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VIVEC (VORTEX INDUCED VIBRATION ENERGY CONVERTER): A NEW AND RENEWABLE APPROACH TO HARNESS THE HYDRO-KINETIC ENERGY OF GEOPHYSICAL FLUID FLOW

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ABSTRACT

The current scenario of ever increasing global energy demand and the issues of global warming and climate change due to increased fossil fuels consumption has led to more focus and research on green, eco-friendly renewable energy sources. Geophysical fluid flows (ocean/river/wind) represent a widely available potential sources of clean and renewable energy, worthwhile to tackle the global energy demand using for example marine turbines, wind turbines or wave energy converters. Since, the energy density in geophysical fluid flows is small, therefore large systems are required to harvest the significant amount of energy.

Any device used to harness the plentiful clean and renewable energy from geophysical flows must have high energy density, be unobtrusive, be robust, have low maintenance, have a 10–20 years of service life and meet life cycle cost targets. The vortex induced vibration based energy converter (VIVEC)satisfies all these criteria. It converts fluid (ocean/river/wind) current hydrokinetic energy to a usable form of energy such as electricity using VIV (vortex induced vibration). VIV is the motion induced on bluff body placed in the fluid flow due to alternate vortex shedding behind it, which produces alternating lift forces on it perpendicular to fluid flow. Normally, VIV is unwanted and reduced by careful design, in order to prevent mechanical failure of the vibrating structures such as high rise buildings, offshore structures, etc. However, in energy harnessing application from fluid flow, rather than eliminating these vibrations, VIV will be exploited to transform these vibrations into a valuable resource of energy.VIVEC is based on the concept of maximizing instead of spoiling vortex shedding and exploiting rather than suppressing VIV. It incorporates optimal damping and maintains VIV over a broad range of vortex shedding frequency and thereby facilitates optimal energy conversion.The phenomenon of VIV occurs over very broad ranges of Reynolds (Re) number except for three transition regions

where VIVs are suppressed. This makes, the VIVACE can extract energy with high power conversion ratio even from water currents as slow as 0.25 m/s. Thus making ocean/river current kinetic energy a more accessible and economically viable resource of renewable energy.

This paper presents a review on the capability of VIV phenomenon in generating alternative energy from water currents and underlying concepts of the design and operation of the VIVACE converter.A mathematical model is developed, and design scalability for a wide range of application is presented. In order to maximize the potential of energy harnessing, the effects of lock in phenomenon and different geometries of bluff bodies are discussed.

Keywords: Bluff bodies, Vortex Induced Vibration, Vortex Shedding, Lock in, Renewable Energy, Green Tech.

I INTRODUCTION

The global demand for energy is large and ever growing. There have been a worldwide efforts to meet the ever growing energy demand through scalable renewable energy sources viz. solar, wind, hydro etc. Many hydrokinetic energy harvesting technologies exist presently, but are unable to truly fulfil thisdemand due to theirs' self-limitations. The Earth's water bodies constitute a vast portion of theplanet more than two third part and slow and steady moving water represents a vast, but as yet unexploitedrenewable energyresource.

Most of water kinetic energy is currently harnessed from water flow by means of a dam and ahydro-turbine generator. The turbine generator is the most mature technology for flow kinetic energy harvesting. However, the efficiency of conventional hydro-turbine generators reduces as their sizes are increased due to the increased frictional losses in the bearings and the reduced surface area of the turbine blades. Furthermore, rotating parts of turbine such as bearings, blades etc. suffers from fatigue and wearing and tearing, and especially when they are miniaturised[1, 8]. Therefore mankind are in search of new and less ecologically intrusive technology to meet growing energy demand.

One promising new technology to overcome these problems is the hydroelectric power extraction system based on energy harvesting from flow induced vibration, the flow here can be both liquid flow and as well as air flow. There are three main types of energy harvesting systems of this type. These are energy harvesting from vortex-induced vibration (VIV) of bluff bodies, flutter energy harvesting systems and energy harvesting with Helmholtz resonators[1]. Out of these technologies the energy harvesting from vortex induced vibration is more promising due to its salient features.

Flow-induced vibration, as a discipline, is very important in our daily life, especially in mechanical engineering. For example, it can be witnessed in civil structures, viz. slender chimneys stacks, tall buildings, bridges or electric power lines, to name a few. It is also common in offshore structures or in the tubes of heat exchange systems. Generally, VIV is considered as an undesirable effect, as it may severely affect the structural integrity or the reliability of performance. It was first observed by Leonardo da Vinci circa 1500 A.D., in the form of "Aeolian Tones". Since then generally, engineers/scientists try to avoid flow-induced vibration in buildings and structures subjected to fluid flow to minimize fatigue caused by vortex induced vibrations and to reduce possible damage. Furthermore, Von Karman at Cal Tech proved that the Tacoma (USA) Narrows Bridge collapsed in 1940 due to VIV of thunder storm [1, 2]. Recently, such vibration has been investigated as an energy source that can be used to generate electrical energy and the idea been proposed to enhance the vibrations in order to extract useful kinetic energy from the surrounding flow [1]. Because of its practical and scientific interest, VIV has led to a great number of fundamental studies and research [1]. Two types of flow-induced vibration are studied so far: vortex-induced vibration and flutter.

This technology works by securing a cylinder (bluff body) horizontally with elastic strings, in flowing water and limiting it to a single degree of freedom; thereby allowing it to move only upand down perpendicular to the fluid flow. Fluid flow over this cylinder creates analternating vortex pattern behind the cylinder which exerts alternating lift forces on it, pushing it up anddown and so to make it vibrate. This vibratory motion is then converted into electricity via a power take off mechanism[1].

This technology is superior to traditional hydro-kinetic energy extraction technology in several ways. Most turbinebased kinetic energy harvesters only operate efficiently at flow currents greater than 2 m/s, while surfaceoscillation energy converters only give high output over a small range of water wave frequencies. A vortexinduced vibration based energy converter could potentially operate in slow moving waterways over awide range of frequencies. Further, Large scale tidal and dam type systems are environmentally obtrusive and require huge capital investment. The energy generators based on VIV concept are capable of producing energy fromflow (water/air) without altering the local environment, posing any hazard to nearby residents, modifying the landscape in any visible way, or interfering with flow (water/air) traffic in any slow moving flow currents (0.5-5 knots).Scalability and versatility are two of the paramount strengths of this technology. VIV generators modulescan range in size from single cylinder to arrays of cylinders of mega-watt power producing plants. Areas ofpotential power production can include not only water bodies and/or rivers such as the Gulf Stream, but also air flow. The average flow speed required for VIV is significantly lower than the turbine based hydrokinetictechnologies[1-5].

II PHYSICS OF VORTEX INDUCED VIBRATIONS (VIV) THEORY

The non-linear resonance phenomenon acknowledged as Vortex-Induced Vibration (VIV) has considerable relevance in several branches of mechanical engineering. Therefore the literature on vortex induced vibrations of circular cylinders is available extensively, and new knowledge has been developed continuously using experiments, field tests, and computer numerical simulations [2]. Currently, many of the aspects of VIV are well known or the next research steps are defined properly. However, some new phenomena still are being revealed as more researchesare being conducted [2]. A brief overview of some aspects of VIV, which are related to the performance of the VIVACE converter, is presented in this section.

In its simplest form, a VIVACE converter module consists of a circular cylinder mounted on springs and connected to a power take-off (PTO) system via a transmission mechanism for energy generation.Therefore, literature related to free vibrations of elastically sustained rigid cylinders; forced vibrations of such structures, with 1 or 2 degrees of freedom of movement is relevant to the operation of the VIVEC (vortex induced based energy converter) [1-2].

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2.1. VIV Theory

Vortex Induced Vibration (VIV) isfluid-structure interaction phenomenon which occurs due to the nonlinear resonance of bluff bodies (cylinders or spheres, etc.) through irregular vortex shedding lock-in.VIV is also termed as synchronization between vortex shedding and bluff body oscillations, is a result of vortex shedding phenomenon which generally occurs nearly on any bluff body when shedding of irregular vortices as shown in Fig. 1, and Fig. 2, behind bluff body submerged into fluid flow results in the fluctuating pressure difference which in turn results in theoscillation of hydrodynamic forces (lift and drag) on the bluff body and causing it to vibrateThe oscillatory motion of the body is due to alternating lift forces which is perpendicular to the direction of the flow [1-9].

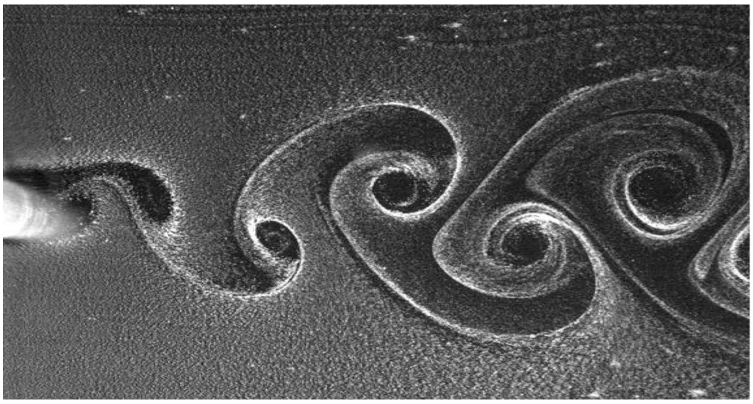


Fig. 1: Vortex shedding periodic pattern [1].

2.2 Reynolds Number

Generally, flow parameter that affects the behaviour ofVIV vortex shedding has been observed to be the Reynolds numberof fluid flow given as:

$$Re = \frac{UD}{\nu} \quad (1)$$

Where U is the free-stream velocity, D is the cylinder diameter and ν is the fluid kinematic viscosity. The regime that is targeted in the design of VIVACE is known as the “fully turbulent vortex street”, with Reynolds number in the range of (300<Re<3x105) [1, 2, 7].

2.3 Strouhal number

The Strouhal Number, St is a non-dimensional parameterthat describes the vortex shedding frequency to the oscillatingflow mechanism.

$$S_t = f_{st}\, D/U \quad (2)$$

Where f_{st} is vortex shedding frequency. Strouhal number will be used as a constant value in the design of VIVACE as the Reynolds number falls in the middle of constant Strouhal number region which is 0.2 for subcritical flow as shown in Fig. 3.

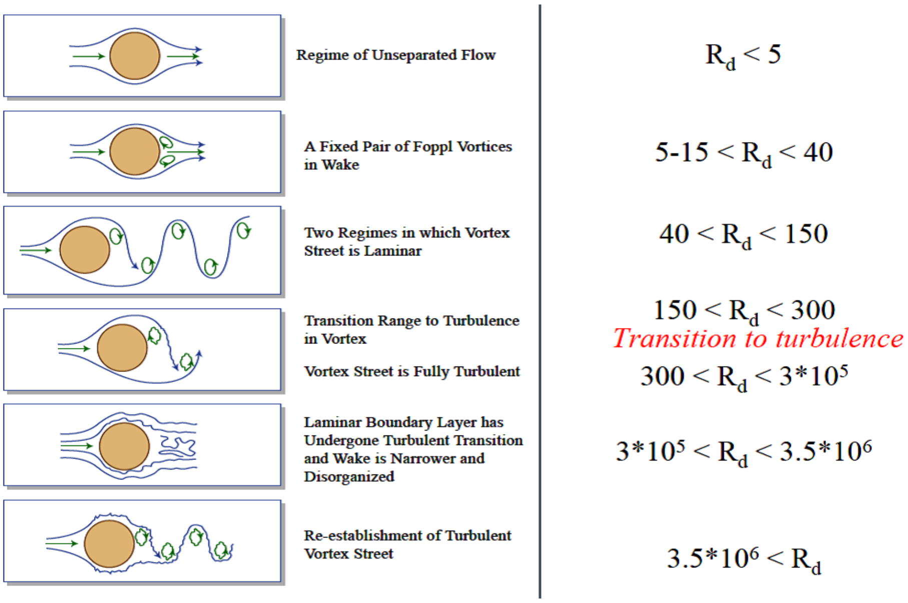


Fig. 2: Formation of vortices for various Reynolds number [7].

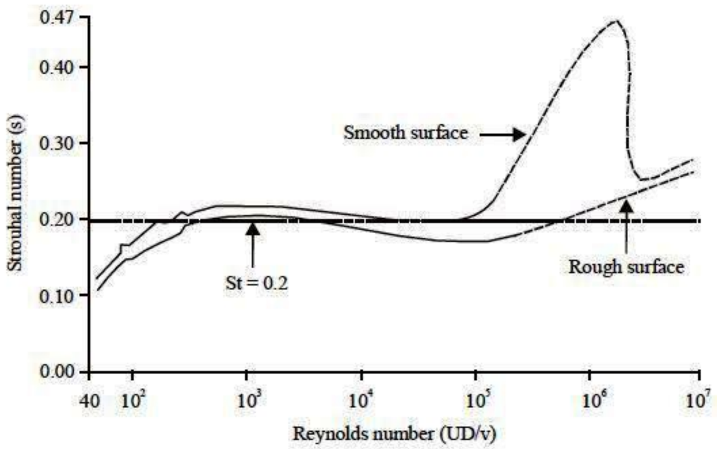


Fig. 3: Curve showing the relationship between Strouhal number and Reynolds number [6].

2.4 Mass Ratio

The mass ratio is defined as the ratio of total oscillating mass of the bluff body under VIV, including all oscillating appendages and 1/3 of the spring mass, to the displaced fluid mass m_d [7]:

$$m^* = m_{osc}/m_d \quad (3)$$

Where

$$m_d = \frac{\pi}{4}\rho D^2 L \quad (4)$$

Where D is the diameter of the cylinder, L is the length of the cylinder, and ρ is the density of the fluid.

2.5 Reduced Velocity Factor

The non-dimensional parameter used to measure vibration of amplitude is known as reduced velocity, U*:

$$U^* = \frac{1}{S_t} = U/D f_{St} \quad (5)$$

Several studies showed that within the range of (3<U*<8), shedding frequency can lock in and shift to match the natural frequency [1-7].

2.6 Frequency Ratio

The frequency ratio, f_{cyl}* is defined as the ratio of the frequency of cylinder oscillation in the fluid, f_{cyl} to the natural frequency of the cylinder in the fluid, $f_{n,fluid}$, i.e. $f_{cyl}^* = f_{cyl} / f_{n,fluid}$. For high mass ratio, for lock-in or synchronization the frequency ratio remains close to unity while, for low mass ratio, it becomes greater than unity means the body oscillates at a distinctly higher frequency and the range of synchronization opens up as seen in Fig. (4).

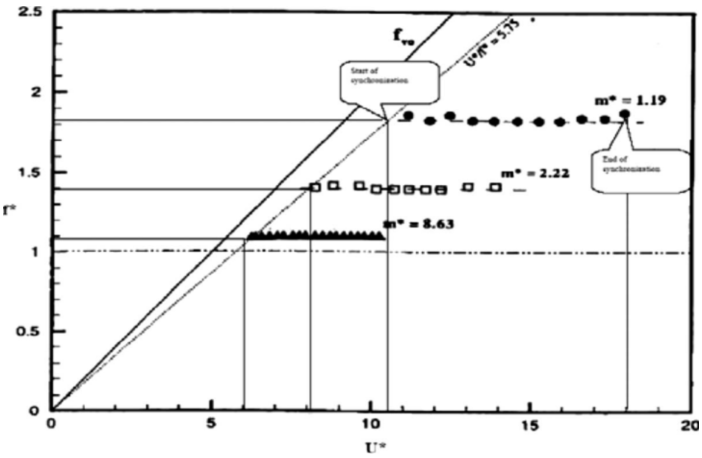


Fig. 4. Lower branch frequency response for different mass ratios versus varying U*; here f*=fcyl [1].

2.7 Lock in Phenomenon

A phenomenon known as “lock in” is a condition when thevortex shedding frequency becomes close to the naturalfrequency of the body.The primary response mode of a VIVACE module, is transverse to the fluid flow. In-line oscillations of smaller magnitude are also detected in VIV and strengthen transverse oscillations [1]. As the free stream velocity U increases, the condition of lock-in for a high mass ratio system is reached when the vortex formation frequency $f_{V,form}$ is close enough to the body’s natural frequency in the fluid $f_{n,fluid}$ as seen in Fig. (4).

Lock-in or vortex synchronization occurs over a broad range of the reduced velocity factor U*, means, for high m* values, till the lock-in occurs $f_{V,form}$ remains equal to f_{St}.

Whensynchronization occurs at 3<U*<8 (Fig. 4.) $f_{V,mode}$ appears and becomes equal to thefrequency of oscillation of the cylinder, f_{cyl}.

Synchronization is defined as the "matching of the frequency of the periodic wake vortex mode with the oscillation frequency of the bluff body", $f_{V,mode} = f_{cyl}$. Correspondingly, the fluid force frequency f_{fluid} must match the oscillation frequency, according to the definition of lock-in given by Sarpkaya [1]. The fluid force frequency comprises a frequency spectrum with $f_{V,mode} = f_{cyl}$ being the dominant frequency at lock-in.

Lock-In has the potential to enlarge theamplitudes of bodies' oscillation which is similar to linearresonance. It is essential to find the range of sheddingfrequency that match with natural frequency, f_n to designenergy harnessing device:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m_{app}}} \quad (6)$$

Where k is spring stiffness and m_{app} is mass applied to the cylinder.

2.8 Bluff Body's Amplitude of Oscillation

The cylinder's amplitude of oscillation in VIV is another important issue. In Fig. (5),the experimental results of Feng's minimum damping case [2] andKhalak and Williamson's case [11] are compared for amplitude of oscillation. FromFig. (5), it is evident that in the Feng's case [2] there are only two branches of amplitude's of oscillation, namely, the "initial" branch and the "lower" branch [11, 13]and there is a hysteretic transition between the two branches. The Feng's experiment was conducted in air so, the massratio m* (m* = 250) is very large. While in thecase of Khalak and Williamson, there are three branches observed with higher amplitudes, namely, theinitial branch, the "upper" branch, and the lower branch, with hysteretic transition between the initial and upper branches and intermittency between the upper and lower branches for the low mass ratio case. In the case of Khalak and Williamson, the mass ratio is low (m*=2.4) as the experiment was conducted in water.

In some recent work [18, 19], however, much higher amplitudes of body's oscillation have been reported for higher Reynolds numbers. And also for free VIV, the wake patterns observed are different from the classical Karman vortex street [2, 13, 14, 19].

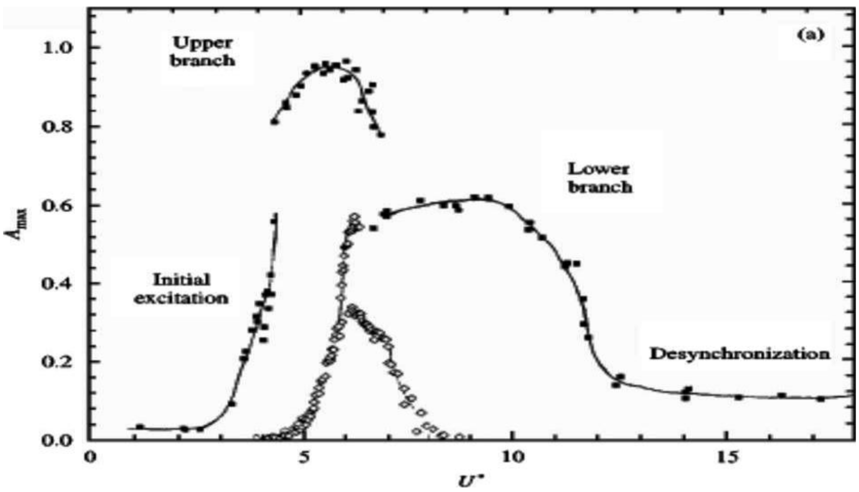


Fig. 5: Amplitude of oscillation Feng's case, for experiments in air by Feng and experiments in water by Khalak and Williamson [11].

2.9 Wake patterns

The wake-mode terminology was first introduced by Williamson and co-workers [2, 13, 20]. The modes of vortex shedding have significant effect on the amplitude of body's oscillation. The wake patterns in the flowing fluid that can be induced by the bluff body motionare discussed below.

(a) The 2S mode: In this mode of body's oscillation, two single vortices are shed per cycle of cylinder oscillationof period T_{osc}. This is like the classicalKarman vortex street. In this mode the frequency of vortex formation $f_{V,form}$ is equal to $f_{osc}=1/T_{osc}$, andthe frequency of vortex wake- mode $f_{V,mode}$ is also equal to fosc. Finally, the fluid forcingfrequency f_{fluid} is also equal to $f_{V,form}$ and f_{osc} [2].

(b) The 2P mode: Brika and Laneville [21, 22] first reported the evidence of the 2P vortex wake mode in free VIV of cable due to vortex shedding [20]. In this mode, two pairs of vortices are shed per cylinderoscillation cycle of period Tosc. The frequency of vortexwake-mode $f_{V,\ mode}$ is always equal to fosc. The frequencyof vortex formation is equal to twice of frequency of wake mode i.e. $f_{V,form}=2f_{V,mode}$. Finally, the fluidforcing frequency f_{fluid} has a major component equal to $f_{osc}= f_{V,\ mode}$ and a secondary one equal to $2f_{V,mode}$ [2].

(c) The P-S mode: In this mode one vortex pair and a single vortex are shedper cylinder oscillation cycle of period Tosc. The frequencyof vortex wake-mode $f_{V,mode}$ is always equal tofosc. The $f_{V,form}$ is equal to $3/2f_{V,mode}$. The fluid forcing frequency f_{fluid} hasa major component equal to fosc= $f_{V,\ mode}$ and a secondary component equal to $3/2f_{V,mode}$ [2].

(d)The 2T mode: In this mode three vortex pairs shed per cylinder oscillationcycle of period Tosc. The frequency of vortexwake-mode $f_{V,mode}$ is always equal to f_{osc}. The frequencyof vortex formation $f_{V,form}=3f_{V,mode}$. And finally, the fluidforcing frequency ffluid has a major component equal tofosc= $f_{V,\ mode}$and a secondary one equal to $3f_{V,mode}$ [2].

Table 1: Definition of dimensional and dimensionless* variables [2].

m_{osc}	Oscillating mass including one-third of spring mass	m_d	$\frac{\pi}{4}\rho_{fluid}D^2L$	C_{sys}	From damping test with transmission disconnected
m^*	m_{osc}/m_d	m_a	$C_a m_d$	C_{tra}	From damping tests with transmission connected minus the C_{sys}
y^*	y/D	y^*_{max}	y_{max}/D	C_{tot}	$= C_{sys}+C_{tra}+C_{gen}+C_{harn}$
y^*_{rms}	y_{rms}/D	$f_{n,fluid}$	$\frac{1}{2\pi}\sqrt{\frac{k}{(m+m_a)}}$	C_{gen}	From damping tests with transmission and generator connected minusC_{tra} minus C_{sys}
f_{St}	$S_t\times U/D$	$f_{V,form}$	Vortex	C_{harn}	From damping tests with

			formation frequency		transmission and generator connected minus C_{tra} minus C_{sys} minus C_{gen}
f_{fluid}	Fluid force frequency	$f_{cyl} = f$	Oscillating cylinder frequency	ξ_{index}	$\frac{C_{index}}{2\sqrt{k(m+m_a)}}$
$f_{v,mode}$	Frequency of vortex mode	f^*_{index}	$f_{index}/f_{n,fluid}$	S_G	$2\pi^3 S_t^{\ 2}(m^*\xi_a)$
ω_{index}	$\frac{1}{2\pi f_{index}} = \frac{T_{index}}{2\pi}$	T_{index}	$\frac{1}{f_{index}}$	$c_y(t)$	$\frac{F_{transverse}\ in\ phase\ with\ velocity}{\frac{1}{2}\rho U^2 DL}$
U^*	$\frac{U}{f_{n,fluid}D}$				

2.10 The aspect ratio (L/D)

The aspect ratio (L/D) is the ratio of length to diameter of the cylinder in VIV. It is an importantdesign parameter of VIVACE operation as the wake properties depend on it [2, 13].The aspect ratio along with VIV has amajor influence on the correlation length of the flow along the cylinder. Evidently, overall forces on the oscillating cylinder increases with correlation length. Theoretically, under VIV of the cylinder, correlationlength becomes infinite.

High Reynolds numbers along with the designof end plate also affect the correlation length and the maximumlift exerted [2, 23]. It was found that higher Reynolds number Re,produces higher lift forces and amplitudesof oscillation [18]. And mostly reported amplitudes of oscillation are between 0.54 and 1.13 times of cylinder diameters [13]. The power extracted by VIVACE from fluid flow and the amplitude of oscillation are directly proportional to C_L the coefficient of lift.Furthermore, $C_L$increases with the Reynolds number[2].

Proximity of a cylinder suspended with springs to free surface andbottom affects VIV of cylinder. Also the freesurface would impact VIV converters in shallow water flows such asin rivers. When the flow is bounded by a surface, itsignificantly alters thewake patterns and the forces exerted on the cylinder [2].

3.0 VIV Based Energy converter

Even though the phenomenon of VIV has been known centuries beforeno device has ever been built to generate significant level of electricity or other forms of usable energy efficiently from a fluid flow using VIV. A device was designed and patented by Clark [2] in 1999 which is based on a formula attributed to Rayleigh similar to the Strouhalformula for St=0.2. Clark's device, however, could not perhaps work because its design andway of operation suppresses VIV in many ways. Yoshitake et al. [2]got little success as they generated miniscule amounts of energy as a by-product of suppressing VIV of towering structures in airflow.The energy converter device based on VIV for aquatic energy harnessing is originally designed by Prof. Michael Bernitas and patented through University ofMichigan in 2008 which is known as VIVACE (vortex induced vibration aquatic clean energy).

The VIVACE converter design is based on the very simple concept of enhancing instead of spoiling vortex shedding and maximizing under significant damping rather than suppressing VIVs. VIVACE is scalable and can extract energy from currents of velocities from 0.25 m/s to 2.5 m/s and above [1, 5]. It is a robust system as it is less sensitive to environmental conditions because vortex shedding lock-in occurs over a broad and continuous range of frequencies, in contrast to just at natural frequencies as in linear resonance. For stationary cylinders, the Karman vortex street stream occurs over very broad ranges of Reynolds number. The VIV may occur over the same Reynolds number ranges except in two transition regions, where VIV is suppressed [2].

The VIVEC technology is based on four principles which are interrelated: (i) vortex induced vibrations, (ii) Nonlinear resonance, (iii) Correlation length and (iv) Energy generation.

(a) Vortex induced vibrations of a rigid circular cylinder mounted on linear springs: Asymmetric and oscillatory lift forces are created due to alternating vortex shedding. The cylinder in VIV oscillates perpendicularly to its axis and transversely to the fluid flow velocity Fig. (6),thereby it absorbs fluid hydro-kinetic energy. In VIVACE it is needed to enhance VIV to maximize energy harnessing at high damping while maintaining a high VIVamplitude and synchronization range.

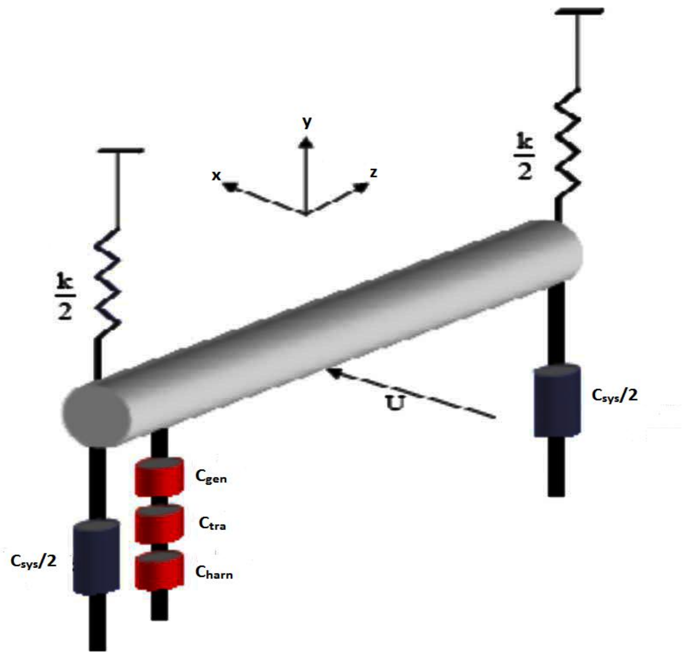


Fig. 6: Simple schematic of a VIVACE module with coordinate system [2].

(b) Nonlinear resonance: Nonlinear phenomenon is inherently related to VIV. Vortex shedding locks onto the oscillation frequency of the cylinder in VIV over a broad range of frequency around the cylinder's natural frequency in the fluid. The vortex shedding frequency f_{St}, which is calculated for a stationary cylinder in a steady flow, ceases to exist. Thereafter frequency $f_{V,mode}$ appears and becomes equal to the cylinder's oscillating frequency f_{cyl}. Furthermore the vortex formation frequency $f_{V,form}$ depends on the wake pattern formed in the fluid as explained in Sec. 2.7- 2.9. Therefore, this phenomenonis called as synchronization—between vortex shedding and cylinder's oscillations and it depends on the oscillating mass ratio and may occur not just at the natural frequency but over a broad range of frequency around the cylinder's natural frequency in the fluid,. Also, the amplitude of oscillation of the cylinder in VIV is self-limiting. Because of these last two properties, VIV is obviously a self-excitednonlinear resonance phenomenon.

(c) Correlation length: In VIV, the correlation length of the flow past the cylinder is defined as the length of the cylinder along which vortex shedding occurs in phase within certain tolerance. Higher correlation length implies higher total vortex induced forces. For a circular cylinder under VIV, the theoretical correlation length is infinite. In practice, aspect ratio between the ranges 7 to 20 is recommended [2]. The actual choice of aspect ratio depends on other issues tailored to suit the specific application and the requirements/restrictions on the values of U, m*, m_d, and required power output.

(d) Electrical generation: The mechanical oscillation energy of a cylinder in VIV can be converted into electricity and other forms of usable energy. The PTO (power take off) system consists of a magnetic sliders attached to the cylinder which moves up and down as the cylinder oscillates, over a rail containing a coil housed in cylinder's housing or gear-belt system through which cylinder is connected to a generator and generate electric current, which can be transmitted to utilization or stored into the grid [1, 6].Alternatives such as a linear generator can also be used.

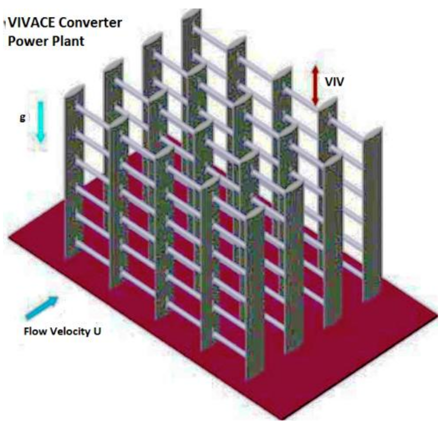
The oscillation energy of the cylinder can also be converted to mechanical energy, say, by using hydraulic system to pump water or to pressurize it for desalination. Irrespective of the final form of energy converted, the energy converter induces mechanical damping back to the VIV system. The power generation is maximised by adding electrical damping in the form of electrical resistance to the system. However, too much damping suppresses VIV and decreases the amplitude as shown in Griffin's plot [13], therefore results in zero harnessed energy. Furthermore, too little damping results in little harnessed energy and/or much amount of energy left in the VIV letting it to reach its self-limiting value of amplitude, which consecutively results in intermittent VIV. Increase in damping can be offset by reducing the mass ratio m*.

The design optimization process requires that high VIV amplitudes are maintained, under high damping, over a broad range of synchronization. The peak response amplitude y_m of oscillation varies as a, function of m* and the damping ratio [13], and the product of m* with damping coefficient is the controlling parameter to the maximum amplitude of oscillation [13, 14]. The oscillating mass m*, the stiffness k of the supporting springs, and the total system damping, can be adjusted to obtain a high degree of flexibility of the system over the broad range of VIV synchronization.

IV PHYSICAL MODEL OF VIVEC

A simple schematic of a single module of the VIVEC converter is depicted in Fig. (6).The module consists of the following elements: a circular rigid/hollow cylinder of diameter D and length L, two (or) four supporting linear springs each of stiffness k/2 (or k/4), system damping c_{sys}, one or more generators, generator damping c_{gen}, transmission damping c_{tra}, and the energy generating damping c_{harn}. The cylinder is suspended with springs with its axis in the z direction perpendicular to the flow velocity U, which is in the x-direction. The cylinder is free to oscillate in the y-direction, perpendicular to both its axis in z and the flow velocity in x direction.

As discoursedearlier, the design of VIVACE converter is modular, scalable, and flexible in terms of geometry and configuration. Therefore, depending on intended application, converters of different sizes can be designed by assembling a number of modules of various sizes and properties in a variety of configurations. Fig. (7)depicts aversion of a small array of VIVACE converter for an offshore power plant application. The supporting piles house all the transmission and electricity generating components, and are also hydro-dynamically faired to avoid their own VIV. The oscillating cylinders are connected by small pins to sliding bearings on a steel rod with springs and damping to provide an elastic support to attain VIV of the cylinders. The next stage of development, aims to use a hydraulic system to connect multiple VIVACE modules to one generator. Alternatives such as a hydraulic system or a linear generator can also be used.



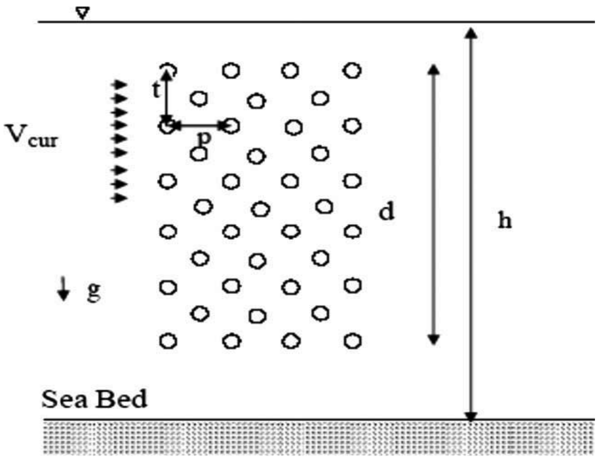


Fig. 7: Small VIVACE converter power plant.

Fig. 8: Geometry, dimensions, and arrangement of cylinders in a VIVACE converter assembly [2].

In addition to the system parameters used to define a module, for a VIVEC assembly, the following geometric variables need to be defined, as shown in Fig. (8): t=vertical distance between centers of cylinders, and p=horizontal distance between centers of cylinders, h=water depth, d=draft of the VIVACE converter assembly.

V MATHEMATICAL MODEL

A simple mathematical model of the system dynamics and fluid forces is discussedin this section. The oscillation of the cylinder in the y-direction see Fig. (6), is modelled by a second order linear equation as:

$$m_{osc}\, y'' + c_{tot} y' + ky = F_{fluid}\, \hat{y} \quad (7)$$

Where y is the displacement perpendicular to the flow and the cylinder axis in y direction, m_{osc} is the oscillating system mass including all appendages and one-third of the spring mass, k is the spring stiffness, c_{tot} is the total damping coefficient, and F_{fluid} is the force exerted by the fluid on the body in the y-direction. The fluid force can be separated into viscous and inviscid components [13,14,17]as follows:

$$m_{osc}\, y'' + c_{tot} y' + ky = F_{viscous}\, \hat{y} + F_{inviscid}\, \hat{y} \quad (8)$$

The inviscid force may be defined in terms of the inviscid added mass m_a. The added mass is defined as "the impulse given to the fluid during an incremental change of body velocity, divided by thatincremental velocity" [1]. Furthermore:

$$F_{inviscid}\, \hat{y} = \; -m_a\, y'' \quad (9)$$

$$F_{viscous}\, \hat{y} = \frac{1}{2} c_y(t) \rho U^2 DL \quad (10)$$

Using eqn. 4 the mass of the displaced fluid m_d, theequation of motion reduces to:

$$(m_{osc} + m_a) y'' + c_{tot} y' + ky = \frac{2}{\pi D} c_{y(t)}\, m_d U^2 \quad (11)$$

By using the non-dimensional variables defined in Table 1 the above equation can be made non-dimensional as:

$$(m^* + C_a).\left(\frac{y''^*}{f_{n,fluid}^2} + \frac{4\pi \xi_{tot} y'^*}{f_{n,fluid}} + 4\pi^2 y^*\right) = \frac{2}{\pi} c_y(t) U^{*2} \quad (12)$$

At system resonance, the cylinder dynamics is periodic and for certain wake structures nearly sinusoidal. Hence, VIV can be modelledby a linear oscillator model. However, outside of the resonance range, the model has to be modelled by a nonlinearresonance model. As a simple approximation to the resonant response without experiments, traditionally it is assumed a sinusoidal form for the fluctuating transverse force coefficient and amplitude [2].

$$y^* = y^*_{max} \sin\left(2\pi f_{fluid}\, t\right) \quad (13)$$

$$c_y(t) = C_y \sin\left(2\pi f_{fluid}\, t + \phi\right) \quad (14)$$

Where $f_{fluid}$is the fluid forcing frequency, ϕ is the phase difference between the fluid forcing and the displacement which is near about $\pi/2$ for linear systems at resonance. Here f_{fluid} is assumed equal to f_{cyl}, for the ensuing analysis, which is the dominant fluid forcing frequency, andmatching of these two frequencies defines resonance. The phase differencebetween the fluid force and the body displacement ϕ, is important in determining the energy harnessing from the fluid by VIVEC. From Eqn. (12) and eqn. (14), it can be deduced that $C_y \sin\phi$ has to be positive for VIVEC to generate energy. For forced vibration, it was found [2] that for A/D=0.3, $C_y \sin\phi$ is positive for reduced velocity factor in the range of $5.5<U^*<8.0$. Furthermore, as the amplitude of the forced oscillation increases above 0.85 D, the value of $C_y \sin\phi$ becomes negative. In free vibration tests, Cy sin ϕ can never be found negative and for lower mass ratio the value of $C_y$sinϕ is found positive for higher values of dimensionless amplitude up to 1.13 [13].

(i) The power in the flowing fluid over the projected area of the cylinder in the direction perpendicular to the flow direction can be calculated from Bernoulli's equation as the product of the acting force with the velocity as:

$$Power\ in\ fluid = \frac{1}{2}\rho U^3 DL \quad (15)$$

(ii) The fluid power in VIVEC is obtained by dividing the work done by the period of oscillation cycle. And the work done by the fluid force acting on the cylinder during a oscillation cycle is obtained by taking the inner vector product of the force by the displacement vector dx and integrating over a cycle. Then, the fluid power in VIVACE is:

$$P_{VIVEC-fluid} = \frac{W_{VIVEC-fluid}}{T_{cyl}} = \frac{\int_0^{T_{cyl}} F_{fluid} y' dt}{T_{cyl}} \quad (16)$$

The fluid force exerted on the cylinder is given by the right hand side of Eqn. (11). Therefore multiplying this force by theinstantaneous velocity, and integrating, and averagingit over the vibration cycle period T_{osc}, and further assuming $f_{fluid}=f_{v,mode}=f_{cyl}=1/T_{cyl}$ for synchronization, and after simplification using non-dimensional variable m_d, we have:

$$P_{VIVEC-fluid} = \frac{1}{2}\rho\pi C_y U^2 f_{cyl} y_{max} DL \sin\phi \quad (17)$$

(iii) Mechanical Power in VIVEC is obtained from Eqn. (11), by multiplying the left hand side of it by the instantaneous velocity, and integrating and averagingthe product over the cycle period T_{cyl} and then substituting the sinusoidal expression for y from Eqn. (13) into it and simplifying, we have:

$$P_{VIVEC-mech} = 8\pi^3 (m_{osc} + m_a)\xi_{tot} \left(y_{max} f_{cyl}\right)^2 f_{n,fluid} \quad (18)$$

(iii) Equating Fluid Power in VIVEC to Mechanical Power in VIVEC.Since the VIVEC fluid power given by Eqn. (17) and the mechanicalpower in Eqn. (18) are equal because they are integrals of the twosides of Eqn. (11). Therefore, after equating and simplifying we have:

$$C_y U^2 \sin\phi = 4\pi^3 D(m^* + C_a)\xi_{tot} y_{max} f_{cyl} f_{n,fluid} \quad (19)$$

We can measure three of the four quantities C_y, sin ϕ, y_{max}, and ζ_{tot} from experiments and solve for the remaining fourth quantity.

(iv) Upper Limit of VIVEC Power Conversion. The actual power converted by the VIVEC can be obtained only by measurements. However,the maximum harness-able power, can be calculated either in fluid form (Eqn. 17) or in mechanical form (Eqn. 18) as:

$$h_{UL-VIVEC} = \frac{P_{VIVEC-fluid}}{power\ in\ fluid} = \frac{\frac{1}{2}\rho\pi C_y U^2 f_{cyl} y_{max} DL \sin f}{\frac{1}{2}\rho U^3 DL} \quad (20)$$

Where, for a given VIVEC module, U, D, L, ρ, and m* are specified; and f_{cyl}, y_{max}, and ϕ are measured from experiments; and then C_y is calculated from the following equation:

$$-(m_{osc} + m_a)\omega_{cyl}^2 y_{max} + k y_{max} = \frac{1}{2}\rho C_y U^2 DL \sin\left(\frac{\pi}{2} + f\right) \quad (21)$$

The value of lift coefficient C_y(also denoted as C_L) increases with Reynolds number, increases with lower mass ratio, and decreases with increasing damping [11]. The value of C_y becomes higher as m_{osc} decreases and Re increases [2].

(v) Harness-able Energy using VIVEC. The power that can be harnessed using the VIVECis the power reaching to the generator, which is equal to the power extracted from the fluid by VIVEC minus the power dissipated by the structural, transmission, and internal generator losses. To simplify calculations, it is assumed that the damping force due to the generator, which is responsible for the mechanical energy to electrical energy conversion, is in phase with the velocity term, and its behaviouris same as structural and transmission damping. The total damping, in non-dimensional form, is:

$$\xi_{tot} = \xi_{tra} + \xi_{sys} + \xi_{gen} + \xi_{harn} \quad (22)$$

From Eqn. (19) and after manipulation we obtain the expression for the harnessed power as:

$$P_{VIVEC-harn} = \frac{\pi}{4}\rho D^2 L\left(2C_y U^2 f_{cyl} y^*_{max}\sin(\phi) - 8\pi^3(m^* + C_a) \times \left(\xi_{tra} + \xi_{sys} + \xi_{gen}\right)\left(y_{max} f_{cyl}\right)^2 f_{n,fluid}\right) \quad (23)$$

Thus, the ratio of harnessed power to power available is:

$$\eta_{VIVEC} = \frac{P_{VIVEC-harn}}{power\ in\ the\ fluid} = \frac{P_{VIVEC-harn}}{\frac{1}{2}\rho U^3 DL} \quad (24)$$

Even though this ratio implies efficiency, it would be controversial to define it as efficiency[1]. It is more appropriate to use energy density (kw/m^3) for comparison of renewable energy converters[1]. Even at these early stages of development, the power conversion ratio achieved experimentally for the VIVACE is 0.22 for U=0.840 m/s[2]. And the upper limit of the VIVACE converter power conversion ratio obtained is 0.37 [2].

VI SIMPLIFIED INITIAL MATHEMATICAL MODEL CALCULATION

In this section a simplified mathematical model is developed to calculate the parameters of oscillating cylinderto describe the oscillator dynamics of fluid-oscillatorinteraction in order to estimate the possibledynamic performance of a VIV. Here many approximations, sometimes crude, have been taken to calculate the initial model parameters of VIVEC. This simplified model is much less complex and quick as opposed to the detailed mathematical model described in section 5. The model parameters calculated using this simplified model are not much accurate however, the result obtained can predict the performance of VIVEC operation and can be used as initial model parameters (/variables) in the VIVEC design.

The Strouhal number which isrelated to the Reynolds number is calculated by using Eqn. (15) to validate the initial estimation of Strouhal number of about 0.2 as discussed previously [1, 14].

$$S_t = 1.98\left[1 - \left(\frac{19.7}{Re}\right)\right] \quad (25)$$

For synchronisation to happen it is a must to match the vortex shedding frequency and natural frequency of oscillation of body in the fluid to provide large amplitude of oscillation. The vortex shedding frequency is calculated from Eqn. (5).

$$f_{St} = \frac{(S_t U)}{D} (26)$$

To calculate the natural frequency of oscillation of the cylinder in the fluid, the following cylinder properties have to be determined.

$$Added\ mass = m_{add} = mass\ of\ all\ appendages + \frac{1}{3}\ mass\ of\ spring\ (27)$$

$$Mass\ of\ pipe = m_{pipe} = \rho_{cyl} \times L + m_{add} (28)$$

$$Volume\ of\ displaced\ fluid = m_{dis} = \frac{\pi}{4} D^2 L (29)$$

$$Mass\ of\ displaced\ fluid = m_{dis} = \rho_{fluid} \times volume\ of\ cylinder (30)$$

$$Total\ oscillating\ mass = total\ applied\ mass\ to\ the\ VIVEC\ system =$$

$$m_{app} = m_{osc} = m_{pipe} + m_{dis} (31)$$

Spring stiffness, k value is calculated by using Eqn. (6) and letting the body's natural frequency equal to vortex shedding frequency for lock-in condition requirement, though it is a rough approximation.

$$k = [f_{St} \times 2\pi]^2 \times m_{app} (32)$$

The oscillating fluid force F(t) i.e. the lift force, exerted on cylinder is assumed to be a sinusoidal function with frequency equivalent to natural frequency of oscillation of the cylinder, though it is a crude approximation, and is given by:

$$F_L = 0.5 \rho_{fluid} U^3 DL. C_L (33)$$

The coefficient of lift force C_L is assumed to be 0.6 for water based on previous study [1, 3].

The amplitude of oscillation of the cylinder as a function of time, y(t) is given by Eqn. below:

$$y(t) = \left[F_L \sin(\omega_n t + (\pi/2)) / \left[k \sqrt{\left(1 - \left(f_{St}/f_{n,fluid}\right)^2\right)^2 + 4\xi^2 \left(f_{St}/f_{n,fluid}\right)^2} \right] \right] (34)$$

Where damping coefficient, ζ is total damping coefficient and is assumed to be the 0.06 [1, 3].

Then, the velocity of the cylinder v (t) is obtained by differentiating Eqn. (22) as given by Eqn. (35) below:

$$v(t) = \frac{d}{dt}(y(t))$$

$$v(t) = \left[F_L \omega_n \cos \omega_n t + (\pi/2))/\left[k\sqrt{\left(1-\left(f_{St}/f_{n,fluid}\right)^2\right)^2 + 4\xi^2\left(f_{St}/f_{n,fluid}\right)^2}\right]\right] \quad (35)$$

Power extracted by VIVEC is calculated by taking the product of the maximum velocity and force of lift experienced by the cylinder. The frequency of power P (t) is double the frequency of velocity or lift force since it contains product of sine term as given below:

$$P(t) = v(t) \times F_L \sin(\omega t) \quad (36)$$

Mean power extracted by cylinder is calculated byEqn. (37) below:

$$P_{rms} = P_{max}/\sqrt{2} \quad (37)$$

The theoretical maximum power available in the fluid is calculated on Eqn. (38) below.

$$P_{fluid} = 0.5\rho_{fluid} U^3 DL) \quad (38)$$

Efficiency of the VIVEC is calculated as (17) below.

$$\eta = P_{rms}/P_{fluid} \quad (39)$$

VII RESULT AND DISCUSSION

We did the initial model calculation of VIVEC for wind energy conversion. The Reynolds Number targeted in these calculation is within the range of ($300<Re<3x10^5$) which is known as fully turbulent regime. At this range Strouhal number is about 0.2 for smooth surfaces, which correspond to the fully turbulence vortex range. The cylinder and flow parameters investigated are listed in Table 2.

Table 2: Cylinder and flow parameters

Cylinder Parameters								
Diameter (m)	0.025	0.025	0.025	0.051	0.051	0.051	0.076	0.076
Length (m)	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Density (kg/m^3)	0.476 kg/m^3 (3.4 mm thickness)							
Air Properties								
Velocity, U (m/s)	5 m/s average air velocity in Bhopal							
Density (kg/m^3)	1.225							
Kinematic Viscosity (m^2/s)	$1.568*10^{(-5)}$							

Based on the initial model calculation obtained, the theoretical results are tabulated in Table 3.

Table 3: Theoretical calculations and results

Air			Cylinder		System			Dynamics of body (Cyl)				Power		
Re	St	f_{St} = fn	M_{cyl}	Vol (Cyl) (m^3)	M_{dis} (kg)	M_{app} (kg)	K (N/m)	Fl (N)	y(t) max	v(t) max	P(t) max (w)	P rms (w)	P air (w)	η
8694	0.2	39	0.238	0.0003	0.0003	0.238	14213	0.6	0.0003	0.084	0.05	0.03	0.97	0.035
8694	0.2	39	0.2380	0.0003	0.0003	0.238	14213	0.6	0.0003	0.084	0.05	0.03	0.97	0.035
8694	0.2	39	0.238	0.0003	0.0003	0.238	14213	0.6	0.0003	0.084	0.05	0.03	0.97	0.035
17388	0.2	19	0.238	0.001	0.0012	0.239	3575.3	1.2	0.0027	0.332	0.39	0.27	1.94	0.141
17388	0.2	19	0.238	0.001	0.0012	0.239	3575.3	1.2	0.0027	0.332	0.39	0.27	1.94	0.141
17388	0.2	19	0.238	0.001	0.0012	0.239	3575.3	1.2	0.0027	0.332	0.39	0.27	1.94	0.141
26082	0.2	13	0.238	0.0023	0.0028	0.241	1600.5	1.8	0.0091	0.743	1.3	0.92	2.92	0.315
26082	0.2	13	0.238	0.0023	0.0028	0.241	1600.5	1.8	0.0091	0.743	1.3	0.92	2.92	0.315
26082	0.2	13	0.238	0.0023	0.0028	0.241	1600.5	1.8	0.0091	0.743	1.3	0.92	2.92	0.315

In the design of VIVEC it is important to find the range of vortex shedding frequencies that matches with natural frequency of body. As to achieve the lock in condition is the main goal of its design, thereforesuitable value of stiffness of springs for each testing cylinder of different diameters is chosento match the vortex shedding frequency with natural frequency of cylinder. It is evident from table II that spring stiffness decreases as the diameter of cylinder is increased. The difference in stiffness of springs are due to the different mass applied for each of the cylinders. Lock-in condition has the potential to enlarge the amplitudes of oscillation of cylinder under VIV, which can result in higher power generation.

From Table II, it is evident that Reynolds number Re, generally increases with an increase of cylinder diameter while other parameters are kept constant, which result in higher cylinder amplitude. Velocity of cylinder is found by differentiating the equation of displacement, y (t) of cylinder with respect to time. Since we have targeted the range of Reynolds number within

The range (300<Re<3x105), therefore the diameter of cylinder chosen to fall within this range of Re. It is also clearly evident that the maximum power of the system increases as the diameter of cylinder is increased. This is due to the increase of Re which is directly proportional to the cylinder diameter. The increase of Re will result in higher velocity of the cylinder, which in turn increases the power generation rate. The highest efficiency obtained in this result is for the cylinder of 0.076 m diameter. It showed that power efficiency of cylinder is increased as the diameter of cylinder increases. The random and fluctuating behaviour of the vortex shedding affects the efficiency of the power generation [1].

Maximum power generated by the each cylinder is quite small due to lower amplitude of cylinder oscillation attained. Therefore, a single unit of VIV system is insufficient to supply power to a physical household/commercial system. However, large number of VIV module can be installed in an array of cylinders in order to generate sufficient power supply to physical application. Also the VIVEC can be integrated with other renewable energy system such as solar, wind to overcome the limitations of each other and to make a more sustainable renewable energy system.

VIII CONCLUSION

The VIV energy converter is a viable solution to generating alternative green energy. The efficiency of VIVEC depends on L/D ratio, fluid properties and mainly on high VIV amplitudes, under high damping, over a broad range of synchronization. More research is required to increase the VIV amplitudes and optimise the lift coefficient and system damping. On the basis of current literature and test measurements what we performed, we expect significant increase of both numbers in future developments.

Although some renewable technologies such as solar and wind energy technologies are in use, have theirs' self-limitation which can be minimised by use of hybrid systems. Further research on maximizing VIV phenomenon are needed to increase energy extraction rate based on the geometries of the bluff bodies. Different design of VIV convertersrequired to be numerically and experimentally researched to trigger the advancement ofVIV based converter technology to fulfil ever increasing global energy demand by renewable energy sources.

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