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AN ALGORITHM FOR SOLVING LINEAR FRACTIONAL PROGRAMMING PROBLEM

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ABSTRACT

In this paper, an algorithm is developed for solving Linear Fractional Programming (LFP) problem. First, the LFP problem will be converted into single Linear Programming Problem (LPP) and then it can be solved. We then compare our method with other existing methods for solving LPF problems. This method of LFP problem is comparatively simple and easy to understand and apply.

Keywords: Linear Programming, Linear Fractional Programming Problem.

I. INTRODUCTION

In mathematical optimization,linear-fractional programming (LFP) is a generalization of linear programming(LP). Whereas the objective functions in linear programs are linear functions, the objective function in a linear-fractional program is a ratio of two linear functions. A linear program can be regarded as a special case of a linear-fractional program in which the denominator is the constant function one. LFP problems have attracted considerable research and interest, since they are useful in production planning, financial and corporate planning, health care and hospital planning.

Several methods are proposed to solve this problem. Charnes and Cooper (1962) have proposed a method which transforms the LPF into an equivalent LP, they concluded that feasible region is non-empty and bounded. Another method, known as updated objective function method was derived by Bitran and Novaes (1972). This method is used to solve the LFP by solving a sequence of LP only re-computing the local gradient of the objective function. Hasan and Acharjee (2011) also develop a method for solving LFP by converting it into a single LP, but for negative value of β their method failed. Sahaet. al. (2015) has developed an algorithm which overcame the shortfall of Hasan et.al proposal.

II. STEPS IN CONSTRUCTING AN LFP MODEL

Step I: Identify the unknown variables to be determined (decision variables) and represent them in terms of algebraic symbol.

StepII:Identify all the constraints or restrictions in the problem and express them as linear equation or inequalities, which are linear functions of the unknown variables.

Step III: identify the objective function (which is to be maximized or minimized), which is the ratio of two linear functions of the decision variables.

Let the objective function be,

$$Maximize\, Z = \frac{cx+\alpha}{dx+\beta}$$
$$subject\ to\ \ Ax \le b$$
$$and\ \ x \ge 0$$

III. ALGORITHM

Step 1: Assume that β≠0.

Step 2: Check:

(a) If β > 0, then go to step (3), otherwise go to (b)
(b) If β < 0, α ≥ 0, then go to step (4), otherwise go to (c)
(c) If β < 0, α < 0, then go to step (5)

Step 3: Calculate and then go to Step (6),

For the objective function

$$Z = \frac{cx+\alpha}{dx+\beta}$$
$$= \frac{cx+\alpha}{dx+\beta} - \frac{\alpha}{\beta} + \frac{\alpha}{\beta}$$
$$= \frac{c\beta-\alpha d}{\beta(dx+\beta)} + \frac{\alpha}{\beta}$$
$$\therefore\ H(y) = Iy + J$$

For constraints, $Ax \le b$

$$\frac{AyB}{1-yd} \le b$$
$$\Rightarrow\ y \le \frac{b}{A\beta+bd}$$
$$\Rightarrow\ (A\beta+bd) \le b$$
$$\therefore\ Ky \le L$$

Step 4: Calculate and then go to Step (6),

For the objective function

$$Z = \frac{cx+\alpha}{dx+\beta}$$

$$\frac{Z+1}{Z-1} = \frac{cx+\alpha+dx-\beta}{cx+\alpha-dx+\beta}$$

$$= \frac{(c+d)x+\alpha-\beta}{(c-d)x+\alpha+\beta}$$

$$= \frac{c'x+\alpha'}{d'x+\beta'}$$

$where,\ \alpha' = \alpha-\beta,\ \beta' = \alpha+\beta,\ c' = c+d, d' = c-d.$

Now solve this function similarly as in the Step 4, we get

$H'(y) = I'y + J'$

For constraints, following the same procedure as in Step (4), we get

$K'y \le L'$

Step 6: Calculate and then go to Step (6),

For the objective function

$$Z = \frac{cx-\alpha}{dx-\beta}$$

$$= \frac{-cx+\alpha}{-dx+\beta}$$

Now following the same procedure to solve this function as in Step (4), we get

$H_1(y) = I_1 y + J_1$

For constraints, following the same procedure as in Step (4), we get

$K_1 y \le L_1$

Step 6: Express the new LP into its standard form.

Step 7: Find all m X m sub-matrices of the new coefficient matrix by setting n – m variables equal to zero.

Step 8: Test whether the linear system of equation has unique solution or not.

Step 9: If the linear system of equation has got any unique solution, find it.

Step 10: Dropping the solution with negative elements, determine all basic feasible solution.

Step 11: For the maximization (or minimization) of LP the maximum (or minimum) value of F(y) id optimal value of the objective function and the basic feasible solution which yields the value of y.

Step 12: Find the value of *x* using the value of y from the required formula.

Step 13: Finally putting the value of x in the original LFP, we obtain the optimal value of LFP.

IV. CONCLUSION

In this paper, we have developed an algorithm for solving linear fractional programming problem. At first we transform the LFP problems into LPP and then solve them by simplex method. This method presented here will reduce the number of iterations for a problem with larger dimensions. So, we conclude that, this is better than the other well-known methods.

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