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PHASE PLANE ANALYSIS OF A PECULIAR CASE OF ECOLOGICAL AMMENSALISM

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ABSTRACT

In this paper, the stability nature of Ecological Ammensalism with two species(Ammesnal and Enemy species)is discussed. Both the species are strengthened with unlimited resources. This model is developed by a couple of first order non linear differential equations. The behavior of this model is established with Phase plane analysis.

Keywords: Ammensalism, Stability and threshold diagrams.

I. INTRODUCTION

Mathematical modeling in Ecosystem has been giving fruitful solutions for many complex situations in nature. The concept was thoroughly introduced by by Lotka [10] and Volterra [15].Many useful concepts of modeling have been developed by f Meyer [11], Cushing [5], Gause [7], Paul Colinvaux [12], Haberman [8], Pielou [13], Thompson [14], Freedman [6], Kapur [9] etc. Later Pattabhi Ramacharyulu , Acharyulu [1-4] studied the nature of different types of Ammensalism. The behavior of unlimited Ecological Ammensal Model is observed with Phase plane analysis.

II. NOTATIONS ADOPTED

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) :The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{12} :The inhibition coefficient of S_1 due to S_2 i.e The Commensal coefficient.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{12} are assumed to be non-negative constants.

III. BASIC EQUATIONS

The basic equations are given as

$$\frac{dN_1}{dt} = a_1 N_1 - a_{12} N_1 N_2 \quad (1)$$

$$\frac{dN_2}{dt} = a_2 N_2 \quad \text{with initial conditions } N_1(0)=\mathbf{c}_1 \text{ and } N_2(0)=c_2 \quad (2)$$

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In this model, only Fully washed out state is occurred.

The corresponding equilibrium point is $\overline{N}_1 = 0$; $\overline{N}_2 = 0$

By the concept of linearization ,

$$\frac{dU_1}{dt} = a_1 U_1 \text{ and } \frac{dU_2}{dt} = a_2 U_2 \quad (3)$$

and the characteristic equation is $(\lambda - a_1)(\lambda - a_2) = 0$

the roots of which are $\lambda = a_1$, $\lambda = a_2$ i.e. both the roots are negative,

Hence the steady state is **unstable**.

The solutions are obtained as $U_1 = U_{10}\ e^{a_1 t}$ and $U_2 = U_{20}\ e^{a_2 t}$. (4)

Now , the nature of this model is discussed with Phase plane analysis with the considered conditions.

Case(i): When $a_1 = 0.1$, $a_{12} = 0.5$ and $a_2 = 0.5$, The Null clines and Trajectories are shown in the Fig.(1) and Fig.(2) respectively.

In this case, The Eigen values are 0.5 and 1 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.1 & 0 \\ 0 & 0.5 \end{pmatrix}$$

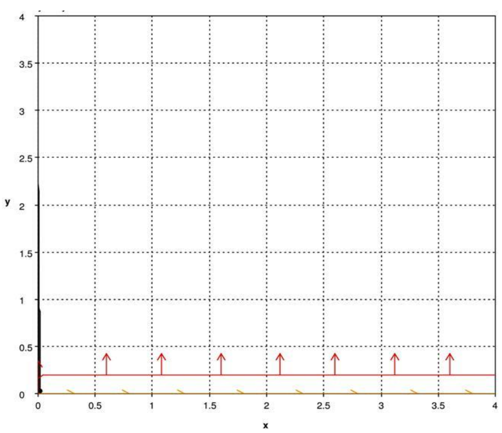

Fig.(1)

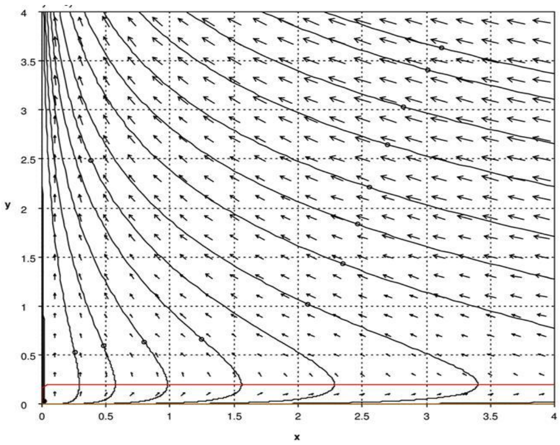

Fig.(2)

Case(ii): When $a_1 = 1$, $a_{12} = 0.5$ and $a_2 = 0.5$, The Null clines and Trajectories are shown in the Fig.(3) and Fig.(4) respectively.

In this case, The Eigen values are 1 and 0.5 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix}$$

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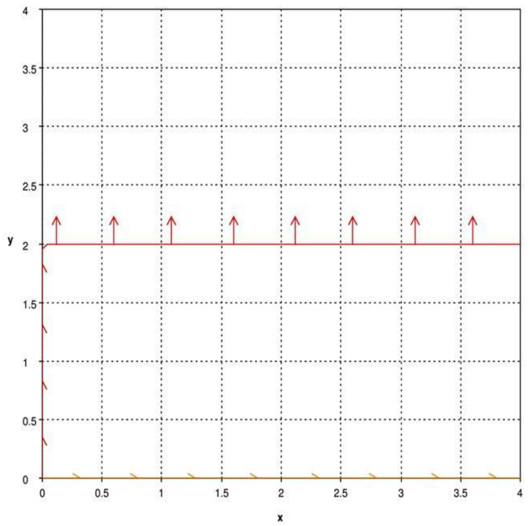


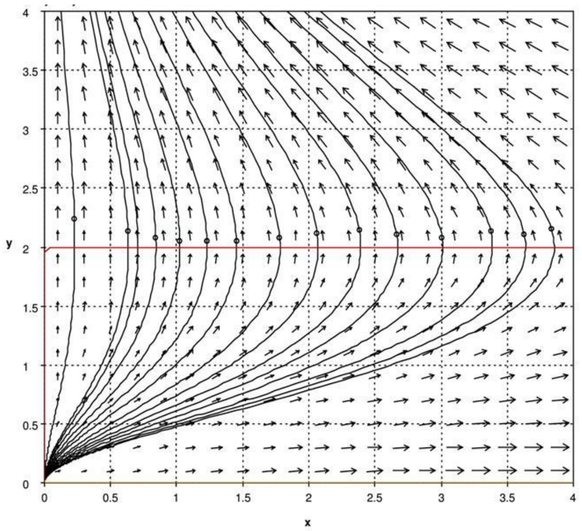
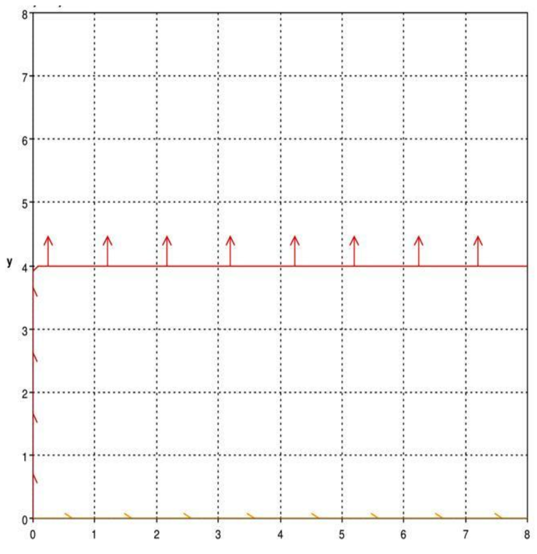


Fig.(3) Fig.(4)

Case(iii).: When a_1= 2, a_{12}=0.5 and a_2=0.5, The Null clines and Trajectories are shown in the Fig.(5) and Fig.(6) respectively.

In this case, The Eigen values are 2 and 0.5 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 2 & 0 \\ 0 & 0.5 \end{pmatrix}$$



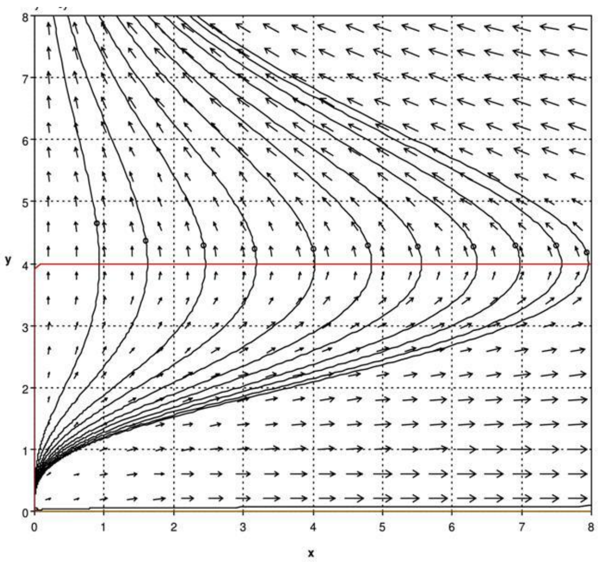
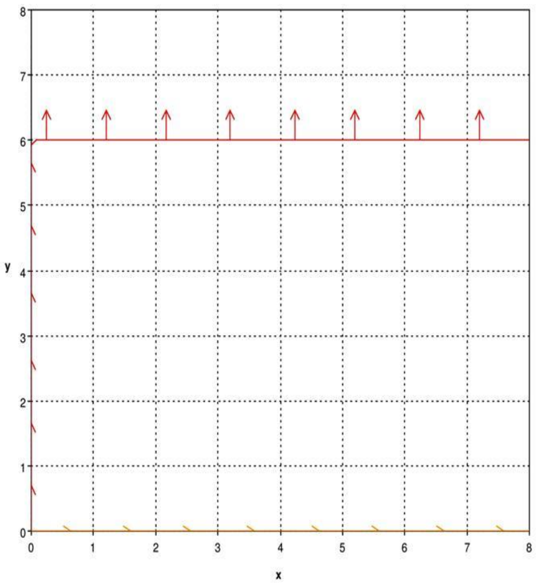


Fig.(5) Fig.(6)

Case(iv): When a_1= 3, a_{12}=0.5 and a_2=0.5, The Null clines and Trajectories are shown in the Fig.(7) and Fig.(8) respectively.

In this case, The Eigen values are 3 and 0.5 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 3 & 0 \\ 0 & 0.5 \end{pmatrix}$$



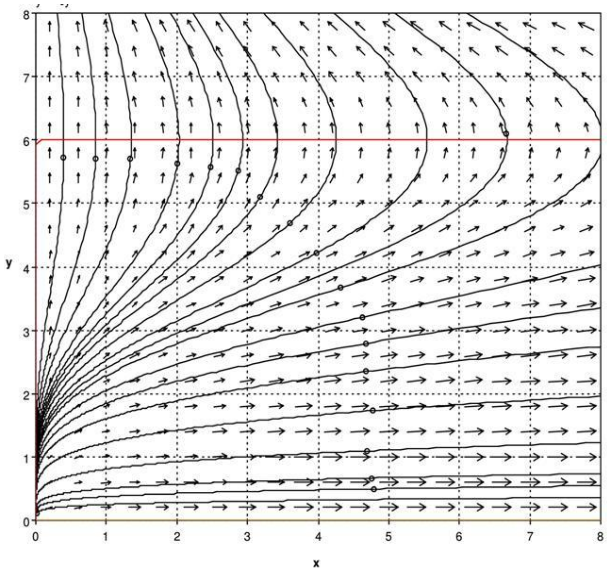
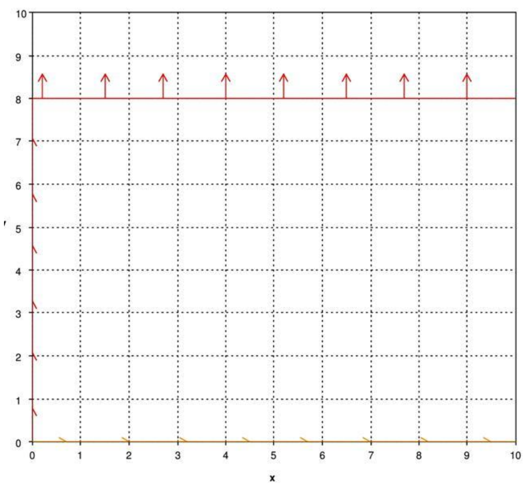


Fig.(7) **Fig.(8)**

Case(v): When a_1= 4, a_{12}=0.5and a_2=0.5, The Null clines and Trajectories are shown in the Fig.(9) and Fig.(10) respectively.

In this case, The Eigen values are 3 and 0.5 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 4 & 0 \\ 0 & 0.5 \end{pmatrix}$$



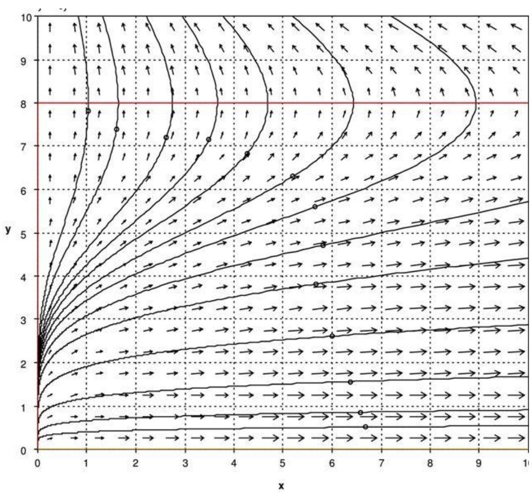


Fig.(9) **Fig.(10)**

Case (vi): When a_1= 8, a_{12}=0.5 and a_2=0.5, The Null clines and Trajectories are shown in the Fig.(11) and Fig.(12) respectively. In this case, The Eigen values are 3 and 0.5 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is $\begin{pmatrix} 8 & 0 \\ 0 & 0.5 \end{pmatrix}$

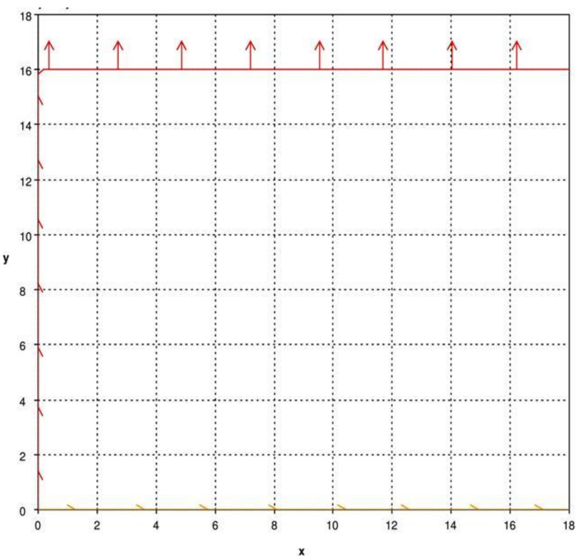


Fig.(11)

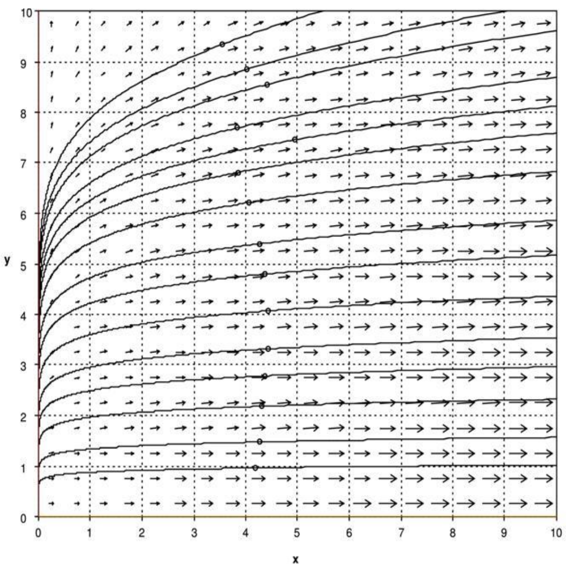


Fig.(12)

IV. CONCLUSIONS

The original nature of unlimited Ecological Ammensalism is unstable. The nature can be changed by increasing the growth rate of Ammensal Species with the fixed growth rate of enemy species to become neutrally stable. No considerable influence is identified by the Ammensal coefficient.

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