



THEORETICAL ANALYSIS AND EXPERIMENTAL DETERMINATION OF NATURAL FREQUENCY OF LATERAL VIBRATION OF A BEAM

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ABSTRACT

Most structural and machine members frequently faces vibration problems due to unbalance in the components arising from operating conditions (like varying forces on a railway sleeper), faulty design, defective manufacture, improper assembly, or poor maintenance. This project aims at theoretically analysing the lateral vibrations of a fixed-fixed beam and predict its first natural frequency. Beams of different widths, thicknesses and lengths have been used and their natural frequencies determined. An experimental setup has been fabricated in the laboratory to simulate the sleeper and its natural frequency has been determined experimentally. The external excitation is provided by using a small variable speed motor. Speed of rotation is measured using a tachometer. Displacement of the plate is obtained by fixing a sketch pen on to the plate and allowing the tip to make its mark on a graph sheet wound on a rotating drum. It is observed that the theoretically predicted values and experimentally obtained results coincide within permissible limits of experimental error, for the various cases studied.

Hence the theory is valid and can be extended to beams of any other isotropic material. Since sleepers were earlier made of wood and recently are of reinforced concrete, theoretical analysis is now done for determination of natural frequency of lateral vibration for these cases. The forcing frequency when a train travelling at 110 kmph moves over the railroad is calculated. It is found that the first natural frequency of vibration is almost 18 times that of the forcing frequency. Since the second, third, etc.. frequencies are much higher, the danger of resonance does not exist. For wooden sleepers the first natural frequency(61.77 Hz) is about 8.1 times that of the forcing frequency (7.64 Hz). Hence concrete sleepers are more safe than wooden sleepers from vibration considerations. The present experimental setup can be used for experimental determination of the natural frequency of any beam of any size by suitable dimensional similitude.

Keywords: *Analysis of Natural Frequency ,Vibration Analysis,Natural Frequency of Railway Concrete Sleeper, Wooden Sleeper, Their Mathematical Analysis*

I. INTRODUCTION

Most human activities involve vibration in one form or the other. For example, we hear our eardrums vibrate and we see because light waves undergo vibration. Breathing is associated with the vibration of lungs walking involves periodic oscillatory motion of legs and hands.

In recent times , many investigations have been motivated by the engineering applications of vibration, such as the design of machines , foundations , structures, engines, turbines, and control systems

Most prime movers have vibration problems due to the inherent unbalance in the engines. The unbalance may be due to faulty designs ,or poor manufacture, imbalance in diesel engines. for example , can cause ground waves sufficiently powerful to create a nuisance in urban area.

The theory of vibration deals with the study of oscillatory motion of bodies and the forces associated with them. A vibratory system , in general , includes a means for storing potential energy(spring or elasticity) , a means for storing kinetic energy (mass or inertia) , and a means by which energy is gradually lost(damper).

Any motion that repeats itself after an interval of time is called vibration. Vibration problems frequently occur where there are unbalanced rotating or reciprocating parts in a machine.

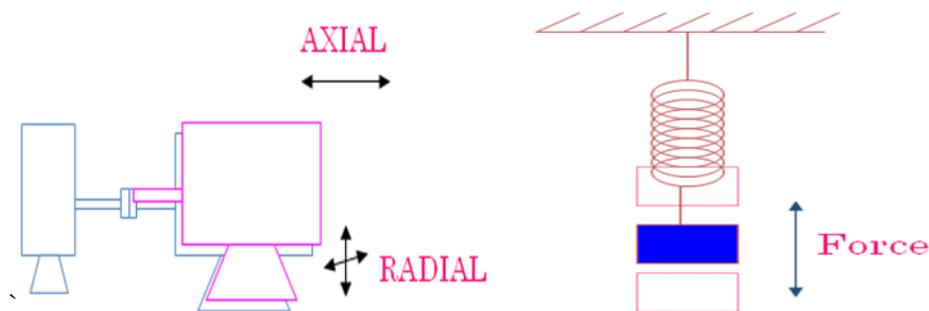


Fig 1.1 Forces in a Vibrating Body

Vibration is simply the continuous motion of a machine or machine component back and forth from its position of equilibrium. The simplest way to understand vibration is to follow the motion of a weight suspended on a spring. Until a force is applied to the weight to cause it to move, there are no vibrations. Vibration is the response of a system to some internal or external excitation or force applied to the system.

Vibration starts when an external force is applied and persists even after the initial disturbing force is removed. If the external force has the same frequency as the natural frequency of vibration of the member, then it experiences extremely large oscillations and this dangerous phenomenon is called resonance. Vibration starts when an external force is applied and persists even after the initial disturbing force is removed. If the external force has the same frequency as the natural frequency of vibration of the member, then it experiences extremely large oscillations and this dangerous phenomenon is called resonance. Resonance results in noise, breakage, accidents, loss of life, loss of production and so on, and must be avoided at all costs.

Determination of natural frequency of a structure or member helps the designer avoid resonance and is thus crucial from safety, economic and social points of view.

1.2 Methods to Control the Vibration

The following methods are to control the vibration.

- 1) Avoiding resonance under external excitations.
- 2) Preventing excessive response of the system, even at resonance by introducing a damping (or) energy dissipating mechanism.
- 3) By reducing the response of the system, by the addition of an auxiliary mass or vibration absorber. From these methods, we discuss the popular and versatile methods for providing vibration absorbers.



II. HEADINGS

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 - 1.1 Importance of the study of vibration
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2. Experimental Setup & Procedure
 - 2.1 Description of Experimental Setup
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 - 4.1 Theory of Lateral Vibration of Beams
 - 4.2 Differential Equation:
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III. INDENTATIONS AND EQUATIONS

Theory of Lateral Vibration of Beams :

$$(1) \rho A(x) dx \frac{\partial^2 w}{\partial t^2}(x, t)$$

$$(2) -(V + dV) + f(x,t)dx + V = \rho A(x) dx \frac{\partial^2 w}{\partial t^2}(x, t)$$

$$(3) (M + Dm) - (V + Dv)dx + f(x,t) dx \frac{dx}{2} - M = 0$$

$$(4) dV = \frac{\partial V}{\partial x} dx \quad \text{and} \quad dM = \frac{\partial M}{\partial x} dx$$

$$-\frac{\partial V}{\partial x}(x,t) + f(x, t) = \rho A(x) \frac{\partial^2 w}{\partial t^2}(x, t).$$

$$(5) C = \sqrt{\frac{EI}{\rho A}}$$

$$(6) \omega = \beta^2 \sqrt{\frac{EI}{\rho A}} = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \dots$$

$$(7) \cos \beta L \cosh \beta L = 1$$

IV. FIGURES AND TABLES

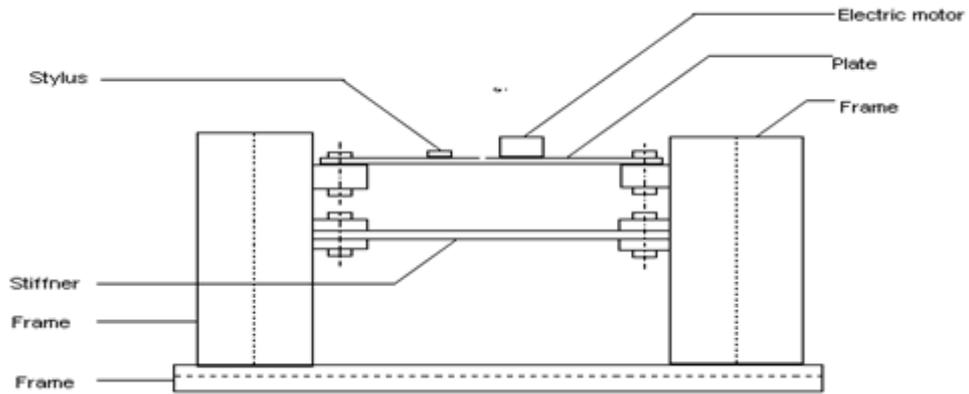


Fig1. Schematic Diagram of Experimental Setup



Fig 3. A Marker Plot The Graph on a Rotor for Resonance of a Beam



Fig 4. Channel of $750 \times 82 \times 40\text{mm}$

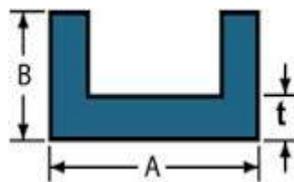


Fig 4.1 Front view of channel



Fig 5. Graph Result without Resonance

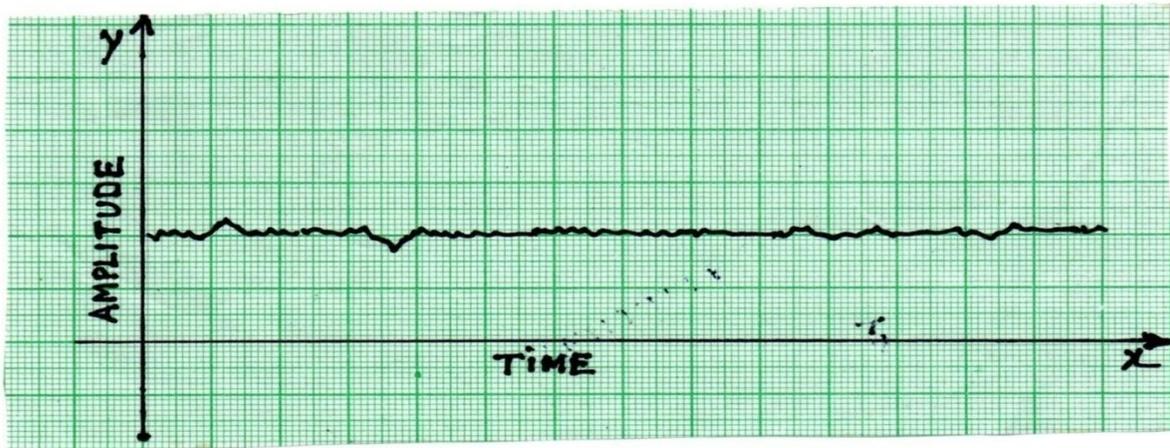


Fig 5.1 Graph Result with Resonance

- Before conducting of experiment, first find natural frequency of beam.
- It is considered as fixed-fixed beam.

$$L = 4.730041$$

β = Frequency related term

$$\rho = 7830 \text{ kg/m}^3$$

ρ = Density of material for steel,

$$= 7830 \text{ kg/m}^3 \text{ for steel}$$

$$= 900 \text{ kg/m}^3 \text{ for wood}$$

$$= 4107 \text{ kg/m}^3 \text{ for concrete}$$

A = Cross sectional Area of beam in

E = Young's modulus N/m^2

I = Moment of inertia, m^4

Plate No.	Dimensions			$\beta_1 = 4.73/L$	ω_1	$f_{1=}$ $\omega_1/2\pi$	Natural Frequency (rpm)		
	b	d	L				I_{st}	2_{nd}	3_{rd}
1.	75 mm	6 mm	750 mm	6.306 7	348	55.41	3325	9182.6	17999
2.	50 mm	6 mm	500 mm	9.46	783.4	124.6 8	7481	20620	40426

3.	50 mm	6 mm	375 mm	12.613	1392.7	221.6	13,305	36,649	71868
4.	90 mm	6 mm	750 mm	6.3067	348.1	55.41	3331	9,182	17,967
5.	40 mm	6 mm	500 mm	9.46	783.4	124.68	7481	20612	4,0422
6.	82mm	5mm	750m m	6.3067	-	36.56	20,949	57,735	113,198

$$I = bd^3/12$$

Table .1: Natural Frequencies for different sizes of plates

PLATE NO	Theoretical value in R.P.M	Experimental value in R.P.M	% of Error value
1.	3,324	3,546	6.6%
2.	7,481	7,230	-3.3%
3.	3,305	3,400	2.87%
4.	3,331	3,536	6.15%
5.	7,481	7,324	-2.09%

Table : 2 Comparison of Theoretical and Experimental values

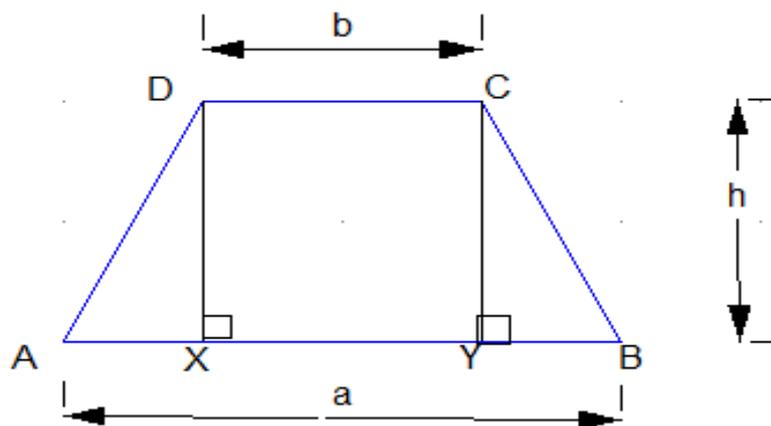


Fig 6. Concrete sleeper dimensions

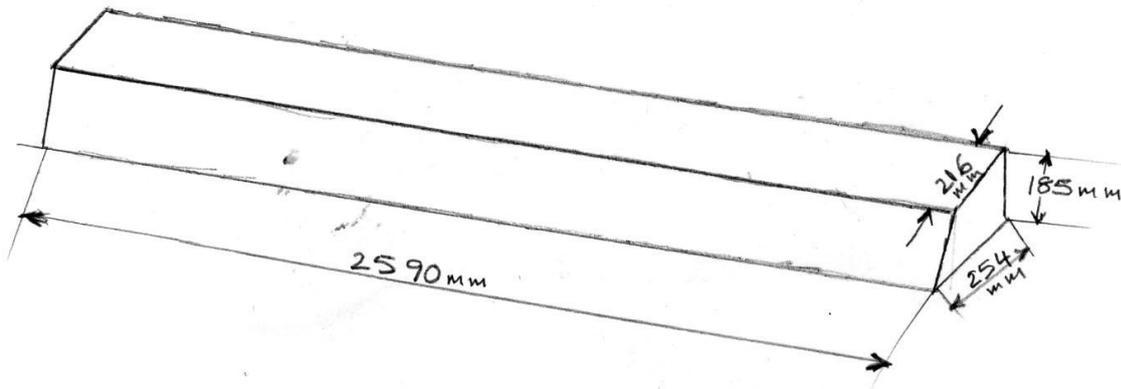


Fig: 7.Trapezoidal concrete beam

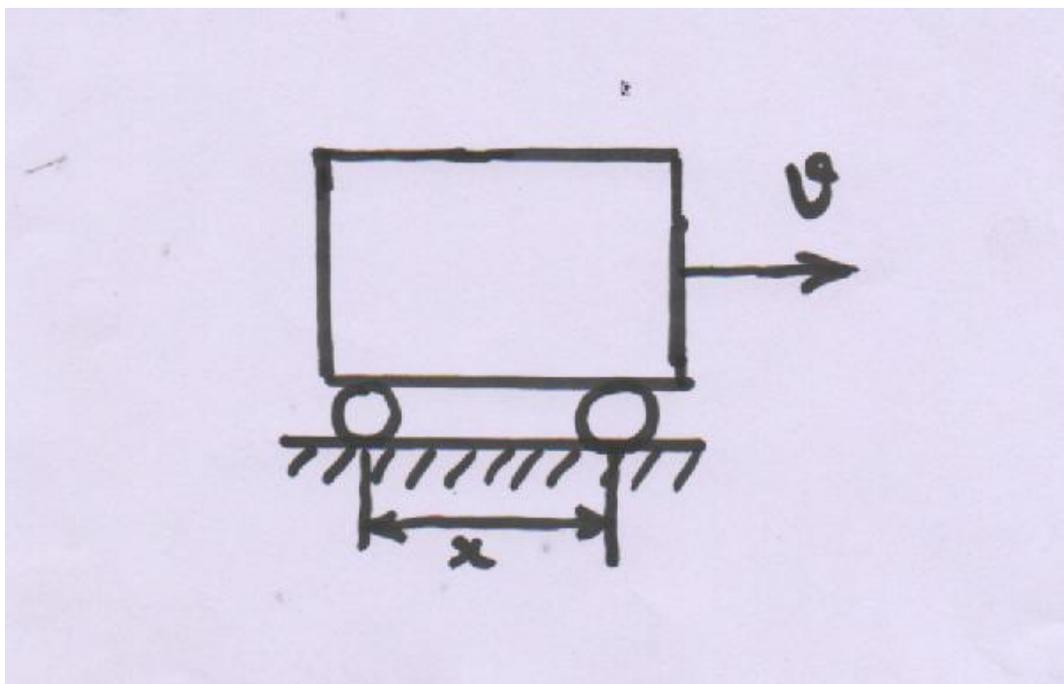


Fig 7.Schematic Diagram of Forcing frequency of a Railway wagon

$$\rho = 4107 \text{ kg/m}^3$$

$$A = 43.475 \times 10^{-3} \text{ m}^2$$

$$E = 17e9 \text{ N/m}^2$$

$$I = bd^3/12 = 1.238e-4 \text{ m}^4$$

$$L = 2.321 \text{ m}$$

$$= 4.73/2.321 = 2.0379 \text{ /m}$$

$$= \rho A /EI = 62.83$$

$$= 735,373.16 \text{ /S}^2$$

$$= 857.538 \text{ rad/s}$$

$$f = \omega/2\pi = 136.5 \text{ cycles/sec}$$

N= 8189 rpm

There is a resonance occurred at the speed at 136.46 rpm.

V. CONCLUSION

The first natural frequency of concrete sleeper is about 9.4 times that of the forcing frequency . Hence resonance will not occur . Since second, third , fourth.. natural frequencies are much higher , the possibilities of resonance at these modes is ever more remote.

Comparing the values for wooden sleeper , we conclude that concrete sleepers are safer than wooden sleepers from resonance point of view.

Using more sophisticated vibration pickups , one can extend this work to more complicated systems than the simple ones studied in this project .

One can also studied experimentally the mode shapes at various forcing frequencies and compare them with the theoretical results.

Those the studied has been limited to fixed-fixed beams , experimental results can be straight away extended to free-free beam.since, the governing frequency equation

$(\cos\beta l \times \cosh\beta l = 1)$ happens to be same for those two kinds of beams. In engineering practice the free-free beam occurs very often , one example is the railway sleepers.

Another common example is an aeroplane during flight.

In this explained how the theoretical value of the natural frequency of lateral vibration of a concrete railway sleeper and also a wooden sleeper may be calculated.

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