SOLVING FUZZY SOLID TRANSPORTATION PROBLEM BASED ON EXTENSION PRINCIPLE WITH INTERVAL BUDGET CONSTRAINT

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ABSTRACT

The solid transportation problem considers the supply, the demand and the conveyance satisfying the transportation requirement in a cost-effective manner. This paper develops a method that is able to derive the fuzzy objective value of the fuzzy solid transportation problem when the cost coefficients, the supply and demand quantities, conveyance capacities are fuzzy numbers and additional constraints on the total budget at each destination which is interval type. We make use of Hu and Wang's Approach based on interval ranking. Based on the extension principle, the fuzzy solid transportation problem is transformed into a pair of mathematical programs that is employed to calculate the lower and upper bounds of the fuzzy total transportation cost at possibility level α *. From different values of* α *, the membership function of the objective value is constructed. Since the objective value is fuzzy, the values of the decision variables derived in this paper are fuzzy as well. An example is illustrated for this model.*

Keywords: Extension Principles, Fuzzy Numbers, Solid Transportation Problem.

I INTRODUCTION

The traditional transportation problem (TP) is a well-known optimization problem in operational research, in which two kinds of constraints are taken into consideration, i.e., source constraint and destination constraint. But in the real system, we always deal with other constraints besides of source constraint and destination constraint, such as product type constraint or transportation mode constraint. In such case, the traditional TP turns into the solid transportation problem (STP). As a generalization of traditional TP, the STP was introduced by Haley [1]. In this paper, we investigate a solution of the fuzzy solid transportation problem with interval valued budget at each destination. An assessment of different results of the model is also presented.

Bellman and Zadeh [1] introduce the notion of fuzziness. Since the transportation problem is essentially a linear program, one uniformly apply the existing fuzzy linear programming techniques (Buckly [2], Chanas et al. [3] And Hadi Basirzadeh [4]) to the fuzzy transportation problem. Unfortunately, most of the existing techniques [2, 3 and 4] only provide crisp solutions. The method of Julien [5] and Parra et al. [6] is able to find the possibility distribution of the objective value, provided all the inequality constraints are of ‘‘≥’’ type or ‘‘≤’’ type. However, due to the structure of the transportation problem, in some cases their method requires the refinement of the problem parameters to be able to derive the bounds of the objective value. There are also studies discussing the fuzzy transportation problem. Obviously, when the cost coefficients, supply and demand quantities are fuzzy numbers, the total transportation cost will be fuzzy as well. In this paper, we develop a solution procedure that is able to calculate the fuzzy objective value of the fuzzy solid transportation problem, where all the parameters are fuzzy numbers. The idea is to apply Zadeh's extension principle [7]. A pair of two-level mathematical programs is formulated [8–9] to calculate the lower and upper bounds of the α-level cut of the objective value. In section 2, we introduce the crisp conversion of the constraints of the respective model using a different order relation of the intervals such as Hu and Wang's Approach [10]. The membership function of the fuzzy objective value is derived numerically by enumerating different values of α. It has been observed that a very less research work is done on the fuzzy transportation problem to minimize the transportation cost using publicly available data which should be more authentic and reliable as compare to crisp data. In the following sections, we first concisely describe the fuzzy solid transportation problem. Then a pair of mathematical programs is formulated to calculate the fuzzy total transportation cost bounds at a specific α level. An example is illustrated to explain the proposed method. Finally, some conclusions are drawn.

II FUZZY SOLID TRANSPORTATION PROBLEM WITH INTERVAL BUDGET CONSTRAINT

Consider m sources and n destinations in a solid transportation problem. At each source, let s_i be the amount of a homogeneous product we want to transport to n destinations to satisfy the demand for d_j units of the product. Here e_k called conveyance denotes the units of this product that can be carries by k different modes of transportation, interval budget at the j^{th} destination, such as the land transportation by car or train, and ocean shipping. A penalty value of the unit shipping cost represents by c_{ijk} of a product from i^{th} origin to j^{th} destination by means of the k^{th} conveyance. We need to determine a feasible way of shipping the available amounts to satisfy the demand such that the total transportation cost is minimized.

Let x_{ijk} denote the number of units to be transported from Source i to Destination j through Conveyance capacities k. The mathematical form of the solid transportation problem with interval valued budget constraint, transportation costs, availabilities and conveyance capacities is given below:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}\, x_{ijk}$$

$$s.t. \quad \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le s_i, \quad i = 1, 2, \ldots..m,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq d_j, \quad j = 1, 2, ...n,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, 2, ...l, \tag{1}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} c_{ijk} x_{ijk} \leq B_j, \quad j = 1, 2, ...n,$$

$$x_{ijk} \geq 0 \quad \forall\ i, j, k.$$

Intuitively, if any of the parameters x_{ijk}, s_i, d_j or e_k is fuzzy, the total transportation cost becomes fuzzy as well and budget constraint is taken with interval value $B_j = [b_{jL}, b_{jR}]$. Then the Model (1) turns into the fuzzy solid transportation problem with interval valued budget constraint.

Suppose the unit shipping cost c_{ijk}, supply s_i, demand d_j, conveyance capacity e_k and budget intervals are approximately known. They it can be represented by the convex fuzzy numbers C_{ijk}, S_i, D_j and E_k respectively, with membership functions $\mu_{\tilde{C}_{ijk}}$, $\mu_{\tilde{S}_i}$, $\mu_{\tilde{D}_j}$ and $\mu_{\tilde{E}_k}$:

$$\begin{aligned}
\tilde{C}_{ijk} &= \{(c_{ijk}, \mu_{\tilde{C}_{ijk}}(c_{ijk})) \mid c_{ijk} \in S(\tilde{C}_{ijk})\},\\
\tilde{S}_i &= \{s_i, \mu_{\tilde{S}_i}(s_i)) \mid s_i \in S(\tilde{S}_i)\},\\
\tilde{D}_j &= \{(d_j, \mu_{\tilde{D}_j}(d_j)) \mid d_j \in S(\tilde{D}_j)\},\\
\tilde{E}_k &= \{(e_k, \mu_{\tilde{E}_k}(e_k)) \mid e_k \in S(\tilde{E}_k)\},
\end{aligned} \tag{2}$$

where $S(\tilde{C}_{ijk})$, $S(\tilde{S}_i)$, $S(\tilde{D}_j)$ and $S(\tilde{E}_k)$ are the supports of $\tilde{C}_{ijk}$, $\tilde{S}_i$, $\tilde{D}_j$ and $\tilde{E}_k$, which denote the universe sets of the unit shipping cost, the quantity supplied by the i^{th} origin, the quantity required by the j^{th} destination, and the capacity carried by the k^{th} conveyance, respectively.

The fuzzy objective function $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{C}_{ijk} x_{ijk}$, which is to be minimized, together with the following constraints, constitutes the fuzzy solid transportation problem:

Using Hu and Wang's approach [7] on budget constraint we have the following crisp conversion.

$$\sum_{i=1}^{m}\sum_{k=1}^{l} C_{ijk} x_{ijk} \leq \frac{(b_{jL} + b_{jR})}{2}, \quad j = 1, 2, ...n, \tag{3}$$

Without loss of generality, all the supply and demand quantities and conveyance capacities are assumed to be convex fuzzy numbers as the crisp values can be represented by degenerated membership functions which have only one value in their domains. In the next section, we shall develop the solution procedure for fuzzy solid transportation problem with fuzzy supply, requirement and conveyance capacity.

III THE SOLUTION PROCEDURE

We are interested in deriving the membership function of the total transportation cost $\tilde{Z}$. Since $\tilde{Z}$ is a fuzzy number, instead of a crisp number, it cannot be minimized directly. To tackle this problem, one can transform the fuzzy solid transportation problem, which is based on Zadeh's extension principle to a family of mathematical programs to be solved.

Based on the extension principle, the membership function $\mu_{\tilde{Z}}$ can be defined as:

$$\mu_{\tilde{Z}}(z) = \sup\min\{\mu_{\tilde{C}_{ijk}}(c_{ijk}), \mu_{\tilde{S}_i}(s_i), \mu_{\tilde{D}_j}(d_j), \mu_{\tilde{E}_k}(e_k)\ \forall i, j, k \mid z = Z(c, s, d, e)\}, \tag{4}$$

Where $Z(c, s, d, e)$ is defined in Model (1). The application of the extension principle to $\tilde{Z}$ may be viewed as the application of this extension principle to the ∝ -cuts of $\tilde{Z}$. Let us denote the ∝-cuts of $\tilde{C}_{ijk}$, $\tilde{S}_i$, $\tilde{D}_j$ and $\tilde{E}_k$ as

$$(C_{ijk})_\alpha = \{c_{ijk} \in S(\tilde{C}_{ijk}) \mid \mu_{\tilde{C}_{ijk}}(c_{ijk}) \geq \alpha\} = [(C_{ijk})^L_\alpha, (C_{ijk})^U_\alpha], \tag{5.1}$$

$$(S_i)_\alpha = \{s_i \in S(\tilde{S}_i) \mid \mu_{\tilde{S}_i}(s_i)) \geq \alpha\} = [(S_i)^L_\alpha, (S_i)^U_\alpha], \tag{5.2}$$

$$(D_j)_\alpha = \{d_j \in S(\tilde{D}_j) \mid, \mu_{\tilde{D}_j}(d_j) \geq \alpha\} = [(D_j)^L_\alpha, (D_j)^U_\alpha], \tag{5.3}$$

$$(E_k)_\alpha = \{(e_k \in S(\tilde{E}_k) \mid \mu_{\tilde{E}_k}(e_k) \geq \alpha\} = [(E_k)^L_\alpha, (E_k)^U_\alpha], \tag{5.4}$$

These intervals indicate where the unit shipping cost, supply, demand, and conveyance lie at possibility level ∝.

In Eq. (4), several membership functions are involved. To derive $\mu_{\tilde{Z}}$ in closed form is hardly possible. According to (4), $\mu_{\tilde{Z}}$ is the minimum of $\mu_{\tilde{C}_{ijk}}, \mu_{\tilde{S}_i}, \mu_{\tilde{D}_j}$ and $\mu_{\tilde{E}_k}, \forall i, j, k.$ We need $\mu_{\tilde{C}_{ijk}}(c_{ijk}) \geq \alpha$, $\mu_{\tilde{S}_i}(s_i)) \geq \alpha$, $\mu_{\tilde{D}_j}(d_j) \geq \alpha$ or $\mu_{\tilde{E}_k}(e_k) \geq \alpha$ and at least one $\mu_{\tilde{C}_{ijk}}(c_{ijk}), \mu_{\tilde{S}_i}(s_i), \mu_{\tilde{D}_j}(d_j), \mu_{\tilde{E}_k}(e_k)\ \forall i, j, k$, equal to ∝ such that $z = Z(c, s, d, e)$ to satisfy $\mu_{\tilde{Z}}(z) = \alpha$. To find the membership function $\mu_{\tilde{Z}}$, it suffices to find the left shape function and right shape function of $\mu_{\tilde{Z}}$, which is equivalent to finding the lower bound Z^L_α and upper bound Z^U_α of the ∝-cuts of $\tilde{Z}$. Since Z^L_α is the minimum of $Z(c, s, d, e)$ and Z^U_α is the maximum of $Z(c, s, d, e)$, they can be expressed as:

$$Z^L_\alpha = \min\{Z(c, s, d, e) \mid (C_{ijk})^L_\alpha \leq c_{ijk} \leq (C_{ijk})^U_\alpha, (S_i)^L_\alpha \leq s_i \leq (S_i)^U_\alpha,$$
$$(D_j)^L_\alpha \leq d_j \leq (D_j)^U_\alpha, (E_k)^L_\alpha \leq e_k \leq (E_k)^U_\alpha \quad \forall i, j, k\},$$

$$Z^U_\alpha = \max\{Z(c, s, d, e) \mid (C_{ijk})^L_\alpha \leq c_{ijk} \leq (C_{ijk})^U_\alpha, (S_i)^L_\alpha \leq s_i \leq (S_i)^U_\alpha,$$
$$(D_j)^L_\alpha \leq d_j \leq (D_j)^U_\alpha, (E_k)^L_\alpha \leq e_k \leq (E_k)^U_\alpha \quad \forall i, j, k\},$$

This can be reformulated as the following pair of two-level mathematical programs:

$$Z_\alpha^L = \min_{\substack{(C_{ijk})_\alpha^L \le c_{ijk} \le (C_{ijk})_\alpha^U \\ (S_i)_\alpha^L \le s_i \le (S_i)_\alpha^U \\ (D_j)_\alpha^L \le d_j \le (D_j)_\alpha^U \\ (E_k)_\alpha^L \le e_k \le (E_k)_\alpha^U \\ \forall i,j,k}} \begin{cases} \min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x_{ijk} \\ s.t.\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le s_i, \quad i = 1,2,\ldots..m, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge d_j, \quad j = 1,2,\ldots..n, \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k, \quad k = 1,2,\ldots..l, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} C^L{}_{ijk}x_{ijk} \le \dfrac{(b_{jL}+b_{jR})}{2}, \quad j = 1,2,\ldots..n, \\ x_{ijk} \ge 0 \quad \forall\ i,j,k. \end{cases} \tag{6a}$$

$$Z_\alpha^U = \max_{\substack{(C_{ijk})_\alpha^L \le c_{ijk} \le (C_{ijk})_\alpha^U \\ (S_i)_\alpha^L \le s_i \le (S_i)_\alpha^U \\ (D_j)_\alpha^L \le d_j \le (D_j)_\alpha^U \\ (E_k)_\alpha^L \le e_k \le (E_k)_\alpha^U \\ \forall i,j,k}} \begin{cases} \min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x_{ijk} \\ s.t.\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le s_i, \quad i = 1,2,\ldots..m, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge d_j, \quad j = 1,2,\ldots..n, \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k, \quad k = 1,2,\ldots..l, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} C^U{}_{ijk}x_{ijk} \le \dfrac{(b_{jL}+b_{jR})}{2}, \quad j = 1,2,\ldots..n, \\ x_{ijk} \ge 0 \quad \forall\ i,j,k. \end{cases} \tag{6b}$$

In Model (6a), the inner program calculates the objective value for each c_{ijk} , s_i , d_j and e_k specified by the outer program, while the outer program determines the values of c_{ijk} , s_i , d_j and e_k that generate the smallest objective value z^L . The objective value is the lower bound of the objective value for Model (3). By the same token, the inner program of Model (6b) calculates the objective value for each given value of c_{ijk} , s_i , d_j and e_k ,while the outer program determines the values of c_{ijk} , s_i , d_j and e_k that produce the largest objective value. The objective value z^U is the upper bound of the objective value for Model (3).

Since the value of ∝ varies in, Model (6a and 6b), it can also be regarded as a pair of parametric programming model.

A necessary and sufficient condition for Model (6a and 6b) to have feasible solutions is $\sum_{i=1}^{m} s_i \ge \sum_{j=1}^{n} d_j$ and $\sum_{k=1}^{l} e_k \ge \sum_{j=1}^{n} d_j$. In the first level of Model (6a and 6b), s_i , d_j and e_k are allowed to vary in the range of $[(S_i)_\alpha^L, (S_i)_\alpha^U]$, $[(D_j)_\alpha^L, (D_j)_\alpha^U]$, and $[(E_k)_\alpha^L, (E_k)_\alpha^U]$, respectively. However, to ensure the transportation problem of the second level to be feasible, it is necessary that the constraint $\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j$ and $\sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} d_j$ be imposed in the outer program.

here $B_j = \dfrac{b_{jL} + b_{jU}}{2}$.

Hence, Model (6a and 6b) becomes:

$$
\begin{array}{l}
Z_\alpha^L = \min \\
(C_{ijk})_\alpha^L \leq c_{ijk} \leq (C_{ijk})_\alpha^U \\
(S_i)_\alpha^L \leq s_i \leq (S_i)_\alpha^U \\
(D_j)_\alpha^L \leq d_j \leq (D_j)_\alpha^U \\
(E_k)_\alpha^L \leq e_k \leq (E_k)_\alpha^U \\
\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j \\
\sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} d_j \\
\forall i, j, k
\end{array}
\left\{
\begin{array}{l}
\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk} \\
s.t. \quad \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq s_i, \quad i = 1, 2, \ldots..m, \\
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq d_j, \quad j = 1, 2, \ldots..n, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, 2, \ldots..l, \\
\sum_{i=1}^{m} \sum_{k=1}^{l} c_{ijk} x_{ijk} \leq B_j \quad j = 1, 2, \ldots..n, \\
x_{ijk} \geq 0 \quad \forall i, j, k.
\end{array}
\right. \tag{7a}
$$

$$
\begin{array}{l}
Z_\alpha^U = \max \\
(C_{ijk})_\alpha^L \leq c_{ijk} \leq (C_{ijk})_\alpha^U \\
(S_i)_\alpha^L \leq s_i \leq (S_i)_\alpha^U \\
(D_j)_\alpha^L \leq d_j \leq (D_j)_\alpha^U \\
(E_k)_\alpha^L \leq e_k \leq (E_k)_\alpha^U \\
\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j \\
\sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} d_j \\
\forall i, j, k
\end{array}
\left\{
\begin{array}{l}
\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk} \\
s.t. \quad \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq s_i, \quad i = 1, 2, \ldots..m, \\
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq d_j, \quad j = 1, 2, \ldots..n, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, 2, \ldots..l, \\
\sum_{i=1}^{m} \sum_{k=1}^{l} c_{ijk} x_{ijk} \leq B_j \quad j = 1, 2, \ldots..n, \\
x_{ijk} \geq 0 \quad \forall i, j, k.
\end{array}
\right. \tag{7b}
$$

In above Model (7a and 7b) will be infeasible when $\sum_{i=1}^{m} S_{\alpha=0}^U \leq \sum_{j=1}^{n} D_{\alpha=0}^L$ for any α level. In other words, a fuzzy transportation problem is feasible if the upper bound of the total fuzzy supply is greater than or equal to the lower bound of the total fuzzy demand. To derive the lower bound of the objective value in Model (7a), we can directly set c_{ijk} to its lower bound $(C_{ijk})_\alpha^L$, $\forall i, j, k$ to find the minimum objective value.

Hence, Model (7a) can be reformulated as:

$$
\begin{array}{l}
Z_{\alpha}^{L} = \min \\
(C_{ijk})_{\alpha}^{L} \le c_{ijk} \le (C_{ijk})_{\alpha}^{U} \\
(S_i)_{\alpha}^{L} \le s_i \le (S_i)_{\alpha}^{U} \\
(D_j)_{\alpha}^{L} \le d_j \le (D_j)_{\alpha}^{U} \\
(E_k)_{\alpha}^{L} \le e_k \le (E_k)_{\alpha}^{U} \\
\sum_{i=1}^{m} s_i \ge \sum_{j=1}^{n} d_j \\
\sum_{k=1}^{l} e_k \ge \sum_{j=1}^{n} d_j \\
\forall i, j, k
\end{array}
\left\{
\begin{array}{l}
\min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk} x_{ijk} \\
s.t. \quad \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le s_i, \quad i = 1, 2, ...m, \\
\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge d_j, \quad j = 1, 2, ...n, \\
\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k, \quad k = 1, 2, ...l, \\
\sum_{i=1}^{m}\sum_{k=1}^{l} c_{ijk} x_{ijk} \le B_j = [b_{jL}, b_{jU}] \quad j = 1, 2, ...n, \\
x_{ijk} \ge 0 \quad \forall i, j, k.
\end{array}
\right. \qquad (8)
$$

Since Model (8) is to find the minimum of all the minimum objective values, one can combine the constraints of inner program and outer program together and simplify the two-level mathematical program to the conventional one-level program as follows:

$$
\begin{array}{ll}
Z_{\alpha}^{L} = \min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} (C_{ijk})^{L} x_{ijk} & \\
s.t. \quad \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le s_i, & i = 1, 2, ...m, \\
\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge d_j, & j = 1, 2, ...n, \\
\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k, & k = 1, 2, ...l, \\
\sum_{i=1}^{m}\sum_{k=1}^{l} (C_{ijk})^{L} x_{ijk} \le B_j & j = 1, 2, ...n, \\
\sum_{i=1}^{m} s_i \ge \sum_{j=1}^{n} d_j & \\
\sum_{k=1}^{l} e_k \ge \sum_{j=1}^{n} d_j & \\
(S_i)_{\alpha}^{L} \le s_i \le (S_i)_{\alpha}^{U} & \\
(D_j)_{\alpha}^{L} \le d_j \le (D_j)_{\alpha}^{U} & \\
(E_k)_{\alpha}^{L} \le e_k \le (E_k)_{\alpha}^{U} & \\
x_{ijk} \ge 0 \quad \forall i, j, k. &
\end{array} \qquad (9)
$$

This model is a linear program which can be solved easily. In this model, since all c_{ijk} have been set to the lower bounds of their ∝-cuts, that is, $\mu_{\tilde{C}_{ijk}}(c_{ijk}) = \alpha$, this assures $\mu_{\tilde{Z}}(z) = \alpha$ as required by (4).

To solve Model (7b) is not so straightforward as Model (7a). The outer program and inner program of Model (7b) have different directions for optimization, one for maximization and another for minimization. A transformation is required to make a solution obtainable. The dual of inner program is formulated to become a maximization problem to be consistent with the maximization operation of outer program. It is well known from the duality theorem of linear programming that the primal model and the dual model have the same objective value. Thus, Model (7b) becomes:

$$Z_\alpha^U = \max_{\substack{(C_{ijk})_\alpha^L \le c_{ijk} \le (C_{ijk})_\alpha^U \\ (S_i)_\alpha^L \le s_i \le (S_i)_\alpha^U \\ (D_j)_\alpha^L \le d_j \le (D_j)_\alpha^U \\ (E_k)_\alpha^L \le e_k \le (E_k)_\alpha^U \\ \sum_{i=1}^m s_i \ge \sum_{j=1}^n d_j \\ \sum_{k=1}^l e_k \ge \sum_{j=1}^n d_j \\ \forall i,j,k}} \begin{cases} \max -\sum_{i=1}^m s_i u_i + \sum_{j=1}^n d_j v_j - \sum_{k=1}^l e_k w_k - \sum_{j=1}^n B_j y_j \\ s.t. \quad -u_i + v_j - w_k - y_j \le c_{ijk} \\ \qquad i=1,2,\ldots\ldots,m,\ j=1,2,\ldots\ldots,n,\ \ k=1,2,\ldots\ldots,l, \\ \qquad u_i, v_j, w_k \ge 0, \quad \forall i,j,k \end{cases} \tag{10}$$

Since $(C_{ijk})_\alpha^L \le c_{ijk} \le (C_{ijk})_\alpha^U,\ \forall i,j,k$, in Model (10), one can derive the upper bound of the objective value by setting c_{ijk} to its upper bound because this gives the largest feasible region. Thus, we can reformulate Model (10) as:

$$Z_\alpha^U = \max_{\substack{(C_{ijk})_\alpha^L \le c_{ijk} \le (C_{ijk})_\alpha^U \\ (S_i)_\alpha^L \le s_i \le (S_i)_\alpha^U \\ (D_j)_\alpha^L \le d_j \le (D_j)_\alpha^U \\ (E_k)_\alpha^L \le e_k \le (E_k)_\alpha^U \\ \sum_{i=1}^m s_i \ge \sum_{j=1}^n d_j \\ \sum_{k=1}^l e_k \ge \sum_{j=1}^n d_j \\ \forall i,j,k}} \begin{cases} \max -\sum_{i=1}^m s_i u_i + \sum_{j=1}^n d_j v_j - \sum_{k=1}^l e_k w_k \\ s.t. \quad -u_i + v_j - w_k \le (c_{ijk})_\alpha^U, \\ \qquad i=1,2,\ldots\ldots,m,\ j=1,2,\ldots\ldots,n,\ \ k=1,2,\ldots\ldots,l, \\ \qquad u_i, v_j, w_k \ge 0, \quad \forall i,j,k \end{cases} \tag{11}$$

Now, since both outer program and inner program perform the same maximization operation, their constraints can be combined to form the following one-level mathematical program:

$$Z_\alpha^U = \max -\sum_{i=1}^m s_i u_i + \sum_{j=1}^n d_j v_j - \sum_{k=1}^l e_k w_k \tag{12.0}$$

$$s.t. \quad -u_i + v_j - w_k \le (c_{ijk})_\alpha^U, \qquad i=1,2,\ldots\ldots,m,\ j=1,2,\ldots\ldots,n,\ \ k=1,2,\ldots\ldots,l, \tag{12.1}$$

$$\sum_{i=1}^m s_i \ge \sum_{j=1}^n d_j, \tag{12.2}$$

$$\sum_{k=1}^l e_k \ge \sum_{j=1}^n d_j, \tag{12.3}$$

$$(S_i)_\alpha^L \le s_i \le (S_i)_\alpha^U \qquad i = 1, \ldots\ldots, m, \tag{12.4}$$

$$(D_j)_\alpha^L \le d_j \le (D_j)_\alpha^U \qquad j = 1, \ldots\ldots, n, \tag{12.5}$$

$$(E_k)_\alpha^L \le e_k \le (E_k)_\alpha^U \qquad k = 1, \ldots\ldots, l, \tag{12.6}$$

$$x_{ijk} \ge 0, \quad \forall\, i, j, k$$

(12.7)This model is a linearly constrained nonlinear program. There are several effective and efficient methods for solving this Model (12.0 – 12.7). Similar to Model (9), since all c_{ijk} have been set to the upper bounds of their ∝-cuts, that is, $\mu_{\tilde{C}_{ijk}}(c_{ijk}) = \alpha$, this assures $\mu_{\tilde{Z}}(z) = \alpha$ as required by (4).

If the total supply and the total conveyance capacity are greater than the total demand at all α values, respectively, i.e., $\sum_{i=1}^{m}(S_i)_{\alpha=0}^{L} \ge \sum_{j=1}^{n}(D_j)_{\alpha=0}^{U}$ and $\sum_{k=1}^{l}(E_k)_{\alpha=0}^{L} \ge \sum_{j=1}^{n}(D_j)_{\alpha=0}^{U}$ then the constraints $\sum_{i=1}^{m} s_i \ge \sum_{j=1}^{n} d_j$ can be deleted from Model (12.0 – 12.7). Multiplying constraints (12.4)– (12.6) by u_i, v_j and w_k, respectively, and substituting $s_i u_i$ by p_i, $d_j v_j$ by q_j, and $e_k w_k$ by r_k, Model (12.0 – 12.7) is transformed into the following linear program:

$$Z_\alpha^U = \max -\sum_{i=1}^{m} p_i + \sum_{j=1}^{n} q_j - \sum_{k=1}^{l} r_k$$

$$\begin{aligned}
s.t. \quad & -u_i + v_j - w_k \le (c_{ijk})_\alpha^U, && i = 1, 2, \ldots\ldots, m,\ j = 1, 2, \ldots\ldots, n,\ k = 1, 2, \ldots\ldots, l, \\
& (S_i)_\alpha^L u_i \le p_i \le (S_i)_\alpha^U u_i && i = 1, \ldots\ldots, m, \\
& (D_j)_\alpha^L v_j \le q_j \le (D_j)_\alpha^U v_j && j = 1, \ldots\ldots, n, \\
& (E_k)_\alpha^L w_k \le r_k \le (E_k)_\alpha^U w_k && k = 1, \ldots\ldots, l, \\
& p_i, q_j, r_k \ge 0, \quad \forall\, i, j, k
\end{aligned} \tag{13}$$

In this case, the upper bound of the total transportation cost z_α^U at ∝ level can be found more easily. Problems (7a) and (7b) are assured to be feasible if the lower bound of the total fuzzy demand is smaller than both of the upper bound of the total fuzzy supply and the upper bound of the total conveyance capacity, i.e., $\sum_{j=1}^{n}(D_j)_{\alpha=0}^{L} \le \sum_{i=1}^{m}(S_i)_{\alpha=0}^{U}$ and $\sum_{j=1}^{n}(D_j)_{\alpha=0}^{L} \le \sum_{k=1}^{l}(E_k)_{\alpha=0}^{U}$.

EXAMPLE

As an illustration of the proposed approach, consider a fuzzy solid transportation problem with two fuzzy supplies, three fuzzy demands, two conveyance capacities and three budget intervals in nature. The notations used in this example is (a, b, c, d) for a trapezoidal fuzzy number with a, b, c and d as the coordinates of the four vertices of the trapezoid and (x, y, z) for the triangular fuzzy number with x, y, z as the coordinates of the three vertices of the triangle. The problem has the following mathematical form:

$$\begin{aligned}
\text{Min}\ & (20, 30, 40)\, x_{111} + 70 x_{112} + 60 x_{121} + 60 x_{122} + 50 x_{131} + 30 x_{132} \\
& + (10, 20, 30)\, x_{211} + 40 x_{212} + 20 x_{221} + 50 x_{222} + 40 x_{231} + 50 x_{232}
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\quad & x_{111}+x_{112}+x_{121}+x_{122}+x_{131}+x_{132}\le(70,80,100,120)\\
& x_{211}+x_{212}+x_{221}+x_{222}+x_{231}+x_{232}\le(60,70,80),\\
& x_{111}+x_{112}+x_{211}+x_{212}\ge(10,30,40,50)\\
& x_{121}+x_{122}+x_{221}+x_{222}\ge(40,50,60),\\
& x_{131}+x_{132}+x_{231}+x_{232}\ge(30,40,60,70),\\
& x_{111}+x_{121}+x_{131}+x_{211}+x_{221}+x_{231}\le(70,80,100),\\
& x_{112}+x_{122}+x_{132}+x_{212}+x_{222}+x_{232}\le(60,70,90),\\
& 20x_{111}+70x_{112}+10x_{211}+40x_{212}\le[3450,3750],\\
& 60x_{121}+20x_{221}+30x_{221}+50x_{222}\le[2585,2615],\\
& 50x_{131}+30x_{132}+40x_{231}+50x_{232}\le[2860,2940],\\
& x_{ijk}\ge 0,\quad i=1,2,\quad j=1,2,3,\quad k=1,2.
\end{aligned}$$

The total Supply $S = \tilde{S}_1 + \tilde{S}_2 = (130,150,170,210)$, the total demand $D = \tilde{D}_1 + \tilde{D}_2 + \tilde{D}_3 = (80,120,150,180)$, and the total conveyance capacity $E = \tilde{E}_1 + \tilde{E}_2 = (130,150,190)$ and the intervals of budgets are [3450, 3750], [2585, 2615] and [2860, 2940]. Since $S \cap D \cap E \ne \varnothing$, Problem has feasible solutions. According to models (9) and (12), the lower and upper bounds of $\tilde{Z}$ at possibility level α can be formulated as:

$$\begin{aligned}
Z_\alpha^L = \min\ & 20x_{111}+70x_{112}+60x_{121}+20x_{122}+50x_{131}+30x_{132}\\
& +10x_{211}+40x_{212}+30x_{221}+50x_{222}+40x_{231}+50x_{232}\\
\text{s.t,}\quad & x_{111}+x_{112}+x_{121}+x_{122}+x_{131}+x_{132}\le s_1,\\
& x_{211}+x_{212}+x_{221}+x_{222}+x_{231}+x_{232}\le s_2,\\
& x_{111}+x_{112}+x_{211}+x_{212}\ge d_1,\\
& x_{121}+x_{122}+x_{221}+x_{222}\ge d_2,\\
& x_{131}+x_{132}+x_{231}+x_{232}\ge d_3,\\
& x_{111}+x_{121}+x_{131}+x_{211}+x_{221}+x_{231}\le e_1,\\
& x_{112}+x_{122}+x_{132}+x_{212}+x_{222}+x_{232}\le e_2,\\
& 20x_{111}+70x_{112}+10x_{211}+40x_{212}\le 3600,\\
& 60x_{121}+20x_{221}+30x_{221}+50x_{222}\le 2600,\\
& 50x_{131}+30x_{132}+40x_{231}+50x_{232}\le 2900,\\
& s_1+s_2\ge d_1+d_2+d_3,\\
& e_1+e_2\ge d_1+d_2+d_3,\\
& 70+10\alpha\le s_1\le 120-20\alpha,\\
& 60+10\alpha\le s_2\le 90-20\alpha,\\
& 10+10\alpha\le d_1\le 50-10\alpha,\\
& 40+10\alpha\le d_2\le 60-10\alpha,\\
& 30+10\alpha\le d_3\le 70-10\alpha,\\
& 70+10\alpha\le e_1\le 100-20\alpha,\\
& 60+10\alpha\le e_2\le 90-20\alpha,\\
& x_{ijk}\ge 0,\quad i=1,2,\quad j=1,2,3,\quad k=1,2.
\end{aligned}$$

$$\begin{aligned}
Z_\alpha^U = \max\quad & -s_1u_1 - s_2u_2 + d_1v_1 + d_2v_2 + d_3v_3 - e_1w_1 - e_2w_2 - 3600y_1 - 2600y_2 - 2900y_3\\
\text{s.t.}\quad & -u_1 + v_1 - w_1 - 20y_1 \le 40 - 10\alpha,
\end{aligned}$$

$-u_1 + v_1 - w_2 - 70y_1 \leq 70,$

$-u_1 + v_2 - w_1 - 60y_2 \leq 60,$

$-u_1 + v_2 - w_2 - 20y_2 \leq 20,$

$-u_1 + v_3 - w_1 - 50y_3 \leq 50,$

$-u_1 + v_3 - w_2 - 30y_3 \leq 30,$

$-u_2 + v_1 - w_1 - (30-10\alpha)y_1 \leq 30-10\alpha,$

$-u_2 + v_1 - w_2 - 40y_1 \leq 40,$

$-u_2 + v_2 - w_1 - 30y_2 \leq 30,$

$-u_2 + v_2 - w_2 - 50y_2 \leq 50,$

$-u_2 + v_3 - w_1 - 40y_3 \leq 40,$

$-u_2 + v_3 - w_2 - 50y_3 \leq 50,$

$s_1 + s_2 \geq d_1 + d_2 + d_3,$

$e_1 + e_2 \geq d_1 + d_2 + d_3,$

$70 + 10\alpha \leq s_1 \leq 120 - 20\alpha,$

$60 + 10\alpha \leq s_2 \leq 90 - 20\alpha,$

$10 + 10\alpha \leq d_1 \leq 50 - 10\alpha,$

$40 + 10\alpha \leq d_2 \leq 60 - 10\alpha,$

$30 + 10\alpha \leq d_3 \leq 70 - 10\alpha,$

$70 + 10\alpha \leq e_1 \leq 100 - 20\alpha,$

$60 + 10\alpha \leq e_2 \leq 90 - 20\alpha,$

$u_1, u_2, v_1, v_2, v_3, w_1, w_2 \geq 0.$

We solve the above two problems by using Lingo [12]. Table.1 lists the α -cuts of the total transportation cost at 11 distinct α values: 0, 0.1, 0.2, 0.3… 1.0 and Fig.1 depict the membership function of the total transportation cost of this example.

The α value indicates the level of possibility and degree of uncertainty of the obtained information. The greater the α value, the greater the level of possibility and the lower the degree of uncertainty is.

Table 1: The α -cuts of the total transportation cost

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Z_α^L	1800	1860	1920	1980	2040	2100	2200	2300	2400	2500	2600
Z_α^U	5700	5543	5392	5247	5108	4975	4848	4727	4612	4503	4100

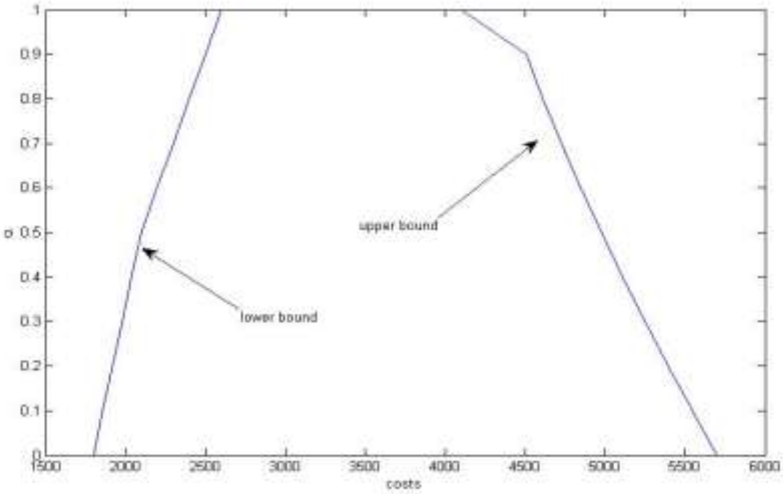

Fig. 1. The membership function of the total transportation cost

Since the fuzzy total transportation cost lies in a range, its most likely value falls between 2600 and 4100, and its value impossible to falls outside the range of 1800 and 5700.

For α =0, the lower bound of Z^* =1800 occurs at x^*_{122} =40, x^*_{132} =30, x^*_{211} =10 with s_1 = 120, s_2 =90, d_1 =10, d_2 =40, d_3 =30, e_1 =100, e_2 =90, and the other decision variables are 0. The upper bound of Z^* =5700 occurs at x^*_{111} =40, x^*_{122} =10, x^*_{132} =70, x^*_{211} =10, x^*_{221} =50 with s_1 = 120, s_2 =60, d_1 =50, d_2 =60, d_3 =70, e_1 =100, e_2 =80, and the other decision variables are 0.

At other extreme end of α =1, the lower bound of Z^* =2600 occurs at x^*_{122} =30, x^*_{132} =40, x^*_{211} =20, x^*_{221} =20 with s_1 = 80, s_2 =70, d_1 =20, d_2 =50, d_3 =40, e_1 =80, e_2 =70, and the other decision variables are 0. The upper bound of Z^* =4100 occurs at x^*_{111} =10, x^*_{122} =10, x^*_{132} =60, x^*_{211} =30, x^*_{221} =40 with s_1 = 80, s_2 =70, d_1 =40, d_2 =50, d_3 =60, e_1 =80, e_2 =70, and the other decision variables are 0. Notably, the values of the decision variables derived in this example are also fuzzy.

IV CONCLUSION

Transportation models have wide applications in logistics and supply chain management for improving service and reduce the cost. We have developed the solution procedure for a fuzzy solid transportation problem with fuzzy supply, requirement, conveyance capacity and budget interval and we put solution using Hu and Wang's approach and fuzzy programming approach. In the present study we solve mathematical problems using Lingo software. In frame work with genuine field problem, the technique could be used as very effectual and promising and in view of a practical significance. The problem can be extended or applied to other similar uncertain models in other areas such as inventory control, ecology, sustainable form management, etc.

REFERENCES

[1] R.E. Bellman, L.A. Zadeh, Decision-making in a fuzzy environment, *Management Science*, 17, 1970, 141–164.

[2] K.B. Haley, "The solid transportation problem", *Operations Research*, 11, 1962, 446–448.

[3] J.J. Buckly, "A Possibilistic linear programming with triangular fuzzy numbers", *Fuzzy Sets and Systems,* 26, 1988, 135–138.

[4] S. Chanas, D. Kuchta "A concept of the optimal solution of the transportation problem with fuzzy cost coefficients", *Fuzzy Sets and Systems, 82*, 1996, 299–305.

[5] Hadi Basirzadeh, "An Approach for Solving Fuzzy Transportation Problem", *Applied Mathematical Sciences,* 5(32), 2011, 1549 – 1566.

[6] A. Baidya, U.K. Bera, M. Maiti, "Multi-item interval valued solid transportation problem with safety measure under fuzzy-stochastic environment*", Journal of Transportation Security*, 6, 2013, 151–174.

[7] M.A. Parra, A.B. Terol and M.V.R. Uria, "Solving the multiobjective possibilistic linear programming problem", *European Journal of Operational Research*, 117, 1999, 175–182.

[8] L.A. Zadeh, "Fuzzy sets as a basis for a theory of possibility", *Fuzzy Sets and Systems*, 1, 1978, 3–28.

[9] R.R. Yager, "A characterization of the extension principle", *Fuzzy Sets and Systems,* 18, 1986, 205–217.

[10] H.J. Zimmermann, Fuzzy Set Theory and Its Applications (third ed., Kluwer-Nijhoff, Boston, 1996).

[11] B.Q. Hu, S. Wang, "A novel approach in uncertain programming part i: New arithmetic and order relation for interval numbers*". Journal of Industrial and Management Optimization*, 2(4), 2006, 351–371.

[12] Lingo User_s Guide (LINDO Systems Inc., Chicago, 1999).