



# OPTIMIZING HETEROGENEOUS FLEETVEHICLE ROUTING PROBLEM

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## ABSTRACT

The objective of this paper is to propose a superior algorithm for Capacitated Vehicle Routing Problem (CVRP). The Vehicle Routing Problem (VRP) was first confronted and described by Dantzig and Ramser (1959) in their article on truck dispatching problem. This problem further has been enhanced as Capacitated Vehicle Routing Problem wherein a set of customer demand is served with a homogeneous fleet of vehicles (single capacity vehicles) from depot or a central node. Clarke and Wright (CW) was the pioneer who proposed a well-structured heuristic savings algorithm for solving Capacitated Vehicle Routing Problem in 1964. Since then several enhancement of the classical CW formulation of CVRP were proposed by researchers by introducing additional parametric terms which accounts for variations in 'distribution', 'distance', 'pick up time' etc. However, it was found that the real time problems were still more complex and are dependent on nonlinear constraints like 'work risks', 'geographical restriction', 'balance workload among routes', 'solution attractiveness' etc. This resulted in a need for more flexible methods to be able to provide a large set of alternative near optimal solution for a Capacitated Vehicle routing problem. It is in this context we propose here a more flexible option for CVRP with introduction of heterogeneous fleet or mixed fleet (variable capacity vehicles, sub-routes within a specific route etc.), which hitherto was nonexistent in a structured manner in earlier researches. Moreover, we have adopted two phase selection procedure which involves sorting the savings values randomly with probability to yield improvement in the savings algorithm proposed in the literature. The above measures have dramatically improved the quality of our solution which we found are among the best.

**Keywords:** Capacitated Vehicle Routing, Heterogeneous Fleet, Route, Heuristic Savings

## I. INTRODUCTION

The main purpose of the paper is to devise an algorithm for a Capacitated Vehicle Routing problem (CVRP) that modifies the classical Clarke–Wright (CW) algorithm [1] to attain the optimal solution. Classically, a CVRP determines a set of vehicle routes to meet the demands of a set of customers who have to travel from their nearest stop to a central point. Here the transportation cost depends on the distance covered from the central point to customers. The constraints associated with CVRP are:

- Each non –depot node is supplied by a single vehicle.
- All vehicles begin and their routes at the depot (node 0)
- A vehicle cannot stop twice at the same non –depot node
- No vehicle can be loaded exceeding its maximum capacity

We propose to modify the above traditional CVRP problem which uses homogeneous fleet, with the introduction of mixed fleet. Our objective is to find a feasible set of vehicle routes that minimizes the total travelling distance with the deployment of vehicles of various capacities.

Here in this paper we have designed an algorithm to provide optimal solution which among others also manages real-time constraints. A hybrid algorithm is introduced here that combines the parallel version of CW heuristic with two phase selection procedure. This method is also flexible as to provide near-optimal solutions for the decision makers to select among vehicle with variable capacities.

For designing the algorithm, as an instance, we have collected the data and the vehicle routes of an Indian Multinational company in Mumbai.

The structuring of rest of our paper is as follows:

Section 2 describes the background and literature survey, Section 3 explains an intuitive description of our proposed approach, Section 4 analyses the experimental results and the technical details and the remaining part of the paper gives an insight to the conclusion and further work in the offing.

## **II. BACKGROUND AND LITERATURE SURVEY**

Vehicle Routing Problem (VRP) was first described by Dantzig and Ramser (1959) in their article on truck dispatching problem[2]. Since VRP is an NP Hard problem, to start with, various approaches and algorithms for tackling realtime instances have been developed. However, these methods such as Branch and Bound, Branch and Cut and Dynamic Programming etc., worked only for small instances, where the number of customers was strictly less than hundred. To surpass this limitation, Clarke and Wright (1964) proposed a heuristic algorithm composed of savings method where the number of customers is more than the number of nodes. CW algorithm is easy to implement, very fast, and obtains quite good solutions and is based on the concept of savings for combining two customers in the same route. According to [1] “A NUMBER of trucks  $x_i$  of capacity  $C_i(i=1. \dots n)$  are available and loads  $q_j$  are required to be delivered to points  $P_j(j= 1. \dots M)$  from a depot  $P_0$ . Given the distances  $d_{y,z}$  between all such points it is required to minimize the total distance covered by the trucks.”

Unfortunately, CW algorithm also had its limitations in terms of visualization of constraints. Researchers have since then introduced improvements to the CW solution to provide near optimal solution. These include proposing new parameters to CW formulation e.g.

- 1) estimating of the maximum savings value and a penalty multiplier ‘ $\alpha$ ’ for solving VRP with backhauls[3]
- 2) introducing route shape ‘ $\gamma$ ’ in the savings formula for solving CVRP to reshape the routes by taking only non-negative values introduced by Gaskell and Yellow [4],[5]
- 3) inclusion of a new term in the Gaskell’s and Yellow’s formula to exploit the asymmetry information between pair of customers regarding their distances to the depot by Paessens [6]
- 4) Enhancement to Paessens formula by introducing a third term which gives priority to customers with large demands by Altinel and Oncan.[7]
- 5) simulation study by Juan et al presented a in routing via the generalized CW,a hybrid algorithm that combining the parallel version of CW with Monte Carlo simulation.[8]

As the literature on VRP is vast and VRP is a proficient research field, a complete review can be reviewed in [9],[10],[11] due to space limitations.

### III. PROPOSED MODIFICATION TO CLARKE–WRIGHT ALGORITHM

Our approach for the proposed modification to the Clarke-Wright Algorithm is primarily focused on adopting parallel version of CW. According to Toth and Vigo, parallel version of CW algorithm provides better results than the sequential algorithm [12]. However, Classical CW uses a homogeneous fleet i.e. single capacity vehicle. This poses a limitation to the optimal solution. Instead, our approach is flexible: we have introduced a competitive approach where we have a mixed fleet of vehicles with varying capacity. The total number of vehicles of various capacity options is also kept variable by us.

Our proposed work modifies parallel version of CW with introduction of the concept of mixed (heterogeneous) fleet of vehicles. Depending on the demand on a particular day or time the user has the option to ply limited number of vehicles so that customers can be merged into the same route satisfying all the constraints of CVRP.

We have calculated all savings values in the savings list as follows:  $S_{ij} = C_{oi} + C_{oj} - C_{ij}$  where  $C_{oi}$  is the travelling distance between depot and customer  $i$  and  $C_{ij}$  is the travelling distance between customer  $i$  and  $j$ .

The savings thus calculated are then ranked in descending order and using the route merging procedure both customers  $i$  and  $j$  combined into the same route if the total demand does not exceed vehicle capacity.

The above savings list has further been modified by the two phase [13] selection procedure where the savings values  $s_{ij}$  are sorted randomly with probability. The flowchart annexed herewith in Fig 1 represents the methodology of our work.

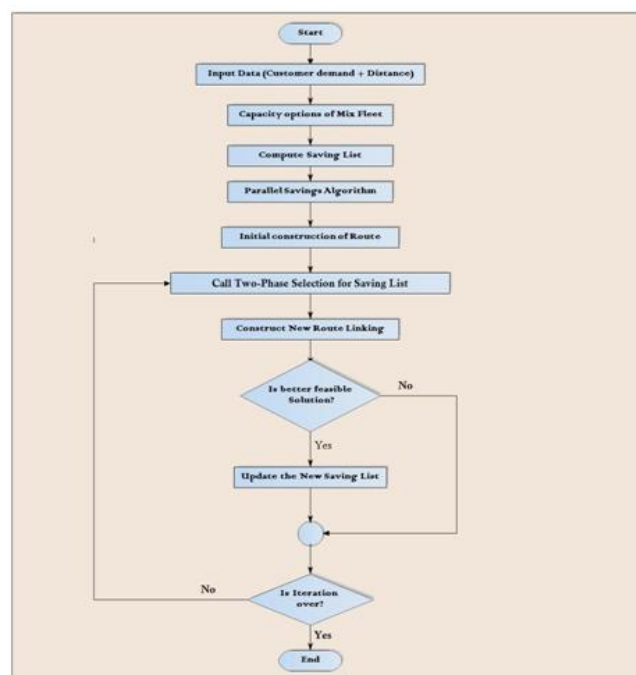


Fig-1: Flow chart of proposed algorithm

The two phase selection procedure was introduced by Pichpibul and Kawtummachai [14],[15] wherein a new savings list is generated by adapting tournament and roulette wheel selections. Adapting the combination of ideas of Goldberg et al [16] and Holland [17] two phase selection procedures has further been applied to generate a new savings list.

In the modified concept, initial savings list is represented by a single genetic chromosome. Each gene of the chromosome represents the savings values between customers  $i$  and  $j$  in the initial saving list. As for example, Fig-2a represents a single chromosome of ten savings values (genes). In Figure 2(b) our approach starts with tournament selection where the savings value is picked out from the set by roulette wheel selection process. The tournament size represented by  $T$  is a random number between two and  $\frac{N}{2} + 1$ , where  $N$  is the number of nodes in a specific vehicle route. For saving number  $n$  with savings value  $s_n$ , its selection probability  $p_n$  and cumulative probability  $q_n$  are calculated as follows:

$$p_n = s_n / \sum_{i \in T} s_i \text{ for } n \in T$$

$$q_n = \sum_{i \in n} p_n \text{ for } n \in T$$

The selection method starts by spinning the roulette wheel with a random number  $r$  between 0 and 1. If  $r \leq q_1$ , then choose the first savings value  $s_1$ ; otherwise choose the  $n$ th savings value  $s_n$  ( $2 \leq n \leq T$ ) such that  $q_{n-1} < r \leq q_n$ . The selected gene is removed from the chromosome which leaves nine savings values as shown in Fig 2(c). This selection procedure is executed until the last gene of the chromosome is selected to be a gene of the new chromosome as shown in Fig 2(d). The new chromosome will replace the previous one only if the new solution is better than the previous.



Fig-2 : Generation of new chromosome

**IV. REAL TIME ANALYSIS: TABULATION OF RESULTS**

As a test instance we have collected the data of vehicle routes of an Indian Multinational company in Mumbai. The company has 7 routes connecting total 75 nodes across Mumbai spanning about 35 kilometers for plying its employees using a homogeneous fleet (fixed capacity vehicles). We proposed a near optimal solution with mixed fleet of vehicles with variable capacity and sub-routes. The mixed fleet comprises vehicle of capacities 30, 40 and 50. The solution was extremely impressive and is under company’s active perusal for implementation.

The above instance was solved by using our proposed heuristic which provides a solution with a total cost of 883.4 units for all 7 routes. The program is implemented in Python code, Version 2.7 (open source) software on Windows XP platform. We used a standard PC with configuration i7 Intel processor with 4GB RAM. The computation to achieve the best solution took less than 10minutes despite this being a heterogeneous fleet which is often time consuming. [18].

The summarization of the final output is illustrated in Table-1.

| Route No.    | Total Demand | Buses Used | Min Total Cost |
|--------------|--------------|------------|----------------|
| 1            | 134.00       | 3          | 150.20         |
| 2            | 83.00        | 2          | 83.60          |
| 3            | 122.00       | 3          | 107.60         |
| 4            | 161.00       | 4          | 190.40         |
| 5            | 160.00       | 4          | 206.80         |
| 6            | 106.00       | 3          | 66.20          |
| 7            | 116.00       | 3          | 78.60          |
| Grand Total: | 883.40       |            |                |

Table-1 : Result of proposed algorithm

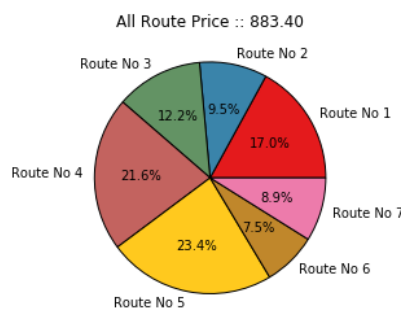


Fig-3 : Pi-chart representation

To verify the efficacy of our algorithm, we used other CVRP instances from the test instances including Auguret’s (1995) data setsA (<http://www.coin-or.org/SYMPHONY/branchandcut/VRP/data/index.htm.old#A>) The solution using our algorithm is far more impressive in terms of overall cost. The company is in active consideration to adopt the above methodology for optimizing routes with a mixed fleet of vehicles.

The allocation of vehicles in each route following our algorithm is summarized below.



| Capacity | Demand | Cost  | Path                |
|----------|--------|-------|---------------------|
| 40       | 39     | 36.00 | [0, 4, 2, 1, 0]     |
| 50       | 46     | 51.60 | [0, 5, 6, 8, 0]     |
| 50       | 49     | 62.60 | [0, 3, 9, 10, 7, 0] |

Final Cost: 150.20

Table-2a : Route1

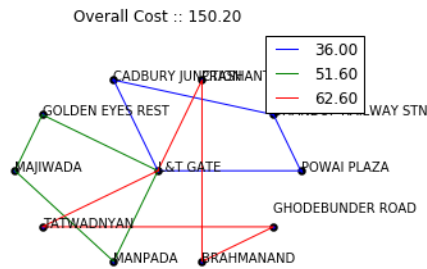


Fig-3a : Route1

| Capacity | Demand | Cost  | Path               |
|----------|--------|-------|--------------------|
| 40       | 37     | 18.00 | [0, 3, 2, 1, 0]    |
| 50       | 46     | 65.60 | [0, 5, 7, 6, 4, 0] |

Final Cost: 83.60

Table-2b : Route2

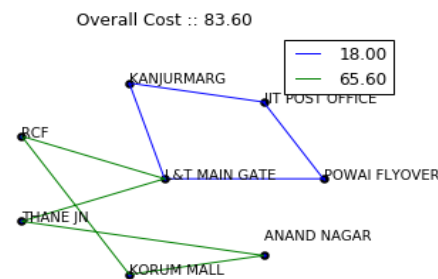


Fig-3b : Route 2



| Capacity | Demand | Cost  | Path            |
|----------|--------|-------|-----------------|
| 40       | 37     | 14.60 | [0, 3, 2, 1, 0] |
| 50       | 45     | 35.60 | [0, 6, 5, 4, 0] |
| 40       | 40     | 57.40 | [0, 7, 8, 9, 0] |

Final Cost: 107.60

Table-2c : Route3

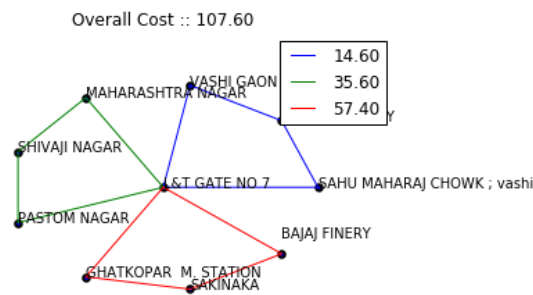


Fig-3c : Route 3

| Capacity | Demand | Cost  | Path              |
|----------|--------|-------|-------------------|
| 50       | 42     | 60.40 | [0, 11, 9, 8, 0]  |
| 40       | 37     | 22.00 | [0, 1, 2, 3, 0]   |
| 50       | 45     | 44.20 | [0, 4, 5, 6, 0]   |
| 40       | 37     | 63.80 | [0, 7, 10, 12, 0] |

Final Cost: 190.40

Table-2d : Route4

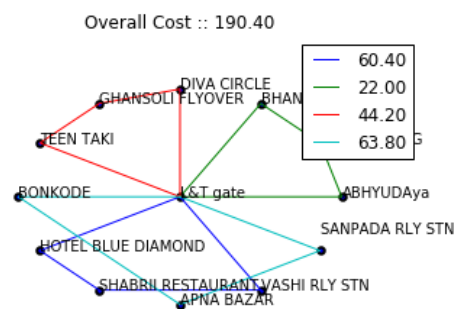


Fig-3d : Route 4



| Capacity | Demand | Cost  | Path                |
|----------|--------|-------|---------------------|
| 50       | 44     | 71.00 | [0, 10, 8, 7, 6, 0] |
| 30       | 29     | 40.60 | [0, 4, 5, 0]        |
| 50       | 45     | 20.00 | [0, 1, 2, 3, 0]     |
| 50       | 42     | 75.20 | [0, 9, 11, 0]       |

Final Cost: 206.80

Table-2e : Route5

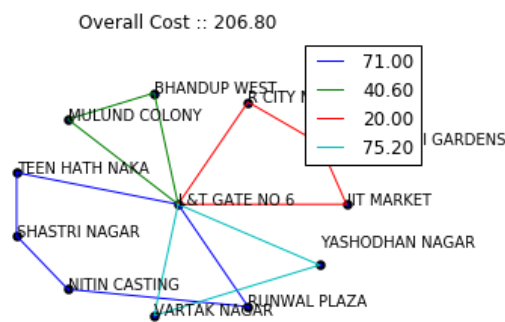


Fig-3e : Route 5

| Capacity | Demand | Cost  | Path               |
|----------|--------|-------|--------------------|
| 50       | 47     | 31.80 | [0, 6, 9, 4, 1, 0] |
| 50       | 46     | 29.20 | [0, 5, 7, 8, 3, 0] |
| 30       | 13     | 5.20  | [0, 2, 0]          |

Final Cost: 66.20

Table-2f : Route6

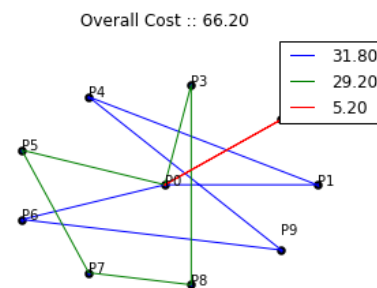


Fig-3f : Route 6





| Capacity | Demand | Cost  | Path                |
|----------|--------|-------|---------------------|
| 50       | 49     | 37.20 | [0, 9, 10, 7, 4, 0] |
| 40       | 40     | 30.20 | [0, 6, 8, 5, 1, 0]  |
| 30       | 27     | 11.20 | [0, 2, 3, 0]        |

Final Cost: 78.60

Table-2g : Route7

Overall Cost :: 78.60

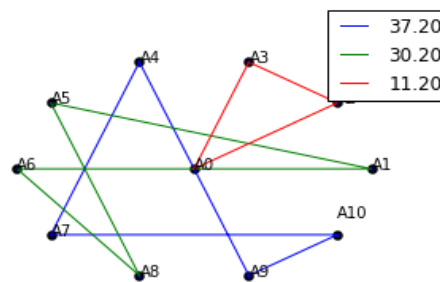


Fig-3g : Route 7

**V. CONCLUSION**

In this study we have improved the initial solution of CW savings algorithm. This paper has sought to give insight into origins of the savings method for the vehicle routing problem incepted 45 years ago. Many extensions of the basic vehicle routing problem , which takes into consideration for constraints for example, length of routes and capacity of vehicles , have been investigated using adaptations of the savings method over years , alongside many other approaches. These approaches were mostly on the homogenous fleet. We introduced a heterogeneous fleet and used two phase selection procedure where in the saving list in CW algorithm is regenerated using Tournament and Roulette wheel selection method as introduced and Goldberg and Holland respectively. This algorithm proves to be effective to deal with realistic situations where the demand of customers can be catered with a mixed fleet.

Our further work is to develop an effective hybrid genetic algorithm on resource constrained shortest path which can provide near optimal solution in a reasonable computation time based on the studies of Christian Prins.[19]

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