



# NEW RELATION BETWEEN VOLUME THERMAL EXPANSION AND THERMAL PRESSURE FOR SOLIDS

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## ABSTRACT

A new relation between volume thermal expansion and thermal pressure for NaCl, KCl, MgO, CaO, Al<sub>2</sub>O<sub>3</sub>, and Mg<sub>2</sub>SiO<sub>4</sub> has been derived. The results from this relation have been compared with experimental data and the results obtained by Kushwah *et.al.*. It has been observed that the new relation is in close agreement with experimental data.

**Keywords:** Equation of State; Bulk Modulus; Thermal Pressure; Thermal Expansion.

## I INTRODUCTION

Volume expansion of solids due to rise in temperature is directly related to thermal pressure<sup>1,2</sup>. Anderson<sup>3</sup> gave a relation for thermal pressure as

$$P(V, T) = P(V, T_0) + \Delta P_{th} \quad \dots (1)$$

where V is the volume, T the temperature and T<sub>0</sub> is initial value of temperature taken here is equal to 300 K. The last term in Equation (1) represents the difference in values of thermal pressure at two temperatures T and T<sub>0</sub>. i.e.

$$\Delta P_{th} = P_{th}(T) - P_{th}(T_0) \quad \dots (2)$$

In Equation (1) and (2), it has been assumed that the thermal pressure is a function of temperature only. At atmospheric pressure, i.e. at P(V, T) = 0, we have:

$$P(V, T_0) = -\Delta P_{th} \quad \dots (3)$$

## II. METHOD OF ANALYSIS

The generalized Kushwah logarithmic EOS can be written as<sup>4</sup>

$$P(1-x)^{K_\infty'} = A_1 \ln(1+x) + A_2 [\ln(1+x)]^2 + A_3 [\ln(1+x)]^3 \quad \dots (4)$$

where  $x = (1 - V/V_0)$ ,  $V_0$  is the volume at atmospheric pressure, the constants  $A_1$ ,  $A_2$ , and  $A_3$  are defined by Kushwah *et al*<sup>5</sup>. as

$$A_1 = K_0 \quad \dots (5)$$

$$A_2 = K_0 \left( -\frac{K_0'}{10} + 1 \right) \quad \dots (6)$$

$$A_3 = K_0 \left( -\frac{K_0'^2}{50} - \frac{K_0'}{5} + 1 \right) \quad \dots (7)$$

where  $K_0$  and  $K_0'$  are isothermal bulk modulus and its first derivative at zero atmospheric pressure.

Values of  $K_0$  and  $K_0'$  for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  are given in Table 1.

**Table 1- Values of input data used in calculations, isothermal bulk modulus  $K_0$  (GPa) and pressure derivative of isothermal bulk modulus  $K_0'$  are at  $T = 300$  K and atmospheric pressure<sup>5</sup>.**

Solids	$K_0$	$K_0'$	$K_\infty'$
NaCl	24.0	5.38	3.23
KCl	17.0	5.46	3.28
MgO	162	4.15	2.49
CaO	111	4.85	2.91
$Al_2O_3$	252	3.99	2.39
$Mg_2SiO_4$	127	5.40	3.24

From Equation (3) and (4)  $\Delta P_{th}$  i.e. difference in thermal pressure can be obtained as

$$-\Delta P_{th} (1-x)^{K_\infty'} = A_1 \ln(1+x) + A_2 [\ln(1+x)]^2 + A_3 [\ln(1+x)]^3 \quad \dots (8)$$

Here  $V/V_0$  is the volume expansion with increase in temperature for solids at atmospheric pressure ( $P = 0$ )

We suggest a new relation for  $\Delta P_{th}$  for solids as

$$\Delta P_{th} = B_0 - B_1 \text{Exp} \left( -\frac{V/V_0}{B_2} \right) \quad \dots (9)$$



where  $B_0$ ,  $B_1$ , and  $B_2$  are constants and values of these constants are given in table 2 and obtained values of thermal pressure  $\Delta P_{th}$  and volume thermal expansion  $V/V_0$  for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  as a function of temperature as given in Table 3.

**Table 2----Values of constants  $B_0$ ,  $B_1$ , and  $B_2$  for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  obtained from mat lab.**

Solids	$B_0$	$B_1$	$B_2$
NaCl	3.7026	2471.01644	0.15377
KCl	2.59103	1866.23596	0.15197
MgO	28.65951	7937.78069	0.17782
CaO	18.42856	7675.73465	0.16579
$Al_2O_3$	50.878	7143.459	0.20226
$Mg_2SiO_4$	14.14074	15549.06156	0.14926

**Table 3 --- values of thermal pressure  $\Delta P_{th}$  and volume thermal expansion  $V/V_0$  for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  as a function of temperature.**

Solids	Temperature T (K)	Thermal pressure $\Delta P_{th}$ (GPa)		Volume thermal expansion $V/V_0$ Ref [3]	Thermal pressure $\Delta P_{th}$ (GPa) Eq. (9)
		Kushwah EOS Eq. (8)	Experimental Ref. [3]		
NaCl	300	0.00	0.00	1.0000	0.00
	350	0.14	0.14	1.0060	0.14
	400	0.28	0.28	1.0123	0.28
	450	0.43	0.43	1.0188	0.43
	500	0.57	0.57	1.0256	0.57
	550	0.71	0.71	1.0328	0.71
	600	0.85	0.85	1.0402	0.85
	650	0.99	0.99	1.0480	0.99
	700	1.13	1.13	1.0561	1.13
	750	1.26	1.27	1.0645	1.27
KCl	300	0.00	0.00	1.0000	0.00
	350	0.09	0.09	1.0054	0.09
	400	0.18	0.19	1.0112	0.19
	450	0.28	0.28	1.0172	0.28
	500	0.37	0.37	1.0235	0.37
	550	0.46	0.47	1.0301	0.47
	600	0.56	0.56	1.0370	0.56
	650	0.65	0.65	1.0442	0.65
	700	0.75	0.75	1.0517	0.75
	750	0.84	0.84	1.0594	0.84

	800	0.93	0.93	1.0675	0.93
	850	1.01	1.02	1.0759	1.02

**Table 3 --- - values of thermal pressure  $\Delta P_{th}$  and volume thermal expansion  $V/V_0$  for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  as a function of temperature**

Solids	Temperature T (K)	Thermal pressure $\Delta P_{th}$ (GPa)		Volume thermal expansion $V/V_0$ Ref [3]	Thermal pressure $\Delta P_{th}$ (GPa) Eq. (9)	
		Kushwah EOS Eq. (8)	Experimental Ref. [3]			
MgO	300	0.00	0.00	1.0000	0.00	
	400	0.53	0.54	1.0033	0.54	
	500	1.16	1.12	1.0073	1.12	
	600	1.76	1.73	1.0112	1.73	
	700	2.38	2.35	1.0153	2.35	
	800	3.02	2.98	1.0196	2.98	
	900	3.66	3.61	1.0240	3.61	
	1000	4.28	4.24	1.0284	4.24	
	1100	4.93	4.87	1.0331	4.87	
	1200	5.58	5.50	1.0379	5.50	
	1300	6.21	6.12	1.0427	6.12	
	1400	6.83	6.74	1.0476	6.74	
	1500	7.48	7.36	1.0528	7.36	
	1600	8.12	7.97	1.0581	7.97	
	1700	8.76	8.58	1.0635	8.58	
1800	9.37	9.20	1.0688	9.20		
CaO	300	0.00	0.00	1.0000	0.00	
	400	0.36	0.36	1.0033	0.36	
	500	0.74	0.74	1.0066	0.74	
	600	1.14	1.13	1.0106	1.13	
	700	1.54	1.53	1.0145	1.53	
	800	1.96	1.94	1.0186	1.94	
	900	2.35	2.34	1.0226	2.34	
	1000	2.74	2.74	1.0267	2.74	
	1100	3.15	3.13	1.0311	3.13	
	1200	3.56	3.53	1.0356	3.53	
	$Al_2O_3$	300	0.00	0.00	1.0000	0.00
		400	0.45	0.45	1.0018	0.45
500		1.00	0.98	1.0040	0.98	
600		1.56	1.55	1.0063	1.55	
700		2.17	2.15	1.0088	2.15	
800		2.79	2.76	1.0114	2.76	
900		3.45	3.43	1.0142	3.43	
1000		4.06	4.01	1.0168	4.01	
1100		4.68	4.64	1.0195	4.64	
1200		5.32	5.27	1.0223	5.27	

	1300	5.97	5.90	1.0252	5.90
	1400	6.56	6.53	1.0279	6.53
	1500	7.19	7.16	1.0308	7.16
	1600	7.81	7.79	1.0337	7.79
	1700	8.38	8.42	1.0364	8.42
	1800	9.01	9.05	1.0394	9.05

**Table 3 --- - values of thermal pressure  $\Delta P_{th}$  and volume thermal expansion  $V/V_0$  for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  as a function of temperature**

Solids	Temperature T (K)	Thermal pressure $\Delta P_{th}$ (GPa)		Volume thermal expansion $V/V_0$ Ref [3]	Thermal pressure $\Delta P_{th}$ (GPa) Eq. (9)
		Kushwah EOS Eq. (8)	Experimental Ref. [3]		
$Mg_2SiO_4$	300	0.00	0.00	1.0000	0.00
	400	0.35	0.36	1.0028	0.36
	500	0.74	0.75	1.0059	0.75
	600	1.16	1.16	1.0094	1.16
	700	1.57	1.57	1.0129	1.57
	800	1.98	1.98	1.0164	1.98
	900	2.37	2.40	1.0199	2.40
	1000	2.80	2.83	1.0238	2.83
	1100	3.22	3.25	1.0277	3.25
	1200	3.67	3.69	1.0320	3.69
	1300	4.11	4.13	1.0363	4.13
	1400	4.54	4.56	1.0407	4.56
	1500	4.97	5.00	1.0451	5.00
1600	5.40	5.43	1.0498	5.43	
1700	5.84	5.87	1.0547	5.87	

### III. CONCLUSION

Comparison of difference in thermal pressure with respect to volume expansion for NaCl, KCl, MgO, CaO,  $Al_2O_3$ , and  $Mg_2SiO_4$  is given in Table 3. Which show that values of  $\Delta P_{th}$  predicted from Equation (9) is accurate and close to experimental data as compared to Equation (8) proposed by Kushwah *et al.*

### REFERENCES

- 1 Kushwah S S & Shanker J, *Physica B*, 253, 1998, 90.
- 2 Shanker J & Kushwah S S, *High Temperature – High pressure*, 33, 2001, 207.
- 3 Anderson O L, *Equation of state of solids for geophysics and ceramic science*, (Oxford University Press, New York), 1995.
- 4 Kushwah S S, Shrivastava H C & Singh K S, *Physica B*, 388,2007,20.
- 5 Kushwah S S & Tomar Y S, *Indian Journal of Pure and Applied Physics*, 49 ,2011, 99