



# PERFORMANCE EVALUATION OF A MODEL REFERENCE ADAPTIVE CONTROLLER (MRAC) ONTWIN ROTOR MIMO SYSTEM

Sukumar Bera<sup>1</sup>, Suman Singha Ray<sup>2</sup>, Chanchal Dey<sup>3</sup>

<sup>1,2</sup>M.Tech Scholar, <sup>2</sup>Assistant Professor, Department of Applied Physics,  
University of Calcutta (India)

## ABSTRACT

Twin rotor multi input multi output system (TRMS) is considered as a prototype laboratory set-up of helicopter, with significant cross coupling and nonlinearities. Such a plant is a well accepted benchmark system to test and explore modern control methodologies. In this paper we have designed and verified the performance of a model reference adaptive controller (MRAC) on a TRMS process. The performance of MRAC during tracking in vertical and horizontal planes is found to be quite satisfactory compared to conventional PID controller. Performance based analysis substantiates the superiority of MRAC technique during both tracking and load regulation phases.

**Keywords:** Model reference adaptive control, Modified MIT rule, Normalized algorithm, TRMS process.

## I. INTRODUCTION

Constant-gain feedback controllers proposed in early fifties fail to provide the better performance for the different operating conditions in high performance aircrafts. Only adaptive controllers that could modify its own behavior through parameter updation depending on load variables and variation in the aircraft dynamics are able to offer acceptable performance. Model reference adaptive control (MRAC) technique is attempted [1] to solve the autopilot design problem for high-performance aircraft. It is considered as an adaptive servo system in which the desired performance is expressed in terms of reference model, which gives the desired response to a command signal. Twin rotor multi input multi output system (TRMS) is considered to be a well-known laboratory prototype with a significant cross coupling and nonlinearities for realising aero-dynamic behaviour of a helicopter.

A number of control strategies [2-5] based on conventional and soft-computing approaches are available in literature towards controlling the TRMS process. A hybrid fuzzy PID controller is developed in [6] for achieving improved responses from a TRMS process. Performance analysis reveals that fuzzy-PID controller reported in [6] outperforms a conventional PID controller. The common difficult task for all the reported works in controlling a TRMS process is to deal with its nonlinear characteristics. Nonlinearity occurs due to variation in process

dynamics, changes in environmental conditions and variation in the nature of the disturbances. As a result the required controller must be adaptive and robust to accommodate these changes.

To overcome the limitations of constant gain feedback controllers, this paper deals with the designing of a MRAC scheme using the modified MIT rule [7]. Here, modified MIT rule is chosen for designing MRAC [8-10] to make the controller insensitive to the changes in the amplitude of command signal. Performance of the proposed MRAC scheme is compared with conventional PID controller through simulation study based on a number of performance indices. A brief description regarding TRMS process is given in section 2. Design of MRAC scheme is provided in section 3. Section 4 provides simulation results of the proposed MRAC in comparison with PID controller. Responses along with performance indices during tracking and load rejection phases substantiate the improved performance of MRAC in comparison with conventional PID controller. Conclusion is given in section 5.

## II. SYSTEM DESCRIPTION

TRMS process is considered as a model of a helicopter with some significant simplifications. The schematic diagram for TRMS is shown in Fig. 1. The TRMS process consists of a tower with a beam attached by two bearings. These bearings allow the beam to move freely in the horizontal and vertical planes within some limits. At the two ends of the beam, rotors are attached which are shifted by  $90^\circ$  from each other allowing them to generate horizontal and vertical thrusts. The main rotor and the tail rotor are used for varying the pitch angle and yaw angle respectively. The two rotors are placed on the opposite sides with a counter balance in between. Counter balance is used for proper balancing to the system. The whole unit is attached to a mechanical support to safely perform experimental studies.

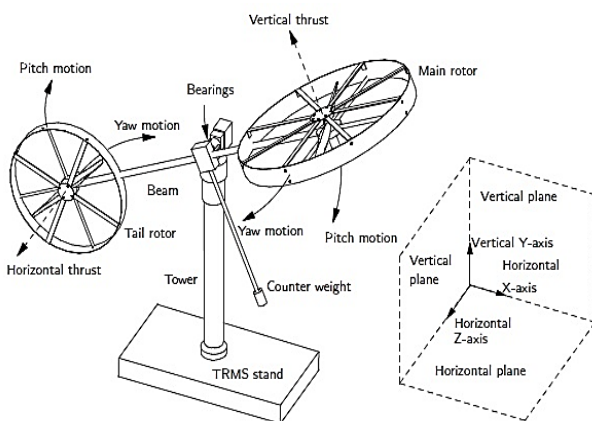


Fig.1: Schematic diagram for TRMS.

The TRMS process consists of a tower with a beam attached by two bearings. These bearings allow the beam to move freely in the horizontal and vertical planes within some limits. At the two ends of the beam, rotors are attached which are shifted by  $90^\circ$  from each other allowing them to generate horizontal and vertical thrusts. The main rotor and the tail rotor are used for varying the pitch angle and yaw angle respectively. The two rotors are placed on the opposite sides with a counter balance in between. Counter balance is used for proper balancing to the system. The whole unit is attached to a mechanical support to safely perform experimental studies.

Dynamics of TRMS has two degrees of freedom - the rotors can rotate about a vertical and horizontal plane. However, it can be transformed to 1DOF by locking either pitch or yaw whatever motion we want to control. The electrical drive unit of TRMS placed under the support allows easy transfer of signals from the sensors to PC and control signal from PC serial port to drive unit via DAQ card [11].

Dynamics of TRMS has two degrees of freedom - the rotors can rotate about a vertical and horizontal plane. However, it can be transformed to 1DOF by locking either pitch or yaw whatever motion we want to control. The electrical drive unit of TRMS placed under the support allows easy transfer of signals from the sensors to PC and control signal from PC serial port to drive unit via DAQ card [11].

## III. MODEL REFERENCE ADAPTIVE CONTROL (MRAC)

### 3.1 Principle of MRAC

The general idea behind the model reference adaptive control (MRAC) [7, 12] is that to design a closed loop controller with parameters which can be updated with the change in response of the system. The output of the system is compared with a given reference model. The deviations between the response obtained from desired

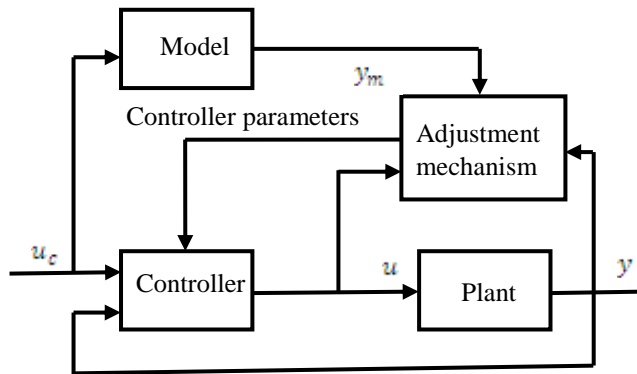


Fig. 2: Block diagram of MRAC.

model and actual process generates an error signal which is used for continuous updating of the controller parameters. Here, the goal is targeted for the parameters to converge to the ideal values that cause the plant response to match the response obtained from the reference model.

The basic block diagram of MRAC scheme is shown in the Fig.2. It has an ordinary feedback loop composed of the process and the controller and another feedback loop that changes the

controller parameters. The parameters of the controller are altered by adjustment mechanism so that plant response attempts to track the responses given by reference model. The algorithm for adjustment mechanism is based on MIT rule. Here, we are using MIT rule with normalized algorithm and the technique is referred as Modified MIT rule to make the controller behaviour independent of command signal amplitudes.

3.2 MIT rule

MRAC control strategy is obtained using gradient decent approach of MIT rule. According to the gradient decent approach, a cost function  $J(\theta)$  is considered in terms of tracking error  $(e)$ . Cost function  $J$  is dependent on  $\theta$  where  $\theta$  is the parameter that will be adapted to minimize the cost function. The tracking error  $(e)$  is defined as difference between the responses obtained from reference model  $(y_m)$  and the plant  $(y)$  to be controlled i.e.

$$e = y - y_m \tag{1}$$

The choice of cost function will determine how the parameters are updated. The typical cost function is given by

$$J(\theta) = \frac{1}{2} e^2 \tag{2}$$

According to the MIT rule, rate of change of  $\theta$  is directly proportional to negative gradient of cost function, as shown in the following equation:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \cdot e \frac{\partial e}{\partial \theta} \tag{3}$$

where  $\theta$  = controller parameter vector,  $\gamma$  = adaptation gain and  $\frac{\partial e}{\partial \theta}$  = sensitivity derivative. Sensitivity derivative determines how the error  $(e)$  is influenced by adjustable parameter  $\theta$ . A controller may contain several parameters that require updating.

There is no particular rule to choose the loss function. We can also choose  $J(\theta) = |e|$  and henceforth the gradient method gives

$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} \text{sign}(e) \tag{4}$$

But here we have chosen  $J(\theta) = \frac{1}{2} e^2$ .



### 3.3 Normalized algorithm

For large values of reference input, system may become unstable when the system is controlled by MRAC using simple MIT rule because it is very sensitive to the changes in the amplitude of the command signal. Hence to overcome this problem, normalized algorithm is used to the MIT rule to develop the control law.

Normalized algorithm modifies the adaptation law in the following manner,

$$\frac{d\theta}{dt} = \frac{\gamma \varphi e}{\alpha + \varphi^T \varphi} \quad (5)$$

Where  $\varphi = -\partial e / \partial \theta$  and  $\alpha (\alpha > 0)$  is introduced to remove the difficulty of division by zero when  $\varphi$  is small.

Eq. (5) is also applicable during the conditions when there is more than one adjustable parameter. With the above modifications using normalized algorithm, the adaptation law is referred as modified MIT rule [13].

Another important fact for designing MRAC is selection of an appropriate reference model. Normally, the reference model is so selected by the designer that it offers the desirable response from the system under all possible operating conditions.

## IV. SIMULATION RESULTS

In this paper, the MRAC technique is implemented for TRMS process in Matlab/Simulink environment. Initially, we consider individual 1DOF model for vertical and horizontal motion of TRMS independently. Thereafter 2DOF model is chosen for TRMS process and its performance is studied in presence of decoupler [11]. Here we use a mixed sinusoidal signal with different frequencies as command signal for pitch  $\psi_{desired}(t)$  and yaw  $\theta_{desired}(t)$  positions. Initially controller performances are studied in absence of any external disturbance and thereafter performances are observed in presence of both band-limited white noise and pulse nature disturbances. To find out the effectiveness of MRAC, we have compared its performance with the conventional PID controller. Performance of the reported controllers are evaluated and compared in terms of set point tracking as well as disturbance rejection phases. The performance indices IAE (Integral Absolute Error), ITAE (Integral

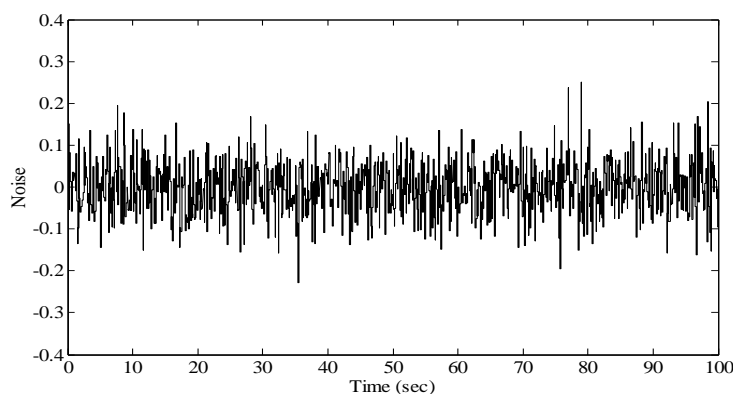


Fig. 3: Band-limited white noise.

Time Absolute Error), and TV (Total Variation of control signal) are computed for each setting. Lesser value of performance indices justifies the superiority of MRAC scheme in comparison with PID controller as reported in following Tables I-II.

The band-limited white noises used in this study are of different power in performance study with 1DOF pitch and yaw and 2DOF pitch and yaw dynamics. Fig. 3 shows a white noise signal with noise power 0.04 and variance 0.01 used in simulation study for TRMS process.

#### 4.1 Comparative study during tracking phase

##### 4.1.1 1DOF pitch response

Comparative performances between MRAC and PID controller for controlling only pitch motion i.e., 1DOF responses during tracking of TRMS in shown Fig.4 and Fig.5 respectively.

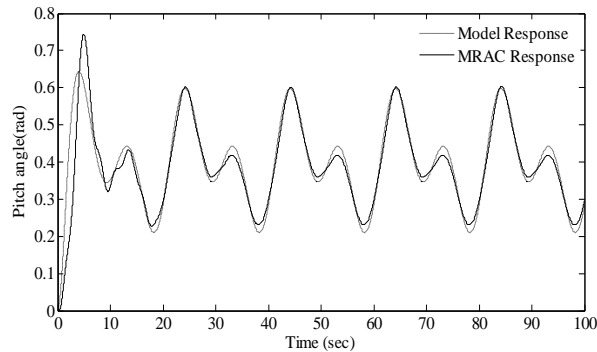


Fig. 4: MRAC response for 1DOF pitch.

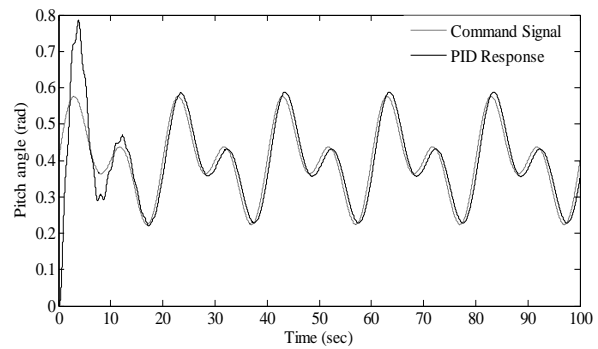


Fig. 5: PID response for 1DOF pitch.

##### 4.1.2 1DOF yaw response

Responses of MRAC and PID controllers for controlling only yaw motion i.e. 1DOF responses of TRMS is shown in Fig.6 and Fig.7 respectively.

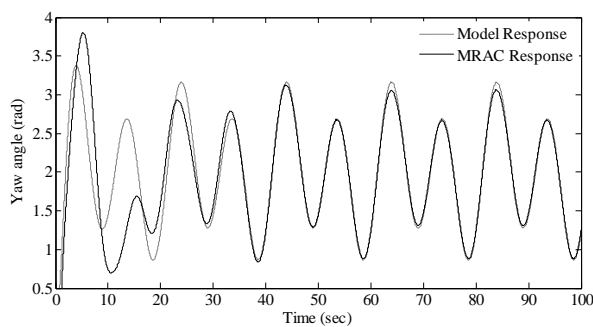


Fig.6: MRAC response for 1DOF yaw.

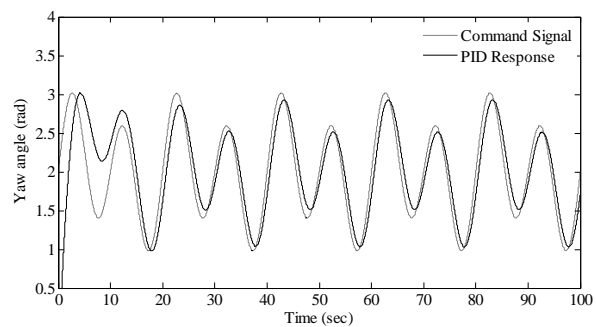


Fig.7: PID response for 1DOF yaw.

##### 4.1.3 2DOF pitch response

Responses of MRAC and PID controllers for controlling 2DOF pitch motion of TRMS is shown in Fig.8 and Fig.9 respectively.

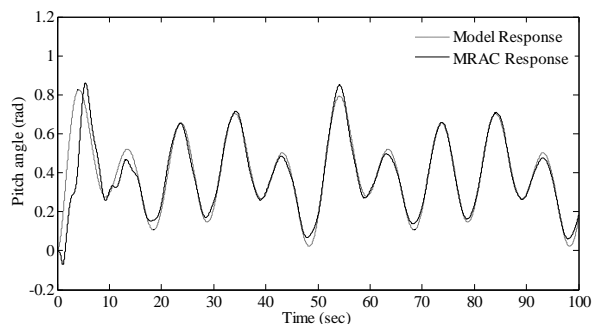


Fig.8: MRAC response for 2DOF pitch.

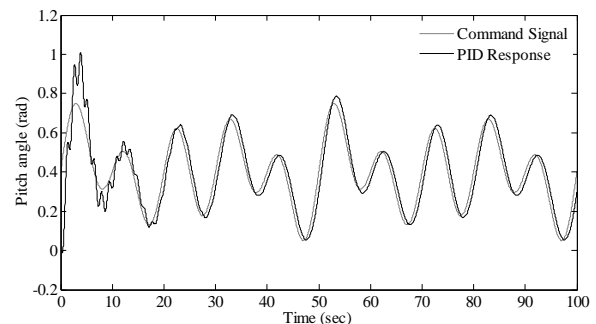


Fig.9: PID response for 2DOF pitch.

##### 4.1.4 2DOF yaw response

Responses of MRAC and PID controllers for controlling 2DOF yaw motion of TRMS is shown in Fig.10 and Fig.11 respectively.

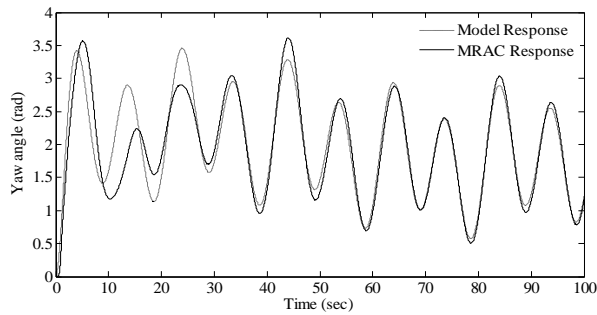


Fig.10: MRAC response for 2DOF yaw.

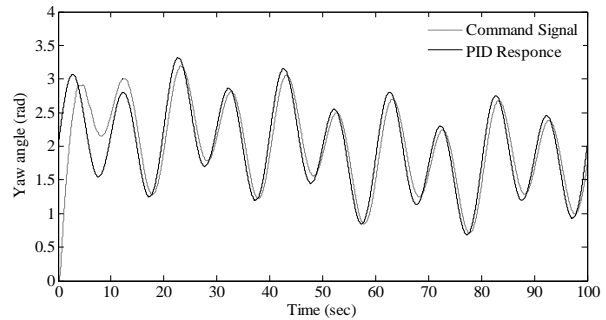


Fig.11: PID response for 2DOF yaw.

For having a clear comparison among the reported controllers in addition to response the values of performance indices (IAE, ITAE and TV) for MRAC and PID controllers are depicted in Table 1.

Table I

TRMS Dynamics	IAE		ITAE		TV	
	PID	MRAC	PID	MRAC	PID	MRAC
1DOF pitch	3.23	2.12	115.70	67.76	0.70	0.20
1DOF yaw	23.84	17.27	866.90	316.10	0.83	0.50
2DOF pitch	5.31	3.41	224.30	83.72	4.07	0.76
2DOF yaw	23.00	18.60	846.80	521.30	1.47	1.94

## 4.2 Performance analysis during tracking in presence of white noise and disturbance

### 4.2.1 1DOF pitch response

Comparative study between MRAC and PID controller for controlling only pitch motion of TRMS in presence of white noise and disturbances is shown in Fig.12 and Fig.13 respectively. Whitenoise power is 0.001 and variance is 0.01. The pulse like disturbances are given at time 50-52s and 70-72s.

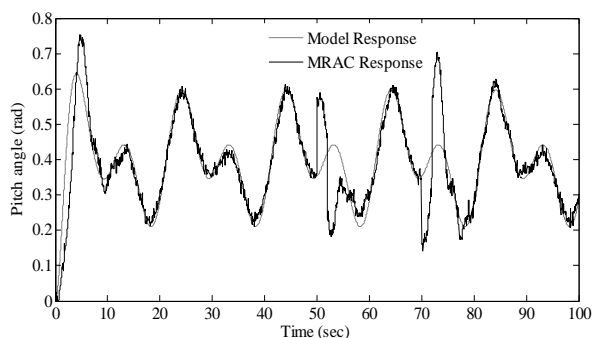


Fig.12: MRAC response for 1DOF pitch in presence of white noise and disturbance.

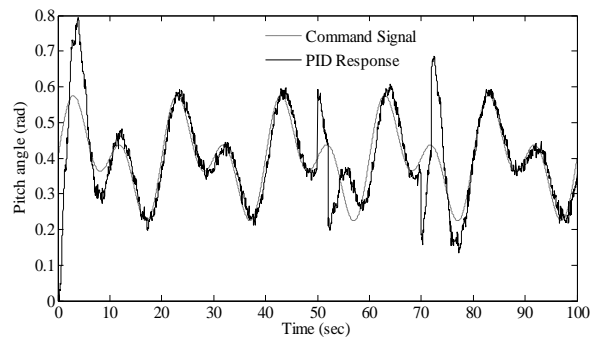
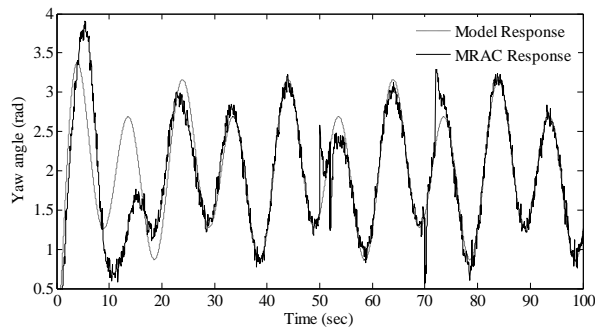


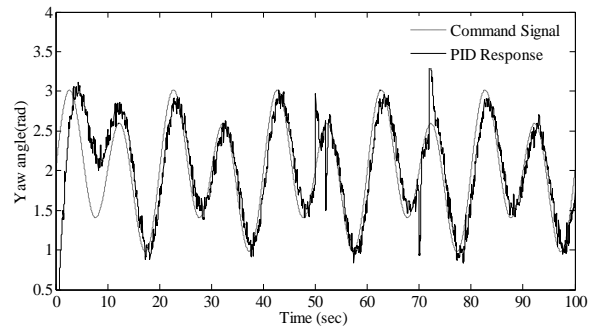
Fig.13: PID response for 1DOF pitch in presence of white noise and disturbance.

### 4.2.2 1DOF yaw response

Performances of MRAC and PID controllers for controlling only yaw motion of TRMS in presence of white noise and external disturbances are shown in Fig.14 and Fig.15. Whitenoise power is 0.04 and variance is 0.01 respectively. Pulse like disturbances are given at time 50-52s and 70-72s in opposite directions.



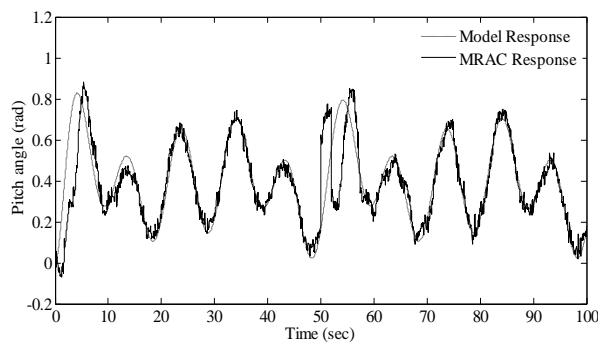
**Fig.14: MRAC response for 1DOF yaw in presence of white noise and disturbance.**



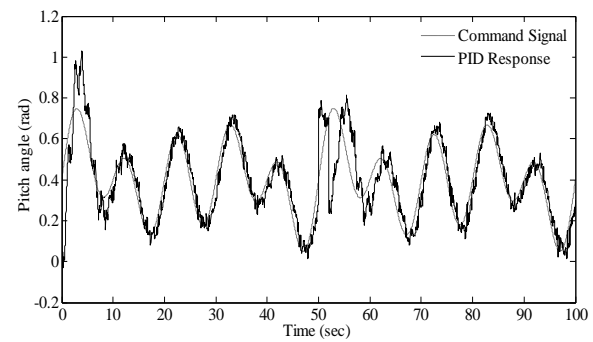
**Fig.15: PID response for 1DOF yaw in presence of white noise and disturbance.**

#### 4.2.3 2DOF pitch response

Similar to previous study, responses of MRAC and PID controllers for controlling 2DOF pitch motion of TRMS in the presence of white noise and disturbances is shown in Fig.16 and Fig.17. Whitenoise power is 0.005 and variance is 0.01. Pulse nature external disturbance is given at time 55-57s.



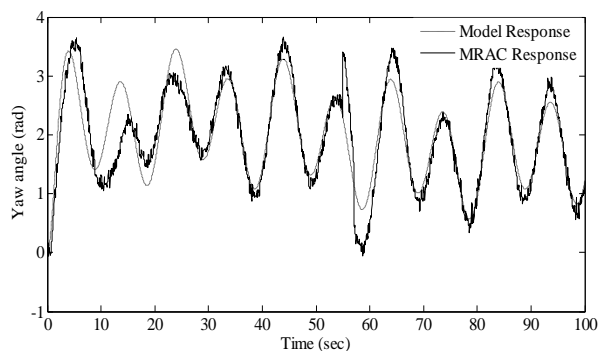
**Fig.16: MRAC response for 2DOF pitch in presence of white noise and disturbance.**



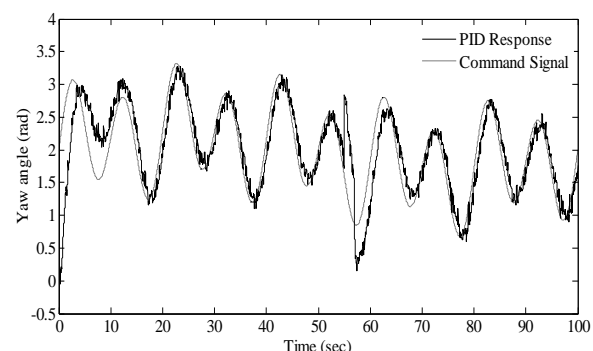
**Fig.17: PID response for 2DOF pitch in presence of white noise and disturbance.**

#### 4.2.4 2DOF yaw response

Comparative study between MRAC and PID controllers for controlling 2DOF yaw motion of TRMS in the presence of white noise and disturbances are shown in Fig.18 and Fig.19 respectively. Whitenoise power is 0.04 and variance is 0.01. The disturbance given at time 55-57s.



**Fig.18: MRAC response for 2DOF yaw in presence of white noise and disturbance.**



**Fig.19: PID response for 2DOF yaw in presence of white noise and disturbance.**



For a clear comparison between MRAC and PID controllers, values of the performance indices (IAE, ITAE and TV) for respective controllers are shown in the Table II.

Table II

TRMS Dynamics	IAE		ITAE		TV	
	PID	MRAC	PID	MRAC	PID	MRAC
1DOF pitch	4.75	3.93	208.20	179.30	2.89	2.69
1DOF yaw	25.94	22.55	1010.00	639.90	848.10	635.60
2DOF pitch	7.23	6.22	333.10	240.50	10.29	4.31
2DOF yaw	27.23	26.76	1103.00	1055.00	10.34	77.31

The related tuning parameters used for MRAC are given in Table III.

Table III

TRMS Dynamics	MRAC constant values		MRAC adaptation gains	
	$\alpha_1$	$\alpha_2$	$\gamma_1$	$\gamma_2$
1DOF pitch	0.001	1.0	-1.9	0.8
1DOF yaw	1.2	1.2	-1.0	1.0
2DOF pitch	0.1	1.0	-1.9	1.4
2DOF yaw	1.0	1.6	-1.4	1.3

V. CONCLUSION

In this paper, model reference adaptive control scheme for controlling the pitch and yaw motions of TRMS process is reported. This controlling method shows better adaptation capability when compared with the conventional fixed gain PID method in terms of IAE and ITAE values. Moreover, it is also found in most of the cases improved responses from MRAC are obtained at a lesser TV values compared to PID. It is also seen from the results that MRAC provides much better results for 1DOF pitch and 1DOF yaw responses compared to PID when noise and disturbance are introduced to the system. Similarly, for 2DOF pitch and yaw responses MRAC also shows satisfactory results compared to PID in presence of noise and disturbance. In future fuzzy rule based inference can be used to select more appropriate values of variable adaptation gains compared to their fixed values so that an overall improved responses can be obtained.

REFERENCES

[1] H. P. Whitaker, J. Yamron and A. Kezer, Design of MRAC system for aircraft, Report no. R-164 Instrumentation Lab, MIT, 1958.



- [2] S. M. Ahmad, A. J. Chipperfield, and M. O. Tokhi, Dynamic modelling and control of a 2 DOF twin rotor multi-input multi-output system, Proc. of Conf. of IEEE Industrial Electronics Society, Nagoya, Japan, 2000, 451–456.
- [3] S. Mondal and C. Mahanta, Adaptive second-order sliding mode controller for a twin rotor multi-input–multi-output system, IET Control Theory & Applications, 6(14), 2012, 2157–2167.
- [4] A. Bayrak, M. H. Salah, N. Nath, and E. Tatlicioglu, Neural network-based nonlinear control design for twin rotor MIMO systems. In Proc. of Int. Symposium on Mechanism and Machine Science, Izmir, Turkey, 2010, 172–178.
- [5] P. Biswas, R. Maity, A. Kolay, K. D. Sharma and G. Sarkar, PSO Based PID Controller Design for Twin Rotor MIMO System, International Conference on Control, Instrumentation Energy and Communication (CIEC), 2014, 106-110.
- [6] A. Rahidehand M. H. Shaheed, Hybrid fuzzy-PID-based control of a twin rotor MIMO system, in Proceedings of the 32nd Annual Conference on IEEE Industrial Electronics (IECON '06), 2006, 49–54.
- [7] K. J. Astrom and B. Wittenmark, Adaptive control, (2nd ed., Dover Publications, New York, 2001).
- [8] P. Swarnkar, S. K. Jain and R. K. Nema, Effect of adaptation gain on system performance for model reference adaptive control scheme using MIT rule. International Conference of World Academy of Science, Engineering and Technology, Paris, 2010, 70-75.
- [9] M. S. Ehsani, Adaptive Control of Servo Motor by MRAC Method, IEEE International Conference on Vehicle, Power and Propulsion, Arlington, 2007, 78 – 83.
- [10] M. Kirar, P. Swarnkar, S. Jain and R. K. Nema, Comparative Study of Conventional and Adaptive Schemes for DC Servomotors, International Conference on Energy Engineering ICEE, Puducherry, India, 2009.
- [11] Twin Rotor MIMO System Manual, (Feedback Instruments Ltd. U. K. 33-949S, 2002).
- [12] S. R. Kolapalli, Study of Discrete time Model Reference Adaptive Control Using Kalman Filter, Jadavpur University, Kolkata, M. Tech, 2010.
- [13] P. Jain and M.J. Nigam, Design of a Model Reference Adaptive Controller Using Modified MIT Rule for a Second Order System, Advance in Electronic and Electric Engineering. ISSN 2231-1297, 3(4), 2013, 477-484.