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CONTRA PRE-γ-CONTINUOUS MAPPINGS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce and investigate some new classes of mappings called contra pre-γ-continuous mappings and almost contra pre-γ-continuous mappings via pre-γ-open sets. Also, some of their fundamental properties are studied.

Keywords: Contra Pre-γ-Continuous, Almost Contra Pre-γ-Continuous.

I. INTRODUCTION

In the recent literature, many topologists had focused their research in the direction of investigating different types of generalized continuity. Dontchev [1] introduced a new class of mappings called contra-continuity. Jafari and Noiri [2, 3] exhibited and studied among others a new weaker form of this class of mappings called contra-α-continuous and contra-pre-continuous mappings. Also, a new weaker form of this class of mappings called contra-semicontinuous mappings was introduced and investigated by Dontchev and Noiri [4]. Contra-δ-precontinuous mapping was obtained by Ekici and Noiri [5]. A good number of researchers have also initiated different types of contra continuous like mappings in the papers (Caldas and Jafari [6]; Ekici [7, 8]; Nasef [9]; Al-Omari and Noorani [10]; El-Magbrabi [11]). Ogata [12] introduced the notion of pre-γ-open sets which are weaker than open sets. The concept of pre-γ-open sets and pre-γ-open maps in topological spaces are introduced by Hariwan Z. Ibrahim [13, 14]. This paper is devoted to introduce and investigate a new class of mappings called contra pre-γ-continuous mappings. Also, some of their fundamental properties are studied.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (simply, X and Y) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure of subset A of X, the interior of A and the complement of A is denoted by $cl(A)$, $int(A)$ and A^c or $X\backslash A$ respectively. A subset A of a space (X, τ) is called regular open [15] if $A = int(cl(A))$. An operation γ [12] on a topology τ is a mapping from τ in to power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [12] if for each $x \in A$, there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open sets in X . Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-open sets are called γ- closed. The τ_γ –interior [16] of A is denoted by $\tau_\gamma\text{-}int(A)$ and defined to be the union of all γ-open sets of X contained in A. A subset A of a space X is said to be pre-γ-open [13] if $A \subseteq \tau_\gamma\text{-} int(cl(A))$.

DEFINITION 2.1.[14] A subset A of X is called pre-γ-closed if and only if its complement is pre-γ-open. Moreover, pre-$\gamma O(X)$ denotes the collection of all pre-γ-open sets of (X, τ) and pre-$\gamma C(X)$ denotes the collection of all pre-γ-closed sets of (X, τ).

DEFINITION 2.2.[14] Let A be a subset of a topological space (X, τ). The intersection of all pre-γ-closed sets containing A is called the pre-γ-closure of A and is denoted by pre-$\gamma Cl(A)$.

DEFINITION 2.3.[14] A subset N of a space (X, τ) is called a pre-γ-Neighborhood (briefly, pre-γ-nbd) of a point $p \in X$ if there exists a pre-γ-open set W such that $p \in W \subseteq N$. The class of all pre-γ-nbds of $p \in X$ is called the pre-γ –neighborhood system of p and denoted by pre-γ-N_p .

DEFINITION 2.4.[14] A mapping $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) pre-γ-continuous if $f^{-1}(V) \in$ pre-γO(X) for every open set V of Y,

(ii) pre-γ-irresolute if $f^{-1}(V) \in$ pre-γO(X) for every pre-γ-open set V of Y.

DEFINITION 2.5. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) contra-continuous [1] if $f^{-1}(V)$ is closed in X for each open set in Y,

(ii) almost contra-continuous [17] if $f^{-1}(V)$ is closed in X for each regular open set of Y.

DEFINITION 2.6. Let A be a subset of space (X, τ) . Then:

(i) the kernal of A [18] is given by $ker(A) = \cap\{U \in \tau : A \subseteq U\}$,

(ii) the pre-γ-boundary of A [19] is given by pre-γb(A) = pre-$\gamma Cl(A) \setminus$ pre-$\gamma int(A)$.

LEMMA 2.1.[20] The following properties are holds for two subsets A, B of a topological space (X, τ) :

(i) $x \in ker(A)$ if and only if $A \cap F \neq \emptyset$, for any closed set F of X containing x,

(ii) $A \subseteq ker(A)$ and $A = ker(A)$, if A is open in X,

(iii) If $A \subseteq B$, then $ker(A) \subseteq ker(B)$.

DEFINITION 2.7. A topological space (X, τ) is said to be:

(i) Urysohn [21] if, for each two distinct points x, y of X, there exist two open sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$,

(ii) ultra Hausdorff [22] if, for each two distinct points x, y of X, there exist two closed sets U and V such that $x \in U$, $y \in V$, and $U \cap V = \emptyset$

(iii) ultra normal [22] if for each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

(iv) weakly Hausdorff [23] if each element of X is the intersection of regular closed sets of X,

(v) strongly S-closed [24] (resp. S-closed [1], S-Lindeloff [25], countably S-closed [26]) if for closed (resp. regular closed, regular closed, countably regular closed) cover of X has a finite (resp. finite, countable, finite) subcover.

III. CONTRA PRE-γ-CONTINUOUS MAPPINGS

DEFINITION 3.1.A mapping $f : (X, \tau) \to (Y, \sigma)$ is called contra pre-γ-continuous, if $f^{-1}(U) \in$ pre-γC(X), for every open set U of Y.

THEOREM 3.1. For a mapping $f: (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:

(i) f is contra pre-γ-continuous,

(ii) for each $x \in X$ and each closed subset F of Y containing $f(x)$, there exist $U \in$ pre-$\gamma O(X)$ such that $x \in U$ and $f(U) \subseteq F$,

(iii) for every closed subset F of Y, $f^{-1}(F) \in$ pre-γO(X),

(iv) $f($pre-γ-$cl(A)) \subseteq ker(f(A))$, for each $A \subseteq X$,

(v) pre-γ-$cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$, for each $B \subseteq Y$.

PROOF.(i) $\Rightarrow$ (ii) Let $x \in X$ and F be any closed set of Y containing $f(x)$. Then $x \in f^{-1}(F)$. Hence by hypothesis, we have $f^{-1}(Y \setminus F)$ is pre-γ-closed in X and hence $f^{-1}(F)$ is pre-γ-open set of X containing x. We put $U = f^{-1}(F)$, then $x \in U$ and $f(U) \subseteq F$.

(ii) $\Rightarrow$ (iii) Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$. Hence by hypothesis, there exists a pre-γ-open subset U containing x such that $f(U) \subseteq F$, this implies that, $x \in U \subseteq f^{-1}(F)$. Therefore, $f^{-1}(F) = \cup\{U : x \in f^{-1}(F)\}$ which is pre-γ-open in X. Then f is contra pre-γ-continuous.

(iii) $\Rightarrow$ (iv) Let A be any subset of X and $y \in ker(f(A))$. Then by Lemma 2.1., there exists a closed set F of Y containing y such that $f(A) \cap F \neq \emptyset$. Hence, $A \cap f^{-1}(F) = \emptyset$ and pre-γ-$cl(A) \cap f^{-1}(F) = \emptyset$. Then $f($pre-γ-$cl(A)) \cap F = \emptyset$ and $y \in f($pre-γ-$cl(A))$. Therefore, $f($pre-γ-$cl(A)) \subseteq ker(f(A))$.

(iv) $\Rightarrow$ (v) Let B be any subset of Y. Then by hypothesis and Lemma 2.1., we have $f($pre-γ-$cl(f^{-1}(B))) \subseteq ker(f(f^{-1}(B))) \subseteq ker(B)$. Thus pre-$\gamma$-$cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$.

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(v) $\Rightarrow$ (i) Let V be any open subset of Y. Then by hypothesis and Lemma 2.1., pre- γ-$cl(f^{-1}(V)) \subseteq f^{-1}(ker(V)) = f^{-1}(V)$.Therefore, $f^{-1}(V)$ is pre-γ-closed in X. Hence f is contra pre-γ-continuous.

DEFINITION 3.2. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) pre-γ-continuous [14], if $f^{-1}(U) \in$ pre$-\gamma O(X)$, for each $U \in \sigma$,

(ii) pre-γ-irresolute [14], if $f^{-1}(U) \in$ pre-$\gamma O(X)$, for each $U \in$ pre-$\gamma O(Y)$,

(iii) pre $*$ - γ -open [19], if $f(U) \in$ pre-$\gamma O(Y)$ for each $U \in$ pre-$\gamma O(X)$,

(iv) pre $*$ - γ -closed [19], if $f(U) \in$ pre$-\gamma C(Y)$ for each $U \in$ pre-$\gamma\, C(X)$.

THEOREM 3.2. If a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra pre-γ-continuous and Y is regular, then f is pre- γ -continuous.

PROOF. Let $x \in X$ and V be an open set of Y containing $f(x)$. Since Y is a regular space, then there exists an open set G of Y such that $f(x) \in G \subseteq cl(G) \subseteq V$.But, if f is contra pre-γ-continuous, then there exists $U \in$ pre-γO(X) such that $x \in U$ and $f(U) \subseteq cl(G) \subseteq V$. Hence, f is pre-γ-continuous.

The next theorems give the conditions under which the composition of two contra pre-γ-continuous mapping is also contra-pre-γ-continuous.

REMARK 3.1. The composition of two contra pre-γ-continuous mappings need not be contra pre-γ-continuous as shown by the following example.

EXAMPLE 3.1 Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau_X = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, $\tau_Y = \{\emptyset, Y, \{d\}\}$ and $\tau_Z = \{\emptyset, Z, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ be an identity map and $g : (Y, \tau_Y) \rightarrow (Z, \tau_Z)$ be defined as $g(a) = a,\ g(b) = b,\ g(c) = a,\ g(d) = d$ and define an operation γ on τ_X by

$$\gamma(A) = \begin{cases} int(cl(A)) & if\ A \neq \{a\} \\ cl(A) & if\ A = \{a\} \end{cases}$$

and define an operation γ on τ_Y is $\gamma(A) = A$. Clearly f and g are contra pre-γ-continuous. But $(g \circ f)$ is not a contra pre-γ -continuous, since $\{a\} \in \tau_Z$ is a open set of Z, $(g \circ f)^{-1}(\{a\}) = \{a, c\} \notin$ pre-γC(X).

THEOREM 3.3. For two mappings $f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ and $g : (Y, \tau_Y) \rightarrow (Z, \tau_Z)$, the following properties are hold:

(i) If f is contra pre-γ -continuous and g is continuous mappings, then $g \circ f$ is contra pre-γ-continuous.

(ii) If f is pre-γ-irresolute and g is contra pre-γ-continuous mappings, then $g \circ f$ is contra-pre-γ-continuous.

PROOF. (i) Let $U \in \tau_Z$ and g be a continuous mapping. Then $g^{-1}(U) \in \tau_Y$. But, f is contra pre-γ-continuous then $(g \circ f)^{-1}(U) \in$ pre-$\gamma C(X)$. Hence $g \circ f$ is contra pre- γ-continuous.

(ii) Let $U \in \tau_Z$ and g be a contra pre-γ-continuous mapping. Then $g^{-1}(U) \in$ pre-γC(Y) . But f is pre-γ-irresolute, then $(g \circ f)^{-1}(U) \in$ pre-$\gamma C(X)$. Hence, $g \circ f$ is contra pre-γ-continuous.

THEOREM 3.4. Let $f : X \to Y$ be a surjective pre- γ -irresolute and pre *- γ -open mapping. Then $g \circ f : X \to Z$ is contra pre-γ-continuous if and only if g is contra pre-γ-continuous.

PROOF. Necessity: Obvious from Theorem 3.3..

Sufficiency: Let $g \circ f : X \to Z$ be a contra pre-γ-continuous mapping and F be closed set of Z . Then $(g \circ f)^{-1}(F) \in$ pre- $\gamma O(X)$ .Since f is surjective pre*-γ-open, then $g^{-1}(F) \in$ pre- $\gamma O(Y)$. Therefore g is contra pre-γ-continuous.

DEFINITION 3.3. A topological space (X, τ) is called:

(i) pre-γ-connected [27] if X cannot be expressed as the union of two disjoint non-empty pre-γ-open sets of X .

(ii) pre-γ-normal, if every pair of disjoint closed sets F_1 and F_2 there exist disjoint pre-γ-open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

(iii) pre- γ- T_1 -space [13], if for every two distinct points x, y of X , there exists two pre-γ-open sets U, V such that x ∈ U, $y \notin U$ and $x \notin V$, $y \in V$.

(iv) pre-γ- T_2 -space or pre-γ-Hausdorff space [13] if for every two distinct points x, y of X , there exist two disjoint pre-γ-open sets U, V such that $x \in U$ and $y \in V$.

THEOREM 3.5. If $f:(X, \tau) \to (Y, \sigma)$ is an injective closed and contra pre-γ-continuous mappings, and Y is ultra normal, then X is pre-γ-normal.

PROOF. Let F_1 and F_2 be two disjoint closed subsets of X . Since f is closed injection, then $f(F_1)$ and $f(F_2)$ are two disjoint closed subsets of Y and since Y is ultra normal space, then there exist two disjoint clopen sets U and V such that $f(F_1) \subseteq U$ and $f(F_2) \subseteq V$. Hence, $F_1 \subseteq f^{-1}(U)$ and $F_2 \subseteq f^{-1}(V)$. Since f is injective contra pre-γ-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are two disjoint pre-γ-open sets of X . Therefore, X is pre-γ- normal.

THEOREM 3.6. If $f:(X, \tau) \to (Y, \sigma)$ is a contra-pre-γ-continuous mapping and X is pre-γ-connected then Y is not a discrete space.

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PROOF. Suppose that Y is a discrete space and U any subset of Y. Then U is open and closed set in Y. Since f is contra pre-γ-continuous, $f^{-1}(U)$ is pre-γ-closed and pre-γ-open in X which is a contradiction with the fact X is pre-γ-connected. Hence, Y is not discrete space.

THEOREM 3.7. If $f:(X,\tau)\to(Y,\sigma)$ is an injective contra pre-γ-continuous mapping and Y is an Urysohn space, then X is pre-γ-T_2.

PROOF. Let $x, y \in X$ and $x \neq y$. By hypothesis, $f(x) = f(y)$. Since Y is an Urysohn space, there exist two open sets U and V of Y such that $f(x) \in U$, $f(y) \in V$ and $cl(U) \cap cl(V) = \emptyset$. Since f is contra pre-γ-continuous, then there exist two pre-γ-open sets P and Q such that $x \in P$, $y \in Q$ and $f(P) \subseteq cl(U)$, $f(Q) \subseteq cl(V)$. Then $f(P) \cap f(Q) = \emptyset$ and hence, $P \cap Q = \emptyset$. Therefore, X is pre-γ-T_2.

COROLLARY 3.1. If $f:(X,\tau)\to(Y,\sigma)$ is an injective contra pre-γ-continuous mapping and Y is an ultra Hausdorff space, then X is pre-γ-T_2.

DEFINITION3.4. A mapping $f:(X,\tau)\to(Y,\sigma)$ is called weakly- pre-γ-continuous, if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in$ pre-$\gamma O(X)$ such that $x \in U$ and $f(U) \subseteq cl(V)$.

THEOREM 3.8. If $f:(X,\tau)\to(Y,\sigma)$ is a contra pre-γ-continuous mapping, then f is weakly-pre-γ-continuous.

.**PROOF.** Let $x \in X$ and $V \in \sigma$ containing $f(x)$. Then $cl(V)$ is closed set in Y. Since f is contra pre-γ-continuous, then $f^{-1}(cl(V)) \in$ pre-$\gamma O(X)$ and containing x. If we put $U = f^{-1}(cl(V))$, then $f(U) \subseteq cl(V)$. Hence, f is weakly-pre-γ- continuous.

REMARK 3.2. The converse of Theorem 3.8. is not true as shown by the following example.

EXAMPLE 3.2. Let $X = Y = \{a,b,c,d\}$ with topologies $\tau_X = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $\tau_Y = \{\emptyset, Y, \{a\}, \{b\}, \{a,b\}\}$. Let $f:(X,\tau_X)\to(Y,\tau_Y)$ be defined as $f(a)=a,\ f(b)=c,\ f(c)=d,\ f(d)=c$ and define an operation γ on τ_X by $\gamma(A) = \begin{cases} int(cl(A)) & if\ A \neq \{a\} \\ cl(A) & if\ A = \{a\}. \end{cases}$

Then a mapping f is weakly pre-γ-continuous. But it is not contra pre-γ-continuous, since $f^{-1}(\{a\}) = \{a\} \notin$ pre-$\gamma C(X)$.

IV. ALMOST CONTRA-PRE- γ -CONTINUOUS MAPPINGS

DEFINITION 4.1.A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is called almost contra pre- γ -continuous if for each $x\in X$ and each open set V of Y containing $f(x)$, there exists $U\in$ pre- $\gamma O(X)$ such that $x\in U$ and $f(U)\subseteq int(cl(V))$, equivalently, $f^{-1}(V)$ is pre-γ-open in X for every regular open set V of Y.

THEOREM 4.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is called almost pre- γ-continuous if and only if for each $x\in X$ and each regular open set V of Y containing $f(x)$, there exists $U\in$ pre- γO(X) containing x such that $f(U)\subseteq V$.

PROOF. Necessity. Let $V\subseteq Y$ be regular open set containing (x). Then $x\in f^{-1}(V)$. But f is almost pre-γ-continuous, then $f^{-1}(V)=U$ is regular open set of X containing x such that $f(U)=ff^{-1}(V)\subseteq V$.

Sufficiency. Let $V\subseteq Y$ be regular open set. We need to prove that $f^{-1}(V)\in$ pre- $\gamma O(X)$. Suppose that $\in f^{-1}(V)$. Then $f(x)\in V$. By hypothesis, there exists $U\in$pre- $\gamma O(X)$ containing x such that $f(U)\subseteq V$. Hence $x\in U\subseteq f^{-1}(f(U))\subseteq f^{-1}(V)$ .Then $f^{-1}(V)=\cup\{U:x\in U\}$ is a pre-γ-open set of X. Therefore, f is almost pre- γ -continuous.

DEFINITION 4.2. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is called almost contra pre- γ-continuous if $f^{-1}(V)$ is pre-γ-closed in X, for every regular open set V of Y.

EXAMPLE 4.1 Let $X=Y=\{a,b,c,d\}$ with topologies $\tau_X=\{\emptyset,X,\{a\}\}$ and $\tau_Y=\{\emptyset,Y,\{a\},\{c\},\{a,c\}\}$. Let $f:(X,\tau_X)\rightarrow(Y,\tau_Y)$ be defined as $f(a)=d,\ f(b)=a,\ f(c)=c,\ f(d)=b$ and define an operation γ on τ_X by γ (A) $=\begin{cases} int(cl(A)) & if\ A\neq\{a\} \\ cl(A) & if\ A=\{a\}. \end{cases}$

Then a mapping f is almost contra pre- γ -continuous.

THEOREM 4.2. For a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$, the following statements are equivalent:

(i) f is almost contra-pre-γ-continuous,

(ii) $f^{-1}(F)$ is pre-γ-open in X, for every regular closed set F of Y, for each $x\in X$ and each regular closed set F of Y containing $f(x)$, there exists $U\in$pre-γ-$O(X)$ such that $x\in U$ and $f(U)\subseteq F$,

(iii) for each $x\in X$ and each regular open set V of Y not containing $f(x)$, there exists a pre-γ-closed set K of X not containing x such that $f^{-1}(V)\subseteq K$.

PROOF. (i) ⇒ (ii) Let F be any regular closed set of Y. Then $Y\setminus F$ is regular open. By hypothesis, $f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)\in$ pre-$\gamma C(X)$. Therefore, $f^{-1}(F)\in$ pre-$\gamma O(X)$.

(ii) $\Rightarrow$ (i) Obvious.

(ii) $\Rightarrow$ (iii) Let F be any regular closed set of Y containing $f(x)$. Then by hypothesis, $f^{-1}(F) \in$ pre-$\gamma O(X)$ and $x \in f^{-1}(F)$. Put $U = f^{-1}(F)$, then $f(U) \subseteq F$.

(iii) $\Rightarrow$ (ii) Let F be any regular closed set of Y and $x \in f^{-1}(F)$. By hypothesis, there exist $U \in$ pre-$\gamma O(X)$ such that $x \in U$ and $f(U) \subseteq F$. Hence, $x \in U \subseteq f^{-1}(F)$. That implies $f^{-1}(F) = \cup\{U : x \in f^{-1}(F)\}$. Therefore, $f^{-1}(F) \in$ pre-$\gamma O(X)$.

(iii) $\Rightarrow$ (i) Let V be any regular open set of Y non-containing $f(x)$. Then $Y \setminus V$ is regular closed set of Y containing $f(x)$. By (iii),there exists $U \in$ pre- $\gamma O(X)$ such that $x \in U$ and $f(U) \subseteq Y \setminus V$. Then $U \subseteq f^{-1}(Y \setminus V) \subseteq X \setminus f^{-1}(V)$ and so $f^{-1}(V) \subseteq X \setminus U$. Since $U \in$ pre-$\gamma O(X)$, then $X \setminus U = K$ is pre-γ-closed set of X not containing x and $f^{-1}(V) \subseteq K$.

(i) $\Rightarrow$ (iii) Obvious.

REMARK 4.1. The composition of two almost contra-pre-γ-continuous mappings need not be almost contra-pre-γ-continuous as shown by the following example.

EXAMPLE 4.2 Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau_X = \{\emptyset, X, \{a\}\}$, $\tau_Y = \{\emptyset, Y\}$ and $\tau_Z = \{\emptyset, Z, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau_X) \to (Y, \tau_Y)$, $g : (Y, \tau_Y) \to (Z, \tau_Z)$ be an identity map and define an operation γ on τ_X by $\gamma(A) = \begin{cases} int(cl(A)) & if\ A \neq \{a\} \\ cl(A) & if\ A = \{a\}. \end{cases}$

and define an operation on τ_Y is $\gamma(A) = A$. Clearly f and g are almost contra pre-γ-continuous. But $(g \circ f)$ is not a contra pre- γ -continuous, since $\{a\} \in \tau_Z$ is a regular open set of Z,

$(g \circ f)^{-1}(\{a\}) = \{a\} \notin$ pre- $\gamma C(X)$.

THEOREM 4.3. For two mappings $f : (X, \tau_X) \to (Y, \tau_Y)$ and $g : (Y, \tau_Y) \to (Z, \tau_Z)$, the following properties are hold:

(i) If, f is a surjective pre*-γ-open and $g \circ f : X \to Z$ is almost contra-pre-γ -continuous, then g is almost contra-pre-γ-continuous.

(ii) If, f is a surjective pre$*$-γ-closed and g ∘ f : X → Z is almost contra-pre-γ-continuous, then g is almost contra-pre- γ-continuous.

PROOF. (i) Let $V \subseteq Z$ be regular closed set. Since, $g \circ f$ is almost contra-pre- γ -continuous, then$(gof)^{-1}(V) \in$ pre- $\gamma O(X)$. But, f is surjective pre*-γ-open, then $g^{-1}(V) \in$ pre-$\gamma O(Y)$. Therefore, g is almost contra-pre-γ-continuous.

(ii) Obvious.

THEOREM 4.4. If $f : (X, \tau) \to (Y, \sigma)$, is an injective almost contra- pre-γ-continuous mapping and Y is weakly Hausdorff, then X is pre-γ- T_1 .

PROOF. Let x, y be two distinct points of X. Since f is injective, then $f(x) \neq f(y)$ and since Y is weakly Hausdorff, there exist two regular closed sets U and V such that $f(x) \in U, f(y) \notin U$ and $f(x) \notin V, f(y) \in V$. Since f is an almost contra-pre-γ-continuous, we have $f^{-1}(U)$ and $f^{-1}(V)$ are pre-γ-open sets in X such that $x \in f^{-1}(U), y \notin f^{-1}(U)$ and $x \notin f^{-1}(V), y \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence X is pre-γ-T_1.

DEFINITION 4.3. A topological space (X, τ) is said to be:

(i) pre-γ-compact if every pre-γ-open cover of X has finite subcover,

(ii) countably pre-γ-compact if every countable cover of X by pre-γ-open sets has a finite subcover,

(iii) pre-γ-Lindelöff if every pre-γ-open cover of X has a countable subcover.

THEOREM 4.5. If $f : (X, \tau) \to (Y, \sigma)$ is a surjective almost contra- pre-γ-continuous mapping, then the following statements are hold:

(i) If X is pre-γ-compact, then Y is S -closed,

(ii) If X is countably pre-γ-compact, then Y is countably S-closed,

(iii) If X is pre-γ-Lindelöff, then Y is S -Lindelöff.

PROOF. (i) Let $\{V_i : i \in I\}$ be any regular closed cover of Y and f be almost contra-pre-γ-continuous. Then $\{f^{-1}(V_i) : i \in I\}$ is pre-γ-open cover of X . But X is pre-γ-compact, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_i) : i \in I_0\}$, hence $Y = \cup \{f f^{-1}(V_i) : i \in I_0\}$ and then $Y = \cup \{V_i : i \in I_0\}$. Hence Y is S-closed.

(ii) Similar to (i).

(iii) Similar to (i).

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