A NEW WINDOW FUNCTION FOR FIR FILTER DESIGN AND SPECTRAL ANALYSIS

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ABSTRACT

A new window function is presented which like the well known Hamming window offers a preferred property for use in signal spectrum analysis: the sum of window coefficients with its shifted version by half of the order is constant for the overlapped region in the time domain. In high orders, the new window has main-lobe width equal to Hamming window. For low orders, the window parameters are modified to have smaller main-lobe width compared to Hamming window, while maintaining smaller maximum side-lobe peak. The results indicate performance improvement of the proposed window compared to Kaiser and Gaussian windows. The FIR filters designed by windowing method show the efficiency of the new window.

Keywords: FIR, Gaussian, Kaiser, Proposed, Window

I.INTRODUCTION

Window functions are widely used in digital signal processing for the applications in signal analysis and estimation, digital filter design and signal processing. FFT windows reduce the effects of leakage but can not only eliminate leakage entirely. In effect, they only change the shape of the leakage. In addition each type of window affects the spectrum in a slightly different way. Many different windows have been proposed over time each with its own advantage and disadvantage relative to the others. Some are effective for specific types of signal types such as random or sinusoidal. Some improve the frequency resolution, that is, they make it easier to detect the exact frequency of a peak in the spectrum. Some improve the amplitude accuracy that is they most accurately indicate the level of the peak. The best type of window should be chosen for each specific application.

Two main applications of the windows in digital signal processing are: data analysis based on Fast Fourier Transform (FFT) and design of Finite Impulse Response filters from Infinite Impulse Response filters. For FFT analysis, windows are employed to suppress the so called "leakage effect", and for FIR filter design according to the "windowing method", Gibbs oscillations are attenuated. Desirable characteristics for a window in the frequency domain are small main-lobe width and side-lobe peak (high attenuation). However, these two requirements are contradictory, since for a given length, a window with a narrow main-lobe has a poor attenuation and vice versa. Also a third preferred property of window when applied in data spectrum analysis is

that, in the time domain, the sum of window function (w[n]) with its shifted version by M/2 samples (M is the window order) would be constant:

$$w[n] + w[n - M/2] = \text{constant}, \quad M/2 \leq n \leq M \tag{1}$$

In this paper, a new window function is presented which can be considered as a special case of the important class of windows, named raised cosine windows. The proposed window has 2~4 dB more side-lobe attenuation than that of Hamming window, while offering approximately the same main-lobe width and still satisfying the property in equation (1). The window parameters are modified to avoid the performance degradation for lower window lengths, which happens for Hamming window.

II.ANALYSIS OF WINDOWING TECHNIQUE

2.1 Types of Windows

There are different types of windows starting from simple type (rectangular window) to more complex type (Kaiser window). The main goal of these windows is to truncate the impulse response of the filter in order to generate fixed length filter.

2.1.1 Kaiser window

Syntax: w = kaiser(L,beta)

Description: It returns an L-point Kaiser window in the column vector w. Beta is the Kaiser window β parameter that affects the sidelobe attenuation of the Fourier transform of the window. The default value for beta is 0.5. Kaiser window is defined by:

$$w[n] = \begin{cases} \dfrac{I_0\left(\beta(1 - [(n - \frac{M}{2})/(\frac{M}{2})]^2)^{\frac{1}{2}}\right)}{I_0(\beta)} & 0 \leq n \leq M \\ 0 & \text{other} \end{cases} \tag{2}$$

2.1.2 Hamming window

Syntax: w = hamming(L)

Description: It returns an L-point symmetric Hamming window in the column vector w. L should be a positive integer. The coefficients of a Hamming window are computed from the following equation. Hamming window is optimized to minimize the maximum side lobe. It is defined by:

$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{M}\right), & 0 \leq n \leq M \\ 0 & \text{other} \end{cases} \tag{3}$$

2.1.3 Gaussian window

Syntax: w = gausswin(N), w = gausswin(N,Alpha)

Description: It returns an N-point Gaussian window in the column vector w. L is a positive integer. The shape of this window is similar in the frequency domain because the Fourier transform of a Gaussian is also a Gaussian.

2.1.4 Taylor window

Syntax: w = taylorwin(n)

Description: A Taylor window allows to make tradeoffs between the mainlobe width and sidelobe level. The Taylor distribution avoids edge discontinuities.Taylor windows are typically used in radar applications, such as weighting synthetic aperature radar images and antenna design. It returns an n-point Taylor window in a column vector w. The values in this vector are the window weights or coefficients. n must be a positive integer.

2.1.5 Blackman window

Syntax: w = blackman(L)

Description: It returns the L-point symmetric Blackman window in the column vector w, where L is a positive integer. Blackman windows have slightly wider central lobes and less sideband leakage than equivalent length Hamming and Hann windows. Blackman window is defined by:

$$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) + 0.08\cos\left(\dfrac{4\pi n}{M}\right), & 0 \le n \le M \\ 0 & other \end{cases}$$

(4)

2.1.6 Rectangular window

Syntax: w = rectwin(L)

Description: It returns a rectangular window of length L in the column vector w. This function is provided for completeness; a rectangular window is equivalent to no window at all. It is defined by:

$$w[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & other \end{cases}$$

(5)

2.1.7 Hanning window

Syntax: w = hann(L)

Description: It returns an L-point symmetric Hann window in the column vector w. L must be a positive integer. The coefficients of a Hann window are computed from the following equation.

$$w[n] = \begin{cases} 0.5 - 0.5\cos\left(\dfrac{2\pi n}{M}\right), & 0 \le n \le M \\ 0 & other \end{cases}$$

(6)

2.2 Windowing Technique

The windowing method involves multiplying the ideal impulse response with a window function to generate a corresponding filter, which tapers the ideal impulse response. Like the frequency sampling method, the windowing method produces a filter whose frequency response approximates a desired frequency response. The windowing method, however, tends to produce better results than the frequency sampling method.

The impulse response of ideal filters is of infinite duration. It is not possible to evaluate the corresponding frequency response and implement the filter by hardware or software. Thus the impulse response must be truncated at both ends with respect to the central. Even the impulse response can be truncated when it is small enough but such a sudden cut off will cause some undesired effects. The window method will reduce them.

In the time domain windowing means to multiply the infinite impulse response $h_D(n)$ by a finite duration window function $w(n)$ to get a truncation. The resulted impulse response $h(n)$ is their product and is given as follows:

$$H(n) = h_D(n)*w(n) , \quad 0 \le n \le M \tag{7}$$

In the window design method we first evaluate the desired filter response $h_D(n)$ from the given desired frequency response $H_D(\omega)$ and then apply an appropriate window. Thus the method should be called Fourier window method. In this method, use is the made of the fact that the frequency response of the filter, $H_D(\omega)$ and the corresponding impulse response, $h_d(n)$ are related by inverse Fourier transform. The subscript D is used to distinguish between ideal and practical impulse response.

$$H_D(n) = \int_{-\pi}^{\pi} H_D(\omega)\, e^{j\omega n}\, d\omega \tag{8}$$

The basic idea behind the Window method of filter design is that the ideal frequency response of the desired filter is equal to 1 for all the pass band frequencies, and equal to 0 for all the stop band frequencies and then the filter impulse response is obtained by taking the Discrete Fourier Transform (DFT) of the ideal frequency response. Unfortunately, the filter response would be infinitely long since it has to reproduce the infinitely steep discontinuities in the ideal frequency response at the band edges. To create a Finite Impulse Response (FIR) filter, the time domain filter coefficients must be restricted in number by multiplying by a window function of a finite width. Many windows are used for truncating the signal and the simplest window function is the rectangular window which corresponds to truncating the sequence after a certain number of terms.
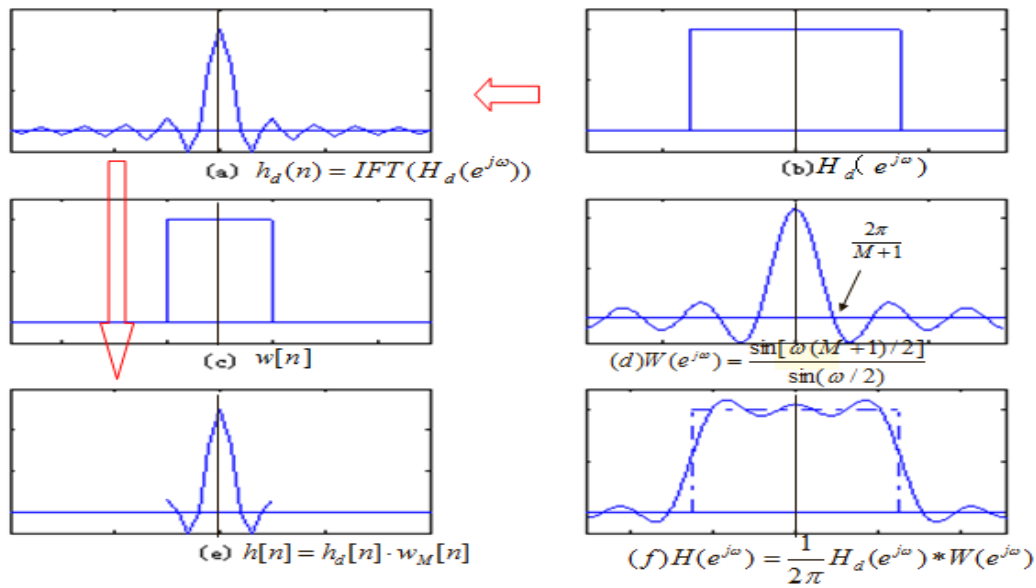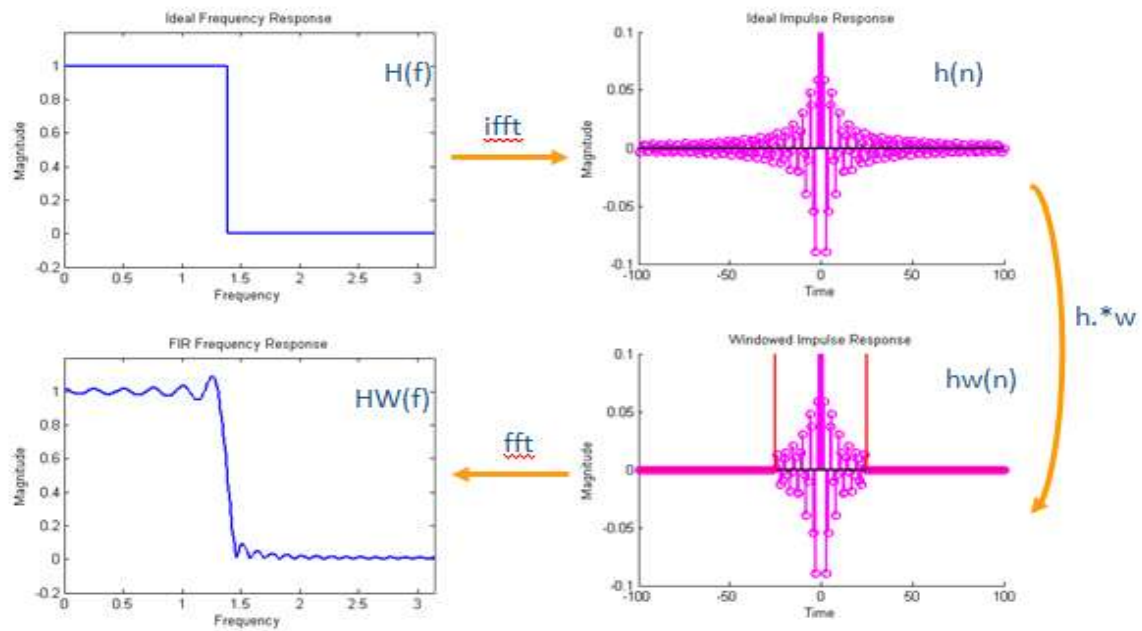
Fig.1 Windowing Technique

Fig. 2 Spectral Response

III. PROPOSED WINDOW

The goal is to modify Hamming window to lower its maximum side-lobe peak, while holding the main-lobe width unchanged, and still satisfying property of equation (4.3). In the following, the derivation of the new

window is explained. All windows described later, have zero valued coefficients outside the interval $0 \leq n \leq m$ Hamming window has the shape of

$$W_H[n] = 0.54 - 0.46 \cos(2\pi n/M), \quad 0 \leq n \leq M \tag{9}$$

This window satisfies the property mentioned in eq. (3), i.e.

$$W_H[n] + W_H[n - M/2] = 0.54 + 0.54, \quad M/2 \leq n \leq M \tag{10}$$

On the other hand, Blackman window is composed of three terms as:

$$W_H[n] = 0.42 - 0.5*\cos(2\pi n/M) + 0.08*\cos(4\pi n/M), \quad 0 \leq n \leq M \tag{11}$$

that is, it has a DC term, a cosine function with frequency 2/M, and its second harmonic. For this window, due to the second harmonic, the property in eq.(3) is not satisfied

$$w_H[n] + w_H[n - M/2] = 0.84 + 0.16*\cos(4\pi n/M), \quad M/2 \leq n \leq M \tag{12}$$

However, it can be found that if the third harmonic is added to the Hamming window function, then the property of eq. (3) will be satisfied. Therefore, the main idea in obtaining the new window is to insert a third harmonic of cosine function into eq. (1). Thus, an extra degree of freedom is obtained in tuning the window coefficients. In this way, the proposed window will be as

$$W[n] = a_0 - a_1*\cos(4\pi n/M) - a_3\cos(6\pi n/M), \quad 0 \leq n \leq M \tag{13}$$

Where for normalization, i.e. $W[M/2] = 1$, one have:

$$a_0 + a_1 + a_3 = 1 \tag{14}$$

The new window is also symmetric about point M/2; thus it has a generalized linear phase, like the other common windows. Checking for the property in eq. (3), one find that

$$W[n] + W[n - M/2] = 2a_0 \tag{15}$$

Another point of view states that eq. (5) is a four-term raised cosine window, with restriction that the third term is zero:

$$W[n] = \sum a_i \cos(2i\pi n/M) \quad 0 \leq n \leq M, k = 3, a_2 = 0 \tag{16}$$

The new window can be analyzed in the frequency domain. Its Fourier transform is:

$$W(\hat{\omega}) = \{a_0 D(\omega) + a_1/2[D(\omega - 2\pi/M) + D(\omega + 2\pi/M)] + a_3/2[D(\omega - 6\pi/M) + D(\omega + 6\pi/M)]\}* \exp(-jM\omega/2) \tag{17}$$

where $D(\omega)$ is Dirichlet kernel.

$$D(\omega) = \sin((M+1)\omega/2)/\sin(\omega/2) \tag{18}$$

Noting the above condition and the normalizing condition in eq. (8), one can apply a simple optimization algorithm to find the optimal values of the window parameters. For sufficiently large orders, the derived window is of the form

$$W[n] = 0.536-0.46\cos*(2\pi n/M)-0.003*\cos(6\pi n/M), \quad 0\leq n\leq M \tag{19}$$

Just like Hamming window, the frequency response of the new window is degraded for low orders; therefore depending on the window order, the above parameters are modified to maintain the efficiency. It shows the dependence of a0 and a1 on M. It can be easily verified that the coefficients are composed of a monotonic function and a DC term. Some simple formulas are tried to present the dependence of these parameters on M, the following formulas approximately fit the data obtained from the optimization:

$$a_0= 0.537 - 0.3/(M+15); \quad a_1 = 0.46 + 0.25/(M+15); \quad a_3 = 1 - a_0 - a_1 \tag{20}$$

IV.RESULTS

The various methodologies are adopted in developing the programs for proposed window, its comparison with other windows, performance evaluation, filter designing and plotting their magnitude response, phase response, pole-zero plot, impulse response, step response, phase delay, group delay and filter information. It discusses the simulation results for window based FIR filters design.
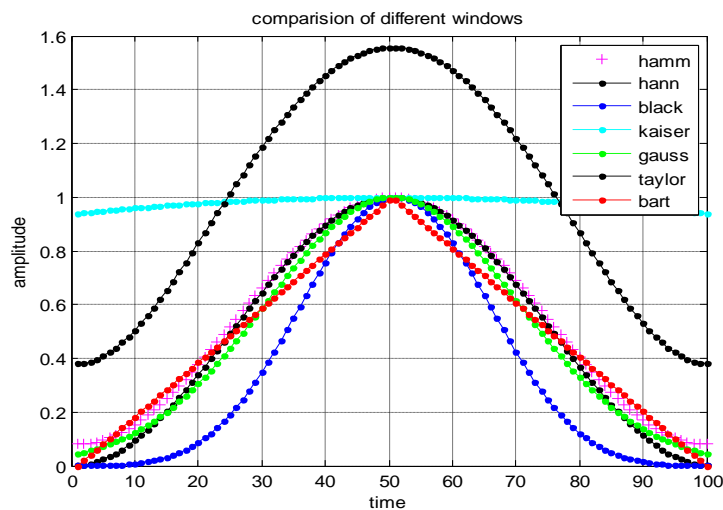

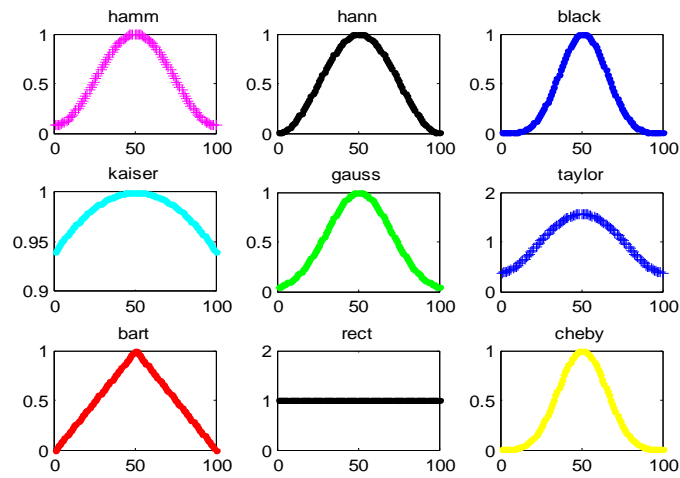**Fig. 3 Time-Magnitude response of different window**

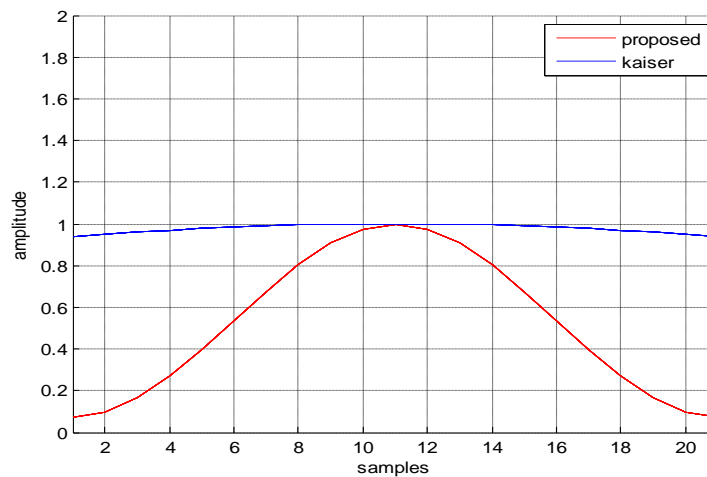**Fig. 4 Plot of different windows in matrix form**

Fig. 5 Time-Magnitude response of Kaiser window & Proposed window
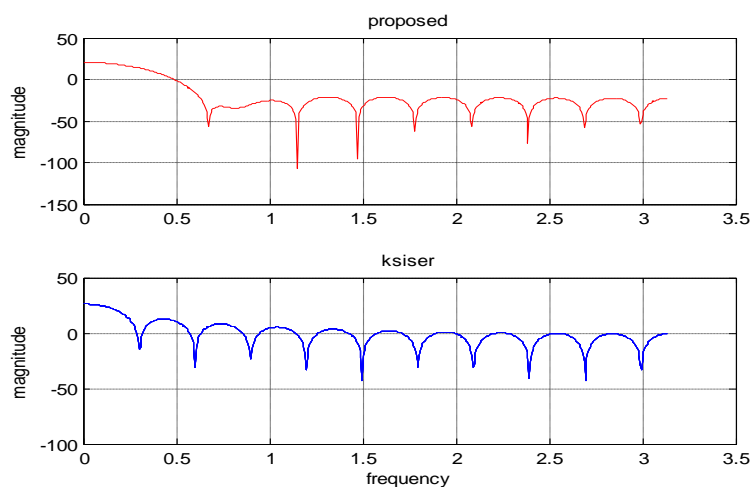

Fig. 6 Frequency-Magnitude response of Kaiser window & Proposed window
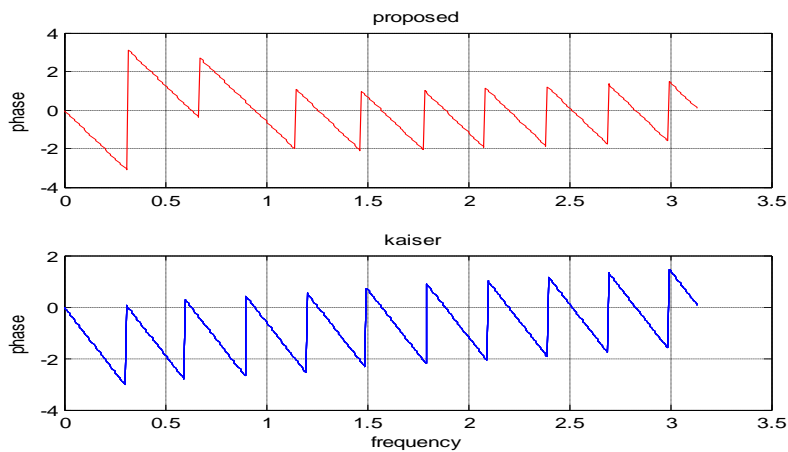
Fig. 7 Frequency-Phase response of Kaiser window & Proposed window
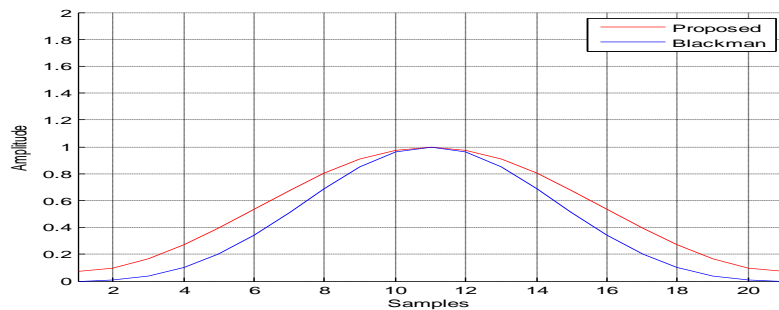

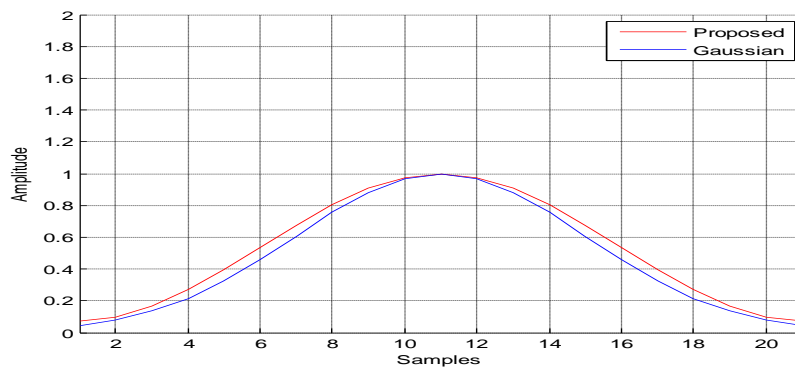**Fig. 8 Time-Magnitude response of Blackman window & Proposed window**

Fig. 9 Time-Magnitude response of Gaussian window & Proposed window
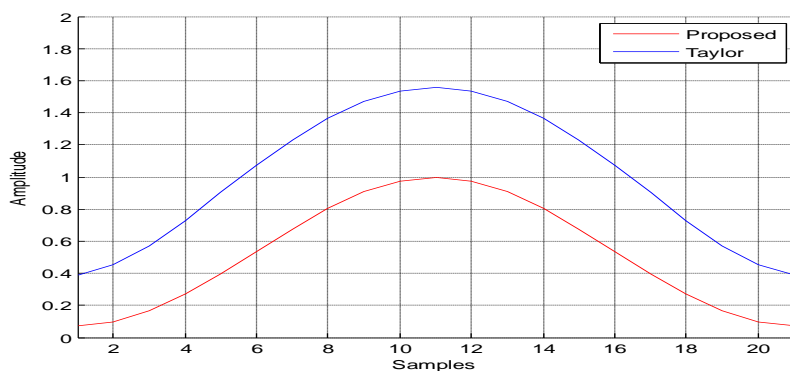

Fig. 10 Time-Magnitude response of Taylor window & Proposed window

V.CONCLUSION

A proposed window is derived for FIR filter designing and spectral analysis which gives better response in comparison with other windows. Performance evaluation is done in comparison with Hamming, Kaiser and Gaussian window. Window based FIR filter is designed. Windowing gives good results for proposed window and it is more convenient. It designs a FIR filter with finite impulse response. It reduces the ripples in the pass-band and the stop-band due to Gibbs'phenomenon. By choosing the window carefully, various trade-offs can be managed so as to maximize the filter-design quality in a given application. The window method for digital filter design is fast, convenient, and robust.

A novel efficient window function has been designed which minimizes the sidelobes. The new window has the main lobe width less than or equal to that of the Hamming window, while offering less maximum side-lobe peak. The performance comparison of the proposed window with the other windows showed the better performance of the proposed window. The average reduction in the side lobe peak of the new window compared to that of the Hamming, Kaiser, and Gaussian windows is 3 dB, 3.3 dB and 8 dB respectively. The FIR filter designed with the proposed window achieves less ripple ratio, than those obtained using the windows like Kaiser, Hamming, Blackman, Gaussian, Hanning, Taylor etc. for all window lengths. The obtained results indicate that the implemented filter has good performance and stability, and during the implementation of various cases, it is possible to reach an optimal design used for final design.

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