



# PERFORMANCE ANALYSIS OF DUAL DIVERSITY RECEIVER OVER CORRELATED $\alpha$ - $\mu$ FADING CHANNEL

Dharmraj<sup>1</sup>, Himanshu Katiyar<sup>2</sup>

<sup>1</sup>Joint Director in Defence Research and Development Organization, CEMILAC, RCMA Lucknow.

He is a research scholar at BBD University Lucknow (India)

<sup>2</sup>Associate Professor in BBDNIIT, Lucknow, (India)

## ABSTRACT

In this paper fading correlation of  $\alpha$ - $\mu$  distribution is discussed. Diversity combining techniques are briefly outlined. Correlation coefficient and joint probability density function for alpha-mu fading is shown. Generalized correlation coefficient for two alpha-mu fading variates is defined. Simulation results of outage and Bit Error Rate (BER) for different  $\alpha$  and  $\mu$  parameters and correlation coefficient is shown and performance is discussed.

**Keywords:**  $\alpha$ - $\mu$  Distribution, Bit Error Rate, Correlation, Diversity Combining, Outage Probability.

## I. INTRODUCTION

The multipath fading in wireless communication is modeled by several distribution such as Rayleigh, Rician, Weibull, Nakagami. In the recent past alpha-mu ( $\alpha$ - $\mu$ ) fading model [1] has been proposed to describe the mobile radio signal considering two important phenomenon of radio propagation non-linearity and clustering. The  $\alpha$ - $\mu$  represents a generalized fading distribution for small-scale variation of the fading signal in a non line-of-sight fading condition. The  $\alpha$ - $\mu$  distribution is flexible, general and has easy mathematical tractability. In fact, the Generalized Gamma Distribution (GGD) also known as Stacy distribution [2] has been renamed as  $\alpha$ - $\mu$  distribution. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely  $\alpha$  and  $\mu$ . The power parameter ( $\alpha > 0$ ) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ( $\mu > 0$ ) is associated to the number of multipath clusters.

In [1,2] the  $\alpha$ - $\mu$  fading distribution and its probability density function has been described. The correlation is a physical phenomenon among channels which affects the performance of a wireless communication system and it can not be neglected in a realistic fading scenarios. In [3] formulation for the  $\alpha$ - $\mu$  multivariate joint density function and its corresponding distribution function are provided. A infinite series representation for the multivariate  $\alpha$ - $\mu$  joint density function is derived in [4,5] allowing for an arbitrary correlation matrix and non-identically distributed variates, whereas in [6] BER and outage probability for the selection combining technique in a correlated  $\alpha$ - $\mu$  fading channel is obtained. The performance analysis of dual selection combining diversity receiver over correlated  $\alpha$ - $\mu$  fading channels with arbitrary parameters is shown in [7], fading between the diversity branches and interferers is correlated and distributed with  $\alpha$ - $\mu$  distribution. In [8] the performance



analysis of signal to interference ratio based selection combining diversity system over  $\alpha$ - $\mu$  fading distributed and correlated channels is reported. Performance of a dual-branch switched and stay combining diversity receiver, operating over correlated  $\alpha$ - $\mu$  fading in the presence of co-channel interference have been analyzed in [9].

The paper is organized as follows. In Section 2, the alpha-mu fading model and its Probability density function is briefly discussed. In Section 3, diversity combining techniques for  $\alpha$ - $\mu$  distribution, maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) is discussed. In Section 4, fading correlation in alpha-mu is explained, expression for correlation coefficient and joint probability density function for alpha mu is shown. Monte-carlo simulation results for outage and BER analysis for different diversity are presented in Section 5. The paper is concluded by Section 6.

## II. THE ALPHA-MU FADING MODEL

In the  $\alpha$ - $\mu$  distribution, it is considered that a signal is composed of clusters of multipath waves. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. As a result, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components. The  $\alpha$ - $\mu$  probability density function (PDF),  $f_R(r)$  of envelope R is given as

$$f_R(r) = \frac{\alpha \hat{r}^{\alpha \mu} \Gamma^{\alpha \mu - 1}}{\hat{r}^{\alpha \mu} \Gamma(\mu)} \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right] \quad (1)$$

where  $\alpha > 0$  is the power parameter, and  $\alpha$ -root mean value of  $R^\alpha$  is given as

$$\hat{r} = \sqrt[\alpha]{E(R^\alpha)} = \sqrt[\alpha]{2\mu\sigma^2}$$

where  $\mu \geq 0$ , is the inverse of the normalized variance of  $\alpha$ - $\mu$  envelope  $R^\alpha$ , and

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad \text{is the Gamma function.}$$

Outage probability of  $\alpha$ - $\mu$  fading channel is given as

$$P_{out} = \frac{\gamma_{thr} \left( \mu, \mu \left( \frac{\gamma_{thr}}{\gamma} \right)^{\alpha/2} \right)}{\Gamma(\mu)} \quad (2)$$

The expression of BER [12] for  $\alpha$ - $\mu$  fading channel is obtained through MGF approach.

## III. DIVERSITY COMBINING TECHNIQUES

Diversity technique provides multiple copies of the same signal on different branches, which undergo independent fading. If one branch undergoes a deep fade, the another branch may have strong signal. In space diversity fading are minimized by the simultaneous use of two or more physically separated antennas. Thus having more than one path to select the SNR at receiver may be improved by selecting appropriate combining technique. Diversity combining methods for uncorrelated fading channels are enumerated below:

### 3.1 Selection Combining (SC)

Selection combining is based on the principle of selecting the best signal among all the signals received from different branches at the receiving end. In this method, the receiver monitors the SNR of the incoming signal using switch logic. The branch with highest instantaneous SNR is connected to demodulator.

SNR of selection combining is given as

$$\gamma_{SC} = \text{Max} (R_1^2, R_2^2) \tag{3}$$

Where  $R_1$  and  $R_2$  represent the fading envelope for two channels seen by two different antennas. PDF of selection combining SNR ( $\gamma$ ) can be obtained by differentiating the CDF of the  $\alpha$ - $\mu$  fading distribution.

$$f_{\gamma_{SC}}(\gamma) = \frac{d}{d\gamma} \left[ \frac{\gamma \left( \mu, \mu \left( \frac{\gamma}{\mu} \right)^{\alpha/2} \right)}{\Gamma(u)} \right]^L \tag{4}$$

Using definition of  $\gamma(s, x)$  i.e. Lower Incomplete Gamma function as

$$\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$$

On solving, we get the PDF of selection combining SNR over  $\alpha$ - $\mu$  fading channel is obtained as

$$f_{\gamma_{SC}}(\gamma) = \frac{L \alpha \mu^\mu \gamma^{\frac{\alpha\mu}{2}-1}}{2 \Gamma(u) \gamma^{-\frac{\alpha\mu}{2}}} e^{-\mu \left( \frac{\gamma}{\mu} \right)^{\alpha/2}} \left[ \frac{\gamma \left( \mu, \mu \left( \frac{\gamma}{\mu} \right)^{\alpha/2} \right)}{\Gamma(u)} \right]^{L-1} \tag{5}$$

This is the most complex scheme in which all branches are optimally combined at the receiver. MRC requires scaling and co-phasing of individual branch. In this all the signals are weighted according to their individual signal voltage to noise power ratios and then summed. Thus MRC produces an output SNR, which is equal to the sum of the individual SNRs. Best statistical reduction of fading is achieved by this method. SNR of MRC is given as

$$\gamma_{MRC} = R_1^2 + R_2^2 \tag{6}$$

### 3.2 Equal Gain Combining (EGC)

A variant of MRC is equal gain combining (EGC), where signals from each branch are co-phased and their weights have equal magnitude. This method also has possibility like MRC of producing an acceptable output signal from a number of unacceptable input signals. SNR improvement of EGC is better than selection combining but not better than MRC. SNR of EGC is given as

$$\gamma_{EGC} = (R_1 + R_2)^2 \tag{7}$$



### 3.3 Switch & Stay Combining (SSC)

SSC further simplifies the complexities of SC. In this in place of continually connecting the diversity path with best quality, a particular diversity path is selected by the receiver till the quality of the path drops below a predetermined threshold. When it happens, then the receiver switches to another diversity path. This reduces the complexities relative to SC, because continuous and simultaneous monitoring of all diversity path in not required. The CDF of SNR of SSC is given [11] as

$$F_{\gamma_{SSC}} = \begin{cases} F_{\gamma}(\gamma_T)F_{\gamma}(\gamma), & \gamma < \gamma_T \\ F_{\gamma}(\gamma) - F_{\gamma}(\gamma_T) + F_{\gamma}(\gamma)F_{\gamma}(\gamma_T), & \gamma \geq \gamma_T \end{cases} \quad (8)$$

PDF is obtained by differentiating (8) as below:

$$f_{\gamma_{SSC}} = \frac{dF_{\gamma_{SSC}}(\gamma)}{d\gamma} = \begin{cases} F_{\gamma}(\gamma_T)f_{\gamma}(\gamma), & \gamma < \gamma_T \\ (1 + F_{\gamma}(\gamma_T))f_{\gamma}(\gamma), & \gamma \geq \gamma_T \end{cases} \quad (9)$$

## IV. FADING CORRELATION IN ALPHA-MU

While studying the performance of diversity techniques, it is assumed that the signals considered for combining are independent of one another. The assumption of independence of channels facilitates the analysis but restricts the understanding of correlation phenomenon.

As per statistics theory variables are said to be correlated, when increase or decrease in one variable is accompanied by increase or decrease in other variable. Correlation is said to be positive when either both variables increase or both variables decrease. Correlation is said to be negative when one variable increases and other variable decreases. The degree of relationship between two correlated variables is measured as Correlation Coefficient ( $\rho$ ).

$$\rho = \frac{\sum(x-X)(y-Y)}{\sqrt{\sum(x-X)^2 \times \sum(y-Y)^2}} \quad 0 \leq \rho < 1$$

$$\rho = \frac{cov(x, y)}{var(x)var(y)} = \frac{cov(x, y)}{\sigma_x \sigma_y} \quad (10)$$

In real-life scenario the assumption of channel independence is not valid due to various reasons such as insufficient antenna spacing in small-size mobile units. Further, for multipath diversity over frequency selective channels, correlation coefficients up to 0.6 between two adjacent paths in the channel impulse response of frequency selective channels have been observed. It has been established by statistical analysis of propagation of several macrocellular, microcellular, and indoor wideband channel responses, that correlation coefficients is many times higher than 0.8, with no significant reduction in correlation even for large path delay differences. Hence, the maximum theoretical diversity gain promised by RAKE reception is not achieved. Therefore effect of correlation between combined signal must be taken into account, while carrying out analysis of performance of diversity techniques.

Let us consider a alpha-mu random variable  $R_1$  with variance  $\sigma^2$ . In order to generate a new  $\alpha$ - $\mu$  random variable  $R_2$  with the same variance of  $R_1$  and a correlation factor  $\rho$  between them, the equation for  $R_2$  can be given as

$$R_2 = \rho R_1 + (1 - \rho^2)^{\frac{1}{2}} v \quad (11)$$

where  $v$  is a alpha-mu random variable with variance  $\sigma^2$ .

Since correlation parameter  $\rho$  is  $0 \leq \rho \leq 1$ , therefore when  $\rho$  is 0; then  $R_2 = v$ , i.e.  $R_2$  is a complex Gaussian random variable but uncorrelated with  $R_1$ . When  $\rho$  is 1; then  $R_2 = R_1$ , i.e.  $R_2$  is fully correlated with  $R_1$ . Taking mean values of  $R_1$  and  $v$  as zero and since both have the same variance  $\sigma^2$ , then the variance of  $R_2$  is also  $\sigma^2$ . Then from equation (10) generalized correlation coefficient  $\rho$  for two  $\alpha$ - $\mu$  fading variates  $R_1$  and  $R_2$  is defined as

$$\rho = \frac{cov(R_1, R_2)}{\sqrt{\sigma_{R_1}^2 \times \sigma_{R_2}^2}} = \frac{cov(R_1, R_2)}{\sigma_{R_1} \times \sigma_{R_2}} \quad (12)$$

For wireless system performance and design study, characterization of the correlation properties of the signals is of utmost importance. Therefore determination of the joint probability density function (joint PDF) is significant.

Let  $R_1$  and  $R_2$  be two  $\alpha$ - $\mu$  fading variates whose marginal statistics are respectively described as below:

$$R_1 \in \{\alpha_1, \mu_1, \hat{r}_1\} \text{ and } R_2 \in \{\alpha_2, \mu_2, \hat{r}_2\} \quad (13)$$

and  $0 \leq \rho \leq 1$  be a correlation parameter. Due to insufficient antenna spacing, the signal envelopes  $R_1$  and  $R_2$  experience correlative  $\alpha$ - $\mu$  fading with joint PDF [7]

$$f_{R_1, R_2}(R_1, R_2) = f_{R_1}(R_1) f_{R_2}(R_2) \sum_{l=0}^{\infty} \frac{l! \Gamma(\mu_1)}{\Gamma(\mu_1 + l)} \rho_{12}^l \times L_l^{\mu_1 - 1} \left( \frac{\mu_1 R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}} \right) L_l^{\mu_2 - 1} \left( \frac{\mu_2 R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}} \right) \quad (14)$$

## V. SIMULATION RESULTS AND DISCUSSIONS

Outage performance and BER for different combining techniques for correlated  $\alpha$ - $\mu$  fading channel is obtained by Monte-Carlo simulation are shown in Fig. 1 to Fig.12. In these simulation 1000000 samples have been considered for a particular  $\alpha$  and  $\mu$  combination.

In Fig.1 & Fig.3 show the outage performance and Fig. 2 & Fig. 4 show the BER performance for  $\alpha=3$ ,  $\mu=1$  when correlation coefficient  $\rho$  increased from 0.1 to 0.6. It is seen that outage and BER performance for a given SNR has deteriorated with increase in  $\rho$ . In Fig.3 & Fig.5 show the outage performance and Fig.4 & Fig.6 show the BER performance for where  $\alpha=3$ ,  $\rho=0.6$  when  $\mu$  increased from 1 to 3, then it is seen that outage and BER performance improves drastically for a given SNR.

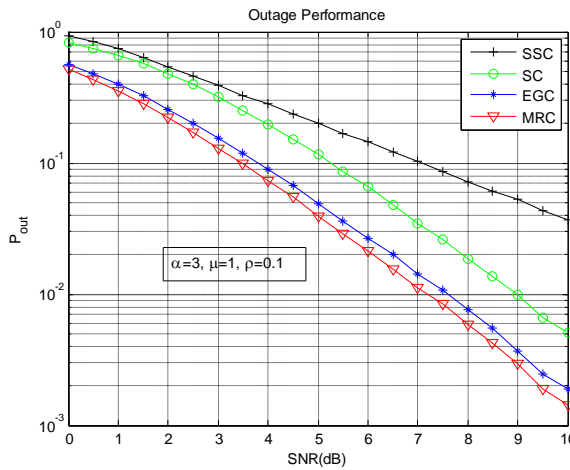


Fig. 1. Outage of  $\alpha$ - $\mu$  fading for  $\alpha=3$ ,  $\mu=1$  and  $\rho=0.1$

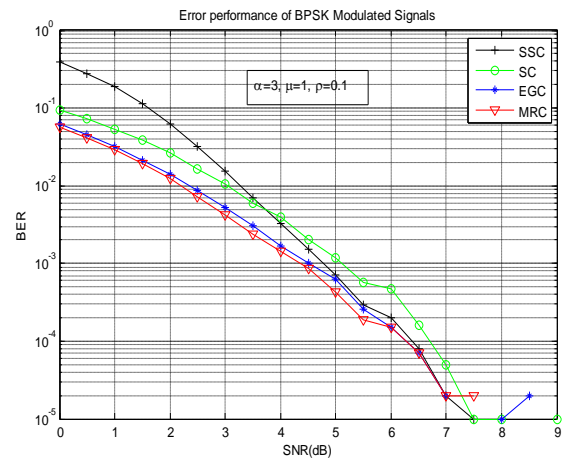


Fig. 2. BER of  $\alpha$ - $\mu$  fading for  $\alpha=3$ ,  $\mu=1$  and  $\rho=0.1$

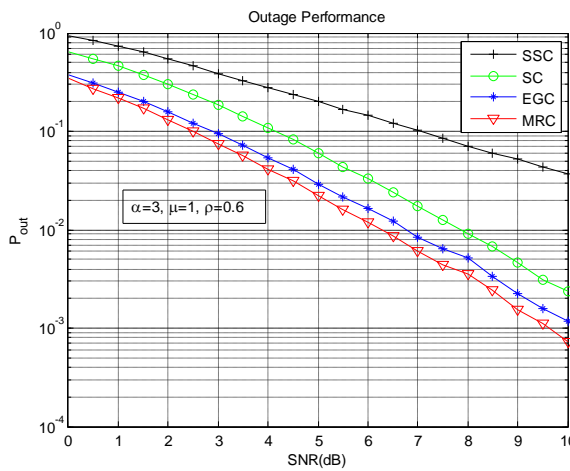


Fig. 3. Outage of  $\alpha$ - $\mu$  fading for  $\alpha=3$ ,  $\mu=1$  and  $\rho=0.6$

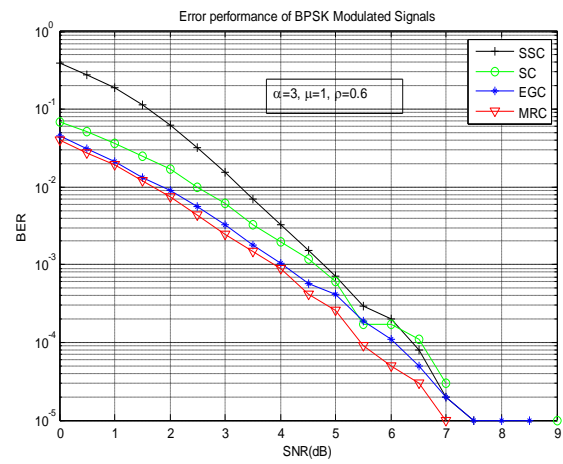


Fig. 4. BER of  $\alpha$ - $\mu$  fading for  $\alpha=3$ ,  $\mu=1$  and  $\rho=0.6$

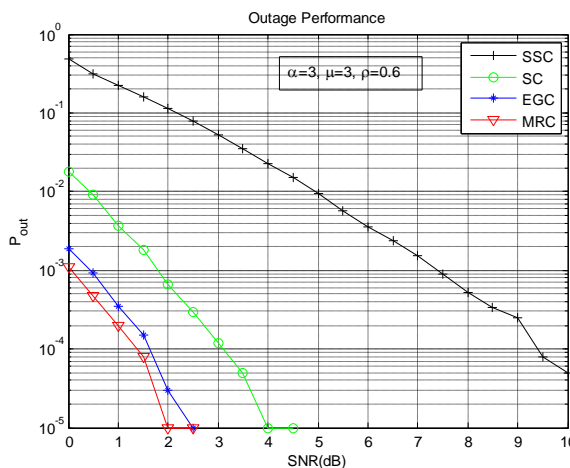


Fig. 5. Outage of  $\alpha$ - $\mu$  fading for  $\alpha=3$ ,  $\mu=3$  and  $\rho=0.6$

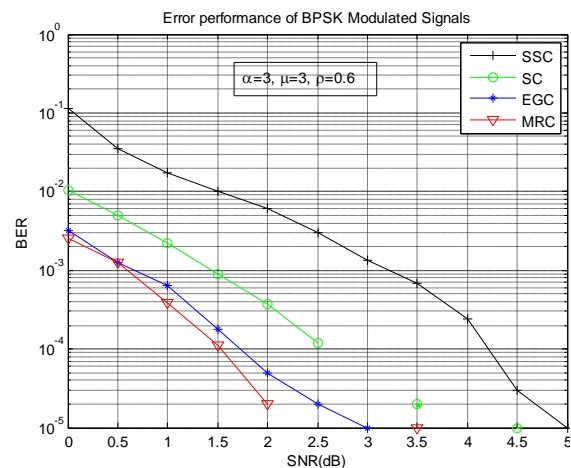


Fig. 6. BER of  $\alpha$ - $\mu$  fading for  $\alpha=3$ ,  $\mu=3$  and  $\rho=0.6$



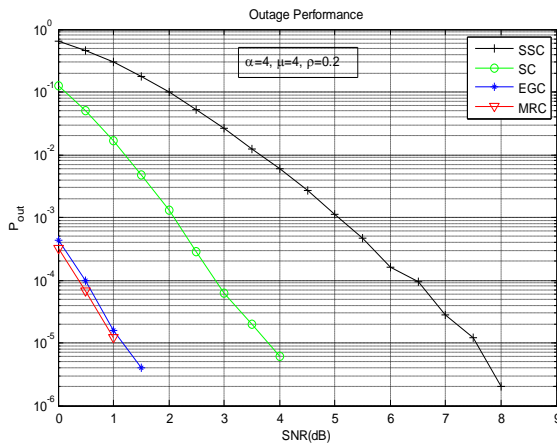


Fig. 7. Outage of  $\alpha$ - $\mu$  fading for  $\alpha=4$ ,  $\mu=4$  and  $\rho=0.2$

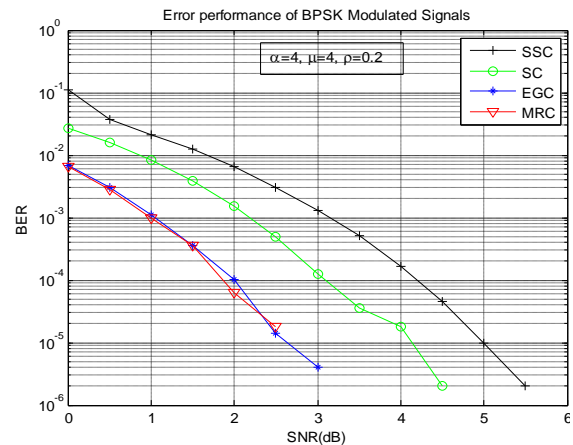


Fig. 8. BER of  $\alpha$ - $\mu$  fading for  $\alpha=4$ ,  $\mu=4$  and  $\rho=0.2$

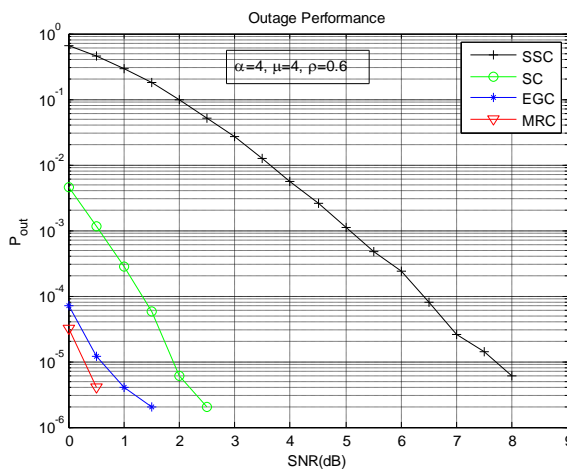


Fig. 9. Outage of  $\alpha$ - $\mu$  fading for  $\alpha=4$ ,  $\mu=4$  and  $\rho=0.6$

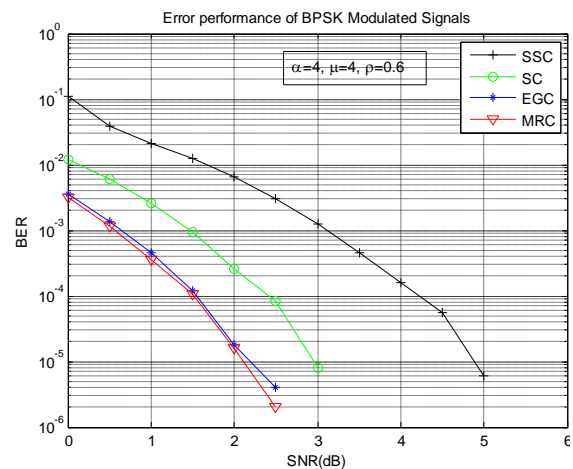


Fig. 10. BER of  $\alpha$ - $\mu$  fading for  $\alpha=4$ ,  $\mu=4$  and  $\rho=0.6$

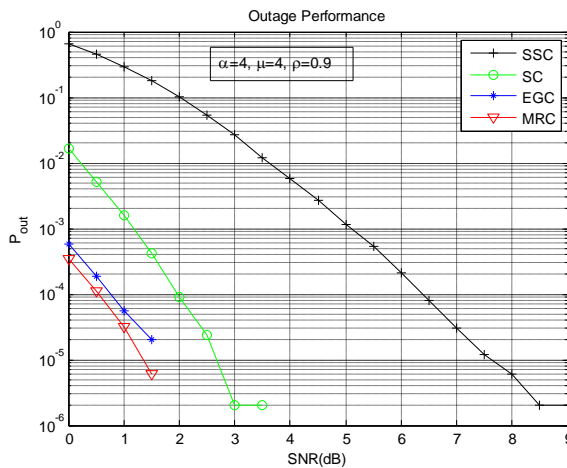


Fig. 11. Outage of  $\alpha$ - $\mu$  fading for  $\alpha=4$ ,  $\mu=4$  and  $\rho=0.9$

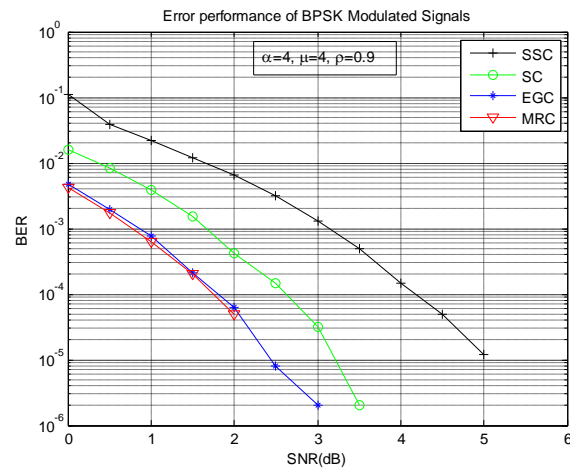


Fig. 12. BER of  $\alpha$ - $\mu$  fading for  $\alpha=4$ ,  $\mu=4$  and  $\rho=0.9$

It is seen that by increasing  $\mu$  from 1 to 3, the improvement in outage and BER is observed. Increase in  $\mu$ , indicates increase in clusters. Thus, inference can be drawn that by increasing the clusters outage and BER improves for the same correlation coefficient. In Fig.7, Fig.9, & Fig.11 show the outage performance and Fig.8, Fig.10, & Fig.12 show the BER performance for  $\alpha=4$  and  $\mu=4$  when  $\rho$  increases from 0.2 to 0.6 to 0.9. It is seen that outage and BER performance deteriorates for a given SNR when  $\rho$  increases. Thus it may be concluded that increase in  $\rho$  has adverse effect on outage and BER for a given  $\alpha$  and  $\mu$  parameters.

It is also observed that in dual diversity correlated fading performance of MRC is better among all and then the EGC, SC and SSC follow the performance subsequently. The MRC is optimal combining scheme but it is at the expense of complexity. MRC requires knowledge of channel amplitude and phase, hence can be used for M-QAM or for any amplitude/phase modulations. On the other hand the SC combiner chooses the branch with highest SNR i.e. output is equal to the signal on only one of the branches, hence it does not require knowledge of the signal phases on each branch as in the case of MRC or EGC. The conventional SC is impractical because it requires the simultaneous and continuous monitoring of all the diversity branches. Therefore the SC is implemented in switched form i.e. SSC, where in place of continuously picking the best branch, receiver remains on a particular branch till its SNR drops below a specified threshold ( $\gamma_T=0.75$ ). That is why SSC performance is slightly poor than SC.

## VI. CONCLUSION

In this paper  $\alpha$ - $\mu$  fading model and probability density function have been briefly discussed. The simulated and analytical results of performance metrics such as outage and BER for alpha mu fading correlation schemes have been illustrated. The effect of  $\alpha$  and  $\mu$  parameters and correlation coefficient variation on BER and outage is brought out. The result obtained in this paper will help researcher to explore correlation concept in  $\alpha$ - $\mu$  generalized fading model.

## VII. ACKNOWLEDGMENT

This work is supported by Defence Research & Development Organisation (DRDO), Ministry of Defence, Government of India. The support is gratefully acknowledged.



## REFERENCES

- [1] Michel Daoud Yacoub, 2002, "The  $\alpha$ - $\mu$  Distribution: A General Fading Distribution", IEEE International Symposium on Personal, Indoor and Mobile Radio Communication PIMRC 2002, vol.2, pp. 629-633.
- [2] Michel Daoud Yacoub, 2007, "The  $\alpha$ - $\mu$  Distribution: A Physical Fading Model for the Stacy Distribution", IEEE Transactions on Vehicular Technology, vol. 56, no. 1, pp. 27-34.
- [3] Rabelo G.S., Yacoub M.D., de Souza R.A.A., Oct 2011, "On the Multivariate  $\alpha - \mu$  Distribution: New Exact Analytical Formulations" IEEE Transactions on Vehicular Technology, vol. 60, no 8, pp 4063-4069.
- [4] Rausley Adriano Amaral de Souza, and Michel Daoud Yacoub, 2008, "The Multivariate alpha-mu Distribution" Proceedings of IEEE Wireless Communications and Networking Conference (WCNC 2008).
- [5] Rausley Adriano Amaral de Souza, and Michel Daoud Yacoub, 2008, "On the Multivariate  $\alpha$ - $\mu$  Distribution with Arbitrary Correlation and Fading Parameters" Proceedings of IEEE International Conference on Communications (ICC 2008).
- [6] Rausley Adriano Amaral, de Souza, Santa Rita do Sapucaí, Gustavo Fraidenraich and Michel Daoud Yacoub, 2006, "On the Multivariate  $\alpha$ - $\mu$  Distribution with Arbitrary Correlation" International Telecommunications Symposium.





- [7] S.R. Panic, M.C. Stefanovic, A.V. Mosaic, 2009, "Performance Analyses of Selection Combining Diversity Receiver over  $\alpha$ - $\mu$  Fading Channels in the presence of Co-channel Interference" IET Communications, vol. 3, no. 11, 2009.
- [8] M Stefanovic, D Krstic, J Anastasov, S Panic, and A Matovic, 2009, "Analysis of SIR-based Triple SC System over Correlated  $\alpha$ - $\mu$  Fading Channels" 5<sup>th</sup> Advanced International Conference on Telecommunications.
- [9] Petar C Spalevic, et all, 2010, "SSC Diversity Receiver over Correlated  $\alpha$ - $\mu$  Fading Channels in the Presence of Co-channel Interference" EURASIP Journal on Wireless Communication and Networking, id 142392.
- [10] Milica Petkovic, et all, 2013, "Outage Probability Analysis of System with dual Selection Combining over correlated Weibull Fading Channel in the Presence of  $\alpha$ - $\mu$  Co-channel Interference" Prezeglad Elektrotechniczny, R 89, nr 8, pp 126-129.
- [11] M K Simon, and M S Alouini, 2005, "Digital Communications over Fading Channels", 2<sup>nd</sup> ed. Hoboken: Wiley.
- [12] Magableh A.M., Matalgah Mustafa M, "Moment Generating Function of the Generalized  $\alpha$ - $\mu$  Distribution with Applications" IEEE Communications Letters, vol. 13, no. 6, pp. 411-413, Jun 2009.

	<p><b>Dharmraj:</b> Working as Joint Director, Scientist 'E', in Defence R&amp;D Orgn (DRDO), CEMILAC, Ministry of Defence, Govt of India, at RCMA, Lucknow. 1985-1995: worked in Indian Air Force, 1995-1999: faculty at Naval College of Engg, Lonavala. 1999 onwards: carrying out airworthiness certification for accessories of Military Aircraft and Helicopters. Received AMIE degree in Electronics and Telecomn Engg, M.E. in Control Systems, M.B.A. in Operations Research and persuing Ph. D. in Wireless Communication from BBD University Lucknow.</p>
	<p><b>Himanshu Katiyar</b> received his B.E. degree in Electronics and Communication Engineering in 2001, M.Tech degree from Madan Mohan Malviya Engineering College, Gorakhpur India in 2004 and Ph.D. degree in wireless communication at the Indian Institute of Technology (IIT), Guwahati, in 2011. At present he is Associate Professor of the Electronics and Communication Engineering Dept at BBDNIIT, Lucknow, Uttar Pradesh, India. He was awarded IETE research fellowship and was project investigator (from September, 2009 to December, 2010) of an IETE sponsored project.</p>