$ i.e., $\rho \neq 0$. Then from we have

$$B(X,Y) = \beta g(X,Y) + \bar{c}g(X,JY) \tag{26}$$

for all $X, Y \in \Gamma(TM)$, where $\beta = \rho^{-1}\{\zeta[\rho] + \rho\tau(\zeta) + \bar{\bar{c}}\}$. As $S(TM)$ is totally umbilical, M is locally a product manifold. From above equations, for any $X, Y, Z \in \Gamma(S(TM))$ we have

$$R^*(X,Y)Z = (\bar{c} - J\bar{\bar{c}} + 2\rho\beta)(g(Y,Z)X - g(X,Z)Y) + (J\bar{c} + \bar{\bar{c}})(\bar{g}(X,JZ)Y - \bar{g}(Y,JZ)X)$$

Again, if we consider X, Y, Z be orthogonal vectors, then we get the following expression

$$R^*(X,Y)Z = (c + 2\rho\beta)(g(Y,Z)X - g(X,Z)Y)$$

where $c = (\bar{c} - J\bar{\bar{c}})$

$$Ric^*(X,Y) = (c + 2\rho\beta)(m-1)g(X,Y), \forall X, Y \in \Gamma(S(TM))$$

. As $m > 2$, so, M^* is an Einstein manifold of constant curvature $(c + 2\rho\beta)$ From (25), we have

$$\zeta[\rho] = \beta\rho - \rho\tau(\zeta) - \bar{\bar{c}}$$

Differentiating (26) and using (12) and (24), for all $X, Y, Z \in \Gamma(S(TM))$, we have

$$\{X[\rho] - \rho^2\eta(X) + \rho\tau(X)\}g(Y,Z) - \{Y[\rho] - \rho^2\eta(Y) + \rho\tau(Y)\}g(X,Z) = 0 \tag{27}$$

Replacing X by ζ in this equation, we have

$$\zeta[\beta] = \beta^2 - \beta\tau(\zeta).$$

Since $(c + 2\rho\beta)$ is a constant, therefore, we get $\beta = 0$ or $(c + 2\rho\beta) = 0$. If $c + 2\rho\beta = 0$, then M^* is a semi-Euclidean space and the second fundamental form C of M^* satisfies $C = 0$. It is a contradiction to $C \neq 0$. Thus we have $\beta = 0$. Consequently, we get $B = 0$ by (26). Thus M is totally geodesic in $\bar{M}$. Also, from the equation (12), we see that $(\nabla_X g)(Y,Z) = 0 \forall X, Y, Z \in \Gamma(TM)$, that is, the induced connection $\tilde{\nabla}$ on M is a metric connection. If $C = 0$, i.e., $\rho = 0$, then, by (25), we have $\bar{\bar{c}} = 0$. Thus we have our main theorem. □

The induced Ricci type tensor $R^{(0,2)}$ of M is defined by

$$R^{(0,2)}(X,Y) = trace\{Z \to R(Z,X)Y\}, \forall X, Y \in \Gamma(TM). \tag{28}$$

Consider the induced quasi-orthonormal frame field $\{\zeta; W_a\}$ on M such that $Rad(TM) = Span\{\zeta\}$ and $S(TM) = Span\{W_a\}$. Using this frame field and the equation (**??**), we obtain

$$R^{(0,2)}(X,Y) = \sum_{a=1}^{m} \epsilon_a g(R(W_a,X)Y, W_a) + \bar{g}(R(\zeta,X)Y, N), \tag{29}$$

for any $X, Y \in \Gamma(TM)$ and $\epsilon_a = g(W_a, W_a)$.

Definition 3.3. *A tensor field $R^{(0,2)}$ of lightlike hypersurfaces M is called induced* Ricci tensor *of M if it is symmetric. A symmetric $R^{(0,2)}$ tensor will be denoted by Ric.*

In general, the induced Ricci type tensor $R^{(0,2)}$ of the non-degenerate submanifolds[12], is not symmetric. So, we are proving the following theorem

Theorem 3.4. *Let $(M, g, S(TM))$ be an $(m+1)(m>2)$-dimensional Radical lighlike hypersurface of a Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$ of constant totally real sectional curvatures $\bar{c}$ and $\bar{\bar{c}}$ with respect to $\bar{g}$. Then M admits an induced symmetric Ricci tensor Ric for totally umbilical $S(TM)$. Both M and the leaf M^* of $S(TM)$ are spaces of constant curvature c.*

Proof. Using (16), (17), (18) and (29), we have $R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}$, $R^{(0,2)}(X,Y) = mcg(X,Y)$, for any $X, Y, Z \in \Gamma(TM)$, by using Theorem 3.2, we have $\beta\rho = 0$. Thus $R^{(0,2)}$ is a symmetric Ricci tensor Ric and M is a space of constant curvature c. Also, from (16) and (19), we have $R^*(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}$, $Ric^*(X,Y) = (m-1)cg(X,Y)$, for $X, Y, Z \in \Gamma(S(TM))$. Also M^* is a space of constant curvature c. □

Recall the following notion of null sectional curvature[2][4]. Let $m \in M$ and ξ be a null vector of T_mM. A plane H of T_mM is called a null plane directed by ξ if it contains ξ, $g_m(\xi, W) = 0$ for any $W \in H$ and there exists $W_0 \in H$ such that $g_m(W_0, W_0) \neq 0$. Then, the null sectional curvature of H, with respect to ξ and the induced connection $\bar{\nabla}$ of M, is defined as a real number

$$K_\xi(H) = \frac{g_x(R(W,\xi)\xi, W)}{g_x(W,W)},$$

It is easy to see that $K_\xi(H)$ is independent of W but depends on ξ in a quadratic manner. Also, we know that[13], an $n(\geq 3)$-dimensional Lorentzian manifold is of constant curvature if and only if its null sectional curvatures are everywhere zero.

Theorem 3.5. *Let $(M, g, S(TM))$ be an $(m+1)(m>2)$-dimensional radical lighlike hypersurface of a Kaehler Norden manifold $(\bar{M}, \bar{J}, \bar{g}, \bar{\bar{g}})$ of constant totally real sectional curvatures $\bar{c}$ and $\bar{\bar{c}}$ with respect to $\bar{g}$ with totally umbilical $S(TM)$. Then every null plane H of T_mM directed by ξ has everywhere zero null sectional curvatures.*

Proof. From (16) and the fact that $\beta\rho = 0$, we show that

$$g(R(X,Y)Z, PW) = c\{g(Y,Z)g(X,PW) - g(X,Z)g(Y,PW)\}$$

for any $X, Y, Z, W \in \Gamma(TM)$. Thus

$$K_\xi(H) = \frac{g_x(R(W,\xi)\xi, W)}{g_x(W,W)} = 0$$

for any null plane H of T_mM directed by ξ. □

REFERENCES

[1] A. Bejanchu, K. L. Duggal, Degenerate hypersurface of Semi-Riemannian manifolds, Bull. Inst. Politehnie Iasi, 37 (1991) 13-22.

[2] J. K. Beem, P. E. Ehrlich, and K. L. Easley, Global Lorentzian Geometry, Marcel Dekker, Inc. New York, Second Edition, (1996).

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

[3] A. Borisov, G. Ganchev, Curvature properties of Kaehlerian manifolds with B-metric, Proc. of 14 Spring Conference of UBM, Sunny Beach (1985), 220-226.

[4] K. L. Duggal, A. Bejancu, Lightlike Submanifolds of Semi-Riemannian Manifolds and Appli-cations, 364 of Mathematics and its Applications, Kluwer Academic Publishers, Dordrecht, The Netherlands, (1996).

[5] K. L. Duggal, B. Sahin, Di erential Geometry of Lightlike Submanifolds, Birkhauser, Verlag, (2010).

[6] G. Ganchev, A. Borisov, Note on the almost complex manifolds with a Norden metric, Compt. Rend. Acad. Bulg. Sci., 39(5) (1986), 31-34.

[7] G. Ganchev, K. Gribachev, V. Mihova, Holomorphic hypersurfaces of Kaehler manifolds with Norden metric, Plovdiv Univ. Sci. Works Math., 23(2) (1985), 221-236.

[8] K. I. Gribachev, D. G. Mekerov, G. D. Dzhelepov, Generalized B-manifolds, Compt. Rend. Acad. Bulg. Sci., 38(3) (1985), 299-302.

[9] D. H. Jin, Lightlike hypersurface with totally umbilical screen distribution, J. Chungcheong Math. Soc., 24(4) (2009), 409-415.

[10] G. Nakova, Some lightlike submanifolds of almost complex manifolds with Norden metric, J.Geom., 103 (2012), 293-312.

[11] G. Nakova, Radical transversal lightlike hypersurfaces of almost complex manifolds with Norden metric, J. Geom., 104 (2013), 539-556.

[12] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, (1983).

[13] A. P. Norden, On a class of four-dimensional A-spaces, Russian Math., 17(4) (1960), 145-157.

[14] B. Sahin, Transversal lightlike submanifolds of indenite Kaehler manifolds, Analele Universitatiide Vest Timisoara Seria Matematica Informatica XLIV(1) (2006), 119-145.

[15] K. Yano, M. Kon, CR-Submanifolds of Kaehlerian and Sasakian Manifolds, Birkhauser, Basel,(1983).