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INVENTORY MODEL FOR DETERIORATING ITEMS FOR QUADRATIC DEMAND WITH PARTIAL BACKLOGGING CONSIDERING VARIABLE HOLDING COST

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ABSTRACT

In many of the inventory models of deteriorating items the demand rate and holding cost has considered as a constant function. But in real life situations they may vary with respect to time. So, in this paper, we develop a deterministic inventory model for deteriorating items in which holding is taken as linear function of time and demand rate is quadratic function of time. Deterioration rate is considered as constant. Shortages are allowed and partially backlogged and backlogging rate is variable and depend on the length of the next replenishment.

Keywords: Holding cost, inventory, Quadratic demand, Partial backlogging, Weibull function

I. INTRODUCTION

In present scenario the effect of deterioration cannot be avoided in any business organization so deterioration is defined by a process in which the stocked items loose part of their value with passage of time. Vegetables, fruits, milk, medicines, fashion goods, alcohol, electronic components, gasoline etc. items are called deteriorating items. Hence deterioration factor has to be given importance while determining the optimal policy for an inventory model.

Deterioration of fashion goods at the goods at the end of the prescribed storage period was first studied by Hadley et al. (8). Ghare and Schrader (7) formulated a mathematical inventory model with constant rate of deterioration and exponentially decaying. Chang and Dye (3) developed an inventory model with time varying demand and partial backlogging. Wu et al. (16) developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Dye et al. (4) find an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. Alamri and Balkhi (2) studied the effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. Dye (6) gave an inventory model to determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging and deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. Teng et al. (15) gave a comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items. Roy (12) developed an inventory model for deteriorating items with time varying holding cost and demand is price dependent. Skouri et al. (14) developed an inventory models with ramp type demand rate, partial backlogging and Weibull's deterioration rate. Shah and Shukla (13) developed a deteriorating inventory model for waiting time partial backlogging when demand and deterioration rate is constant. Abad (1) considers the problem of determining the optimal price and lot size for reseller in which unsatisfied demand is partially backordered. Mishra and Singh (11) gave an inventory model for ramp type demand, time dependent deteriorating items with salvage value and shortages. Hung (9) gave an inventory model with generalized type demand, deterioration and backorder rates.

Many researchers, mentioned above, developed models in which holding cost and demand rate is taken as constant function. But in realistic situation holding cost and demand rate are always not a constant function and depends on time. Mishra and Singh (10) developed a deteriorating inventory model for time dependent demand and holding cost with partial backlogging. Further Yadav and Vats (17) gave a deteriorating inventory model for quadratic demand and constant holding cost with partial backlogging and inflation. In this paper we extended the idea of Mishra and Singh (10) and Yadav and Vats (17) by considering quadratic time dependent demand and holding cost with linear function of time with constant rate of deterioration. In this paper shortages are allowed and partially backlogged; backlogging rate is variable and is dependent on the length of the next replenishment.

II. ASSUMPTIONS AND NOTATIONS

In developing the mathematical model of the inventory system for this study, the following assumptions and notations are used.

2.1 Assumptions

(i) Shortages are allowed and partially backlogged.

(ii) The replenishment rate is infinite.

(iii) The lead time is zero.

(iv) The deterioration rate is constant.

(v) The planning horizon is finite.

(vi) Holding cost is linear function of time $h(t) = \alpha + \beta t, \quad \alpha, \beta \geq 0.$

(vii) The demand rate is quadratic function of time i.e. $D(t) = a + bt + ct^2$ where a, b and c are constants and $a, b, c > 0$.

(viii) During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative inventory is,

$B(t) = \dfrac{1}{1+\gamma(T-t)}$, where γ is backlogging parameter and $(T-t)$ is waiting time $(t_1 \leq t \leq T).$

2.2 Notations

(i) K : the ordering cost per order;

(ii) C : the purchase cost per unit;

(iii) θ : the deteriorating cost;

(iv) $H(t)$: the inventory holding cost per unit per time unit;

(v) K_b : the backordered cost per unit short per time unit;

(vi) K_l : the cost of lost sales per unit;

(vii) T_1 : the time at which the inventory level reaches zero; $t_1 \geq 0$;

(viii) T_2 : the length of period during which shortages are allowed, $t_2 \geq 0$

(ix) T : $(= t_1 + t_2)$ the length of cycle time;

(x) I_m : the maximum inventory level during $[0, T]$;

(xi) I_b : the maximum inventory level during shortage period;

(xii) Q : $(= I_m + I_b)$ the order quantity during a cycle of length T;

(xiii) $I_1(t)$: the level of positive inventory at time t;

(xiv) $I_2(t)$: the level of negative inventory at time t;

(xv) $C(t_1, t_2)$: the total cost per time unit.

III. MATHEMATICAL MODEL

3.1Inventory level without shortage

During the period $[0, t_1]$, the inventory depletes due to demand and deterioration. Therefore differential equation governing the inventory level $I_1(t)$ at any time t during the cycle $[0, t_1]$ is given by

$$\frac{dI_1}{dt} + \theta I_1(t) = -(a + bt + ct^2) \quad ; \quad 1 \leq t \leq t_1 \tag{3.1}$$

With boundary conditions $I_1(t) = 0$ at $t = t_1$

The solution of above equation is

$$I_1(t)e^{\theta t} = -\left[(a + bt + ct^2)\frac{e^{\theta t_1}}{\theta} - (b + 2ct)\frac{e^{\theta t}}{\theta^2} + 2c\frac{e^{\theta t}}{\theta^3} \right] + c_1$$

Now, at $t = t_1$, $I_1(t_1) = 0$

$$c_1 = (a + bt + ct^2)\frac{e^{\theta t_1}}{\theta} - (b + 2ct_1)\frac{e^{\theta t_1}}{\theta^2} + 2c\frac{e^{\theta t_1}}{\theta^3}$$

$$I_1(t) = -\frac{a}{\theta} - \frac{b}{\theta}\left(t - \frac{1}{\theta}\right) - \frac{c}{\theta}\left(t^2 + \frac{2t}{\theta} - \frac{2}{\theta^2}\right) + e^{\theta(t_1 - t)}\left[\frac{a}{\theta} + \frac{b}{\theta}\left(t_1 - \frac{1}{\theta}\right) + \frac{c}{\theta}\left(t_1^2 + \frac{2t_1}{\theta} - \frac{2}{\theta^2}\right)\right] \quad (3.2)$$

When t=0 the level of inventory is maximum and it is denoted by I_m then from equation (3.2)

$$I_m = I_1(0) = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right)\left(e^{\theta t_1} - 1\right) \quad (3.3)$$

3.2 Inventory level with shortage

During the shortage interval $[t_1, T]$ the inventory level depends on demand and a fraction of demand is backlogged. The state of inventory during $[t_1, T]$ can be represented as

$$\frac{dI_2(t)}{dt} = -\frac{(a + bt + ct^2)}{1 + \gamma(T - t)}, \quad t_1 \le t \le t_2 \quad (3.4)$$

With the boundary condition $I_2(t_1) = 0$ at $t = t_1$,

The solution of equation (3.4) is

$$I_2(t) = \frac{a}{\gamma}\log\{1 + \gamma(t_1 + t_2 - t)\} + \frac{b}{\gamma^2}[\{1 + \gamma(t_1 + t_2)\}\log\{1 + \gamma(t_1 + t_2)\} - \{1 + \gamma(t_1 + t_2 - t)\}]$$

$$+ \frac{c}{\gamma^3}\left[\{1 + \gamma(t_1 + t_2)\}^2 \log\{1 + \gamma(t_1 + t_2 - t)\} - 2\{1 + \gamma(t_1 + t_2)\}\{1 + \gamma(t_1 + t_2 - t)\} + \frac{\{1 + \gamma(t_1 + t_2 - t)\}^2}{2}\right] + c_2$$

At $t = t_1$, $I_2(t) = 0$

$$c_2 = -\frac{a}{\gamma}\log(1 + \gamma t_2) - \frac{b}{\gamma^2}[\{1 + \gamma(t_1 + t_2)\}\log(1 + \gamma t_2) - (1 + \gamma t_2)] - \frac{c}{\gamma^3}\left[\{1 + \gamma(t_1 + t_2)\}^2 \log(1 + \gamma t_2)\right.$$

$$\left. - 2\{1 + \gamma(t_1 + t_2)\}(1 + \gamma t_2) + \frac{(1 + \gamma t_2)^2}{2}\right]$$

$$I_2(t) = \frac{a}{\gamma}\log\left\{\frac{1 + (t_1 + t_2 - t)}{1 + \gamma t_2}\right\} + \frac{b}{\gamma^2}\{1 + \gamma(t_1 + t_2)\}\log\left\{\frac{1 + \gamma(t_1 + t_2 - t)}{1 + \gamma t_2}\right\} - \frac{b}{\gamma}(t_1 - t) + \frac{c}{\gamma^3}\{1 + \gamma(t_1 + t_2)\}^2$$

$$\log\left\{\frac{1 + \gamma(t_1 + t_2 - t)}{1 + \gamma t_2}\right\} - \frac{c}{2\gamma^2}(t_1 - t)\{2 + \gamma(3t_1 + 2t_2 - t)\}, \quad 0 \le t \le t_1 \quad (3.5)$$

Therefore, the total cost per replenishment cycle consists of the following components

(a) Inventory holding cost per cycle;

$$C_{HC} = \int_0^{t_1} H(t) I_1(t)\,dt$$

$$= \int_0^{t_1} (\alpha + \beta t) I_1(t)\,dt$$

=

$$\int_0^{t_1}(\alpha+\beta t)\left[-\frac{a}{\theta}-\frac{b}{\theta}\left(t-\frac{1}{\theta}\right)-\frac{c}{\theta}\left(t^2+\frac{2t}{\theta}-\frac{2}{\theta^2}\right)+e^{\theta(t_1-t)}\left\{\frac{a}{\theta}+\frac{b}{\theta}\left(t_1-\frac{1}{\theta}\right)+\frac{c}{\theta}\left(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2}\right)\right\}\right]dt$$

$$\begin{aligned}=\;&\frac{1}{12\theta^5}\Big[12e^{t_1\theta}bt_1\alpha\theta^3+12a\beta e^{t_1\theta}\theta^2-12a\alpha\theta^4t_1-6a\beta\theta^4t_1^2-6b\theta^4\alpha t_1^2-4b\beta\theta^4t_1^3-4b\beta\theta^4t_1^3\\&-6b\beta\theta^3t_1^3-12b\beta e^{t_1\theta}\theta-12b\beta e^{t_1\theta}\theta-12e^{t_1\theta}a\alpha\theta^3-12e^{t\theta}b\alpha\theta^2+12e^{t_1\theta}b\beta t_1\theta^2-12a\theta^3\alpha-12\beta a\theta^2\\&-12a\beta t_1\theta^3+12\theta^2b\alpha+12\beta b\theta+48c\alpha\theta^4t_1-4c\alpha\theta^4t_1^3-24c\alpha\theta^3t_1^2-24\alpha c\theta-3c\beta\theta^4t_1^4-20c\beta\theta^3t_1^3\\&+12c\beta\theta^2t_1^3+12\beta c\theta^2t_1^2+24\beta c\theta^2t_1-24\beta c\theta t_1-24\beta c+12\alpha ct_1^3e^{\theta t_1}-24\alpha c\theta^2e^{\theta t_1}t_1+24\alpha c\theta e^{\theta t_1}\\&+12\beta ct_1^2\theta^2e^{\theta t_1}-2\beta c\theta t_1e^{\theta t_1}+2\beta ce^{\theta t_1}\Big]\end{aligned} \quad (3.6)$$

(b) Backordered cost per cycle;

$$C_{BC}=K_b\left(\int_{t_1}^{t_1+t_2}-I_2(t)dt\right)$$

$$\begin{aligned}C_{BC}=K_b\Bigg[&\frac{1}{6\gamma^4}\Big\{6a\gamma^3t_2+3b\gamma^3t_2^2+6bt_1t_2\gamma^2+6b\gamma^2t_2-6c\gamma t_2+6c\gamma^2t_1^2t_2-6c\gamma^2t_2^3-12c\gamma^2t_1t_2^2\\&+6c\gamma^3t_1t_2^2+2c\gamma^3t_2^3+6a\gamma^2\log\left(\frac{1}{1+\gamma t_2}\right)+6b\gamma^2t_1\log\left(\frac{1}{1+\gamma t_2}\right)+6b\gamma\log\left(\frac{1}{1+\gamma t_2}\right)\\&-6c\{1+\gamma(t_1+t_2)\}\log\left(\frac{1}{1+\gamma t_2}\right)\Big\}\Bigg]\end{aligned} \quad (3.7)$$

(c) Lost sales per cycle

$$C_{LS}=K_l\int_{t_1}^{t_1+t_2}\left\{\left(1-\frac{1}{1+\gamma(t_1+t_2-t)}\right)(a+bt+ct^2)\right\}dt$$

$$\begin{aligned}C_{LS}=K_l\Bigg[&\frac{1}{6\gamma^5}\Big\{6at_2\gamma^5+6bt_1t_2\gamma^5+3bt_2^2\gamma^5+6b\gamma^4t_2+2c\gamma^5t_2^3+6\gamma^5ct_1^2t_2+6c\gamma^5t_1t_2^2\Big\}\\&-6c\gamma^3t_2-9\gamma^4ct_2^2-18c\gamma^4t_1t_2-6a\gamma^4\log(1+\gamma t_2)-6b\gamma^3\log(1+\gamma t_2)-6b\gamma^4t_1\log(1+\gamma t_2)\\&-6\gamma^4t_2\log(1+\gamma t_2)+c\gamma^2\{1+\gamma(t_1+t_2)\}\log(1+\gamma t_2)\Bigg]\end{aligned} \quad (3.8)$$

(c) Purchase cost per cycle=(purchase cost per unit)*(order quantity in one cycle)

$$C_{PC}=C*Q \quad (3.9)$$

The maximum backordered inventory is obtained at $t=t_1+t_2$ then from equation

$$I_b=-I_2(t_1+t_2)$$

$$I_b = \frac{a}{\gamma}\log(1+\gamma t_2) + \frac{b}{\gamma^2}\{1+\gamma(t_1+t_2)\}\log(1+\gamma t_2) - \frac{bt_2}{\gamma}$$
$$+\frac{c}{\gamma^3}\{1+\gamma(t_1+t_2)\}^2\log(1+\gamma t_2) - \frac{ct_2}{2\gamma^2}\{2+\gamma(2t_1+t_2)\} \quad (3.10)$$

Thus the order size during total time interval [0,T],

$$Q = I_m + I_b$$

From equation (3.3) and (3.10),

$$Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right)\left(e^{\theta t_1} - 1\right) + \frac{a}{\gamma}\log(1+\gamma t_2) + \frac{b}{\gamma^2}\{1+\gamma(t_1+t_2)\}\log(1+\gamma t_2) - \frac{bt_2}{\gamma}$$
$$+\frac{c}{\gamma^3}\{1+\gamma(t_1+t_2\}^2\log(1+\gamma t_2) - \frac{ct_2}{2\gamma^2}\{2+\gamma(2t_1+t_2)\} \quad (3.11)$$

Thus $C_{PC} = C * Q$

$$C_{PC} = C\left[\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right)\left(e^{\theta t_1} - 1\right) + \frac{a}{\gamma}\log(1+\gamma t_2) + \frac{b}{\gamma^2}\{1+\gamma(t_1+t_2)\}\log(1+\gamma t_2)\right.$$
$$\left. - \frac{bt_2}{\gamma} + \frac{c}{\gamma^3}\{1+\gamma(t_1+t_2)\}^2\log(1+\gamma t_2) - \frac{ct_2}{2\gamma^2}\{2+\gamma(2t_1+t_2)\}\right] \quad (3.12)$$

(d) Ordering cost,

$$C_{OC} = K \quad (3.13)$$

Therefore, the total cost per time unit is given by

$$C(t_1,t_2,t) = \frac{1}{t_1+t_2}\left[C_{oc} + C_{HC} + C_{BC} + C_{LS} + C_{PC}\right] \quad (3.14)$$

Putting the values from the equations (3.6), (3.7), (3.8), (3.12) and (3.13) in equation (3.14), we get the total cost $C(t_1,t_2,t)$.

The necessary condition for the total cost per time unit, to be minimize is

$$\frac{\partial C(t_1,t_2,t)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial C(t_1,t_2,t)}{\partial t_2} = 0$$

Provided

$$\left(\frac{\partial^2 C(t_1,t_2,t)}{\partial t_1^2}\right)\left(\frac{\partial^2 C(t_1,t_2,t)}{\partial t_2^2}\right) - \left(\frac{\partial^2 C(t_1,t_2,t)}{\partial t_1 \partial t_2}\right) > 0 \text{ and } \left(\frac{\partial^2 C(t_1,t_2,t)}{\partial t^2}\right)^2 > 0$$

IV. CONCLUDING REMARK

The purpose of this study is to present an inventory model for deteriorating items in which demand rate is considered as quadratic function of time and time dependent holding cost and partially backlogging . This paper gives analytical solution of the model that minimize the total inventory cost . The model is very effective for the industries in which the demand rate and holding cost is depending upon the time .

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