MULTIOBJECTIVE LINEAR PROGRAMMING MODEL WITH WEIGHTED INTERVALS IN A MINIMUM CONSENSUS SCENARIO FOR PRODUCTION PLANNING

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ABSTRACT

This paper presentsa multiobjective linear programming (MOLP) model in which the cost coefficients of each decision variable of the objective functions aremodeledin weighted intervals and there exists a minimum consensus scenario among the decision makers. At the present, the different models involved in optimization by linear programming are triedin an independent way, such models as: MOLP,Interval multiobjective linear programming (IMOLP), weighing the cost elements in the objective functions by probabilistic ways, consensus analysis to make a decision.This research presents a model that combines these models in such a way that considers a scenario of minimum consensus among the decision makers that have the responsibility of making the best decision.This model assigns a smooth not probabilistic weight, to the decision variables to avoid a potential management hierarchyto solve a production planning problem.This model could help the decision makers since it reduces the number of efficient solutions provided by a typical MOLP model.

Keywords: Multiobjective Linear Programming, Interval Multiobjective Linear Programming, Weighted Interval Multiobjective Linear Programming, Multicriteria Decision Making, Consensus.

I. INTRODUCTION

The biggest difficulty of the Multicriteria Decision Making (MCDM) is linked to the fact that there is no optimal solution for all the criteria, so negotiations among the different points of view should be carried out to determine an acceptable solution.Nevertheless, it is not an easy problem, as it is explained during the last decades in a large amount of literature.

Herrera and Herrera-Viedma [1] propose an approach that allows to the decision maker to obtain heterogeneous data, which automatically are transformed into a unified scale, before being utilized. Some researcherssuch as Delgado et al. [2]argue that with the purpose to avoid ambiguity, uncertainty or contradiction in the data, the more information is obtained, the better the alternatives are understood.Seyhan [3] mentions that in many applications, the decision maker has multiple alternatives and multiple objectives in conflict,under these circumstances an analysis and systematic evaluation is essential for the adequate planning and control of the organizations.

In many cases, the consensus is not achieved in an adequate way creating situations of natural interests among departments, therefore a methodology should be generated to alleviate this situation since the decision should be taken and executed. Many methods exist to determine the classification of a set of alternatives in terms of a decision criterion set.Although there is a large quantity of works using the theory of decisions, there are very few investigations related to how to evaluate the conflict of havingdifferent classifications for the same set of alternatives,this is a problem that arises when there are more than one decision maker.Works of Iz [4] and Saaty [5] mention that is a typical problem in a group of decision maker.

A decision is the result of an interaction among the actors influenced by a context. During the process taking a decision, the decision maker is influenced by preferences, the description of the alternatives, the conceptualization of the options, the process itself, etc. Sheppard [6] mentions that the weighting process is unstable, subjective and often arbitrary, this has been the presumption that the subtle weighting and the combination of attributes can be achieved only by the mysterious intuitive deliberations of the human intelligence. Zeleny [7]stipulates that it is ambiguous, for example, to expect the decision maker to state that "$\lambda_i = 0.42$", orthat "$0.45 < \lambda_i < 0.5$". More likely, he would express himself in such terms as "$\lambda_i$should be substantially larger than 0.5" or "$\lambda_i$should be in the vicinity of 0.4 but rather larger" or some another fuzzy statement, this theory it was initiated byZadeh [8] at the beginning of the 70's.

In most of the decision making situations, in which the author of the research presented in this paper has participated during the last years, the information available it is changing, since it is based on concepts of continuous improvement in which the situations are dynamics day by day, the cost reductions are constant, the productive capacity should be flexible, in such a way, that must be capable to absorb the changing needs of the clients. As a consequence, the information available generally is established in intervals and, the decisions are taken in most of the cases in nominal basis. On the other hand, the decisions are taken considering the management hierarchy since each managerial activity considers that their priorities are the most important. Bottom line, many variables exist, different metric that should be achieved and, therefore, scenarios exits in which not a global consensus is achieved.Due to the situations that are presented day by day of the businesses, in which the decisions are taken in heuristics basis, there is a need to develop a model that involves several objectives in a minimum consensusscenario in a way that all variables have the similar importance.

II. PROBLEM APPROACH

The model presented in this paper contributesto get a decision under a minimum consensus scenario in which the cost coefficient for each decision variable in the objective functions it is expressed in weighted intervals, giving a tool to the people that have the responsibility to make decisions within the production planning environment. The presented model has the necessary efficient solution validation (or Pareto optimal).

III. MOLP MODELS

3.1 MOLP General Model

The process to optimize systematic and simultaneously a set of objective functions is called multiobjective optimization (MOO), in general form a problem of MOO is formulated in the following way:

Find

$$x = [x_1, x_2, \ldots, x_k]^T \quad (1)$$

To optimize

$$:\mathbf{F}(\mathbf{x}) = [F_1(x), F_2(x), \ldots, F_n(x)]^T \qquad (2)$$

Subject to:

$$:g_j(\mathbf{x}) \leq 0 \; ; j = 1, 2, \ldots, m \qquad (3)$$

$$:h_l(\mathbf{x}) = 0 \; ; l = 1, 2, \ldots, e \qquad (4)$$

Where$k$is the number of design variables, $n$is the number of objective functions, $m$is the number of constraints with inequality and $e$the number of restrictions with equality. $\mathbf{x} \in E^n$ is a scalar vector of design variables x_i,and$\mathbf{F}(\mathbf{x}) \in E^k$is a scalar vector of the objective functions$F_i\,(x){:}E^n \rightarrow E^1$. Many MOO algorithms involve the use of additional restrictions in such a way that the original restrictions shown in the equations (1), (2), (3)and (4) are established as restrictions of the model. The feasible design space is defining as:

$$: X = \left\{ x \mid g_j(x) \leq 0,\; j = 1, 2, \ldots, m;\; h_l = 0,\, l = 1, 2, \ldots, e \right\} \qquad (5)$$

In MOO, an improvement in one objective function, often results in the detriment to another. Consequently, the idea of solution is less straightforward than it is with single objective function optimization. The concept of predominant solution is the Pareto's optimality, and a point is Pareto's optimal if and only if it is impossible to move from that point and reduce at least one objective function without increasing (e.g.detrimentally affecting) any another function.

Typically there are an infinite number of Pareto's optimal points for a problem, and to settle on one point, requires that the decision maker to somehow articulate preferences. The user requires of preferences articulation "a priori" methods to specify his preferences in terms of the relative importance of the function objectives or in terms of goals, before an algorithm of optimization is carried out. The articulation of preferences "a posteriori", involves selecting a solution from a palette of possible solutions, presumably Pareto optimal solutions, after the algorithm is executed.

Approaches to MOO that entail combining all the objective functions into a single scalar function, are called scalarization methods. When such approaches are used with "a priori" articulation of preferences, preferences are modeled as components of the scalar function such as weights in a weighted sum or an exponent in a p norm;these components are called parametric methods.The MOLP models used in the research presented here are described in the next section.

3.2Weighted MOLP Model

The mentioned modelsare combined with MOLP models with the idea of optimize the decision models. Cohon [9]presents a linear multiobjective optimization model with $n$decision variables, $m$constrains and p objectives such as:

$$:\text{Opt } Z(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) = [Z_1(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n), Z_2(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n), \ldots, Z_p(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)] \qquad (6)$$

Subject to:

$$:g_i(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \leq 0 \; ; i = 1, 2, \ldots, m \qquad (7)$$

$$:\mathbf{x}_j \geq 0 ; j = 1, 2, \ldots, n \qquad (8)$$

In the above model the parameters are established and/or known.Cohon [9]presents the following model with objectives weighted as a method to solve the MLOP problem;

$$Opt\ Z(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n;\ \mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p) = \sum_{k=1}^{p} \mathbf{w}_k Z_k(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \quad (9)$$

Subject to:

$$(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \in F_d \quad (10)$$

Where $F_d$represents the feasible region in the decision space and $\mathbf{w}_k$ is the weight of the objective$k$.Zeleny [7]presents the following MOLP model, for$l$objective functions;

$$\theta(\mathbf{x}) = [\theta^1(\mathbf{x}), \theta^2(\mathbf{x}), \ldots, \theta^l(\mathbf{x})] \quad (11)$$

Subject to:

$$\mathbf{x} \in \mathbf{X} \quad (12)$$

Where $\theta(\mathbf{x})$ is the objective function vector,$\mathbf{x} \in E^n$is the decision variable and$X \subseteq E^n$is the set of all the feasible solutions. The problem all non-dominated points$\bar{\mathbf{x}} \in \mathbf{X}$, this means, does not exists other $\mathbf{x} \in \mathbf{X}$such that$\theta(\mathbf{x}) \geq \theta(\bar{\mathbf{x}})and\theta(\mathbf{x}) \neq \theta(\bar{\mathbf{x}})$. Also, Zeleny [7] defines the weighting of the objective functions as:

$$\Lambda = \{\lambda \mid \lambda \in E^l;\ \lambda_i \geq 0,\ i=1, 2, \ldots, l;\ \sum_{i=1}^{l} \lambda_i = 1\} \quad (13)$$

Given that $\lambda \in \Lambda$, be$P(\lambda)$, the following problem is set:

$$\underset{\mathbf{x} \in \mathbf{X}}{Max} \sum_{i=1}^{l} \lambda_i \theta^i(\mathbf{x}) \quad (14)$$

Or to find a point$\hat{\mathbf{x}} \in \mathbf{X}$, such that;

$$\lambda\, \theta(\hat{\mathbf{x}}) \geq \lambda\, \theta(\mathbf{x}) \text{ for all } \mathbf{x} \in \mathbf{X} \quad (15)$$

In this model, the parameters are established and known;based on this, the combination of weights assigned to the objective functions by the MOLP, can be considered as a valid tool for the decision making.Nevertheless, to solve these MCDM problems without utilizing weighted objectives it is too difficult, see[10].Nakayama [11]describesthat the traditional weighted model is not effective due to the MOLP problems; the final decision is made based on the value of the decision maker judgment;consequently, it is important how to obtain the value of that judgment. In many practical cases, the objective function vector is balanced in such way that the judgment value assigned by the decision maker could be incorporated.

The most used balancing technique is the linear weighted sum:

$$\sum_{i=1}^{r} w_i f_i(\mathbf{x}) \quad (16)$$

The decision maker judgment value is reflected by the weighting;although this kind of balances is extensively utilized in many practical problems, it has a disadvantage on it. Especially, itcannot provide a solution in the internal sideof the Pareto surface due to duality absence for non-convex cases. Even for convex cases, for example, in linear cases, even when it is desired to obtain a point in the segment of line between two vertices, only one joint of the Pareto surface is obtained, as large as the one that uses the simplex method;this means that depending on the structure of the problem, the linear sum of the weighting provide a solution that the decision maker wants.

3.3 IMOLPModel

Heindel et al. [13] it taken into account a general model of IMOLP of the form:

$$Max\ CX \quad (17)$$

Subject to:

$$AX = b \quad (18)$$

$$X \geq 0,\ C \in \Phi \quad (19)$$

Where Φ is a set of n x $m$matrices which its element $C_{ij} = [C_{ij}^L, C_{ij}^U]$, for $i = 1, 2,\ldots, n$ and $j = 1, 2,\ldots, m$, A is a p x m matrix, b is a p x 1 vector and X is a 1 x m vector.This system it is called interval linear system,the problem to determinate the solution set of interval linear system is NP-Hard (non-deterministic polynomial-time hard), see[13].

IV. MODELS FOR DEFINING THE WEIGHTS

Among the many methods of multicriteria analysis, Tsamboulas et al. [14]identified the five most convenient methods, after reviewing the methods based on their application following up records and acceptance of the users in practical applications and, also examining methods used, data requirement, user friendliness and the utility of results for different types of problems. Those five methods are: REGIME; ELECTRE, AHP (Analytical Hierarchy Process),MAUT (Multi Attribute Utility Theory) and the ADAM (Attribute-Dynamic Attitude Model) proposed by Zeleny [15,16].The last three methods (AHP, MAUT andADAM) could be classified as additive models,Tsamboulas et al. [14]have also evaluated these five methods based on their adaptation in the management of multidimensional and complex evaluations.A method is considered adequate if has four main characteristics: transparency, simplicity, robustness and responsibility. Tsamboulas et al. [14] have also suggested that the additives methods are the most dependable and of these, the AHP is the method that satisfies all the four main characteristics,besides, based on the appreciation of those five methods, Heindel et al. [13] mentioned that the results of the AHP method can be considered as a solution commitment. Additionally,Aldian and Taylor [17] introduced the method of proportion, which produces more balanced weights. Sayers et al. [18] recommend the use of additives methods, especially the lineal additive method.The lineal additive method is extensively utilized in the decision making; it is a strong method and provides a very intuitive answer.In this method, several impacts of each alternative are weighted utilizing numerical values called weighting criteria. The weighting criteria is summarized to obtain a simple value for each alternative,this method is similar to the cost-benefit analysis, where the monetary weights are applied.

In general, the basic form for a single lineal additive model is(see [19, 20]):

$$S_i = \sum_{j=1}^{n} c_{ij} w_j \ ; \quad \text{for } i=1, 2, 3, \ldots, m \quad (20)$$

Where$S_i$is an evaluation measure for the alternative i, $c_{ij}$is the measure of the alternative$i$respect to the criteria j (generally normalized) and$w_j$is the weighting for the criteria$j$. This implies that the largest value, it is the largest value of $S_i$in the classification.

For defining the weightsw_i, the AHP process and the method of proportioncould be considered, the reader can consult Saaty [21] and Aldian and Taylor [17] respectivelyto review these models.

V. CONSENSUS ANALYSIS AMONG DECISION MAKERS

When it is of interest measuring the level of consensus of the decision makers, in relation to a parameter, two extreme situations can be distinguished: On one hand, if the intervals assigned to the parameter coincide, then a complete consensus exists among the decision makers about that parameter;on the other hand, if the intervals assigned by the decision makers to that parameter are all discordant, then, there is a total absence of consensus (total conflict) between the decision makers and that conflict is so large that the discordant intervals are very distant one of the other.

In practice, in many cases, it can be expected that, relatively,givenanindeterminate parameter *p*, more likely it is in a middle situation of partial consensus; on the one hand, it can be something of overlapping among some of the intervals assigned to the parameter and on the other hand, some other intervals can be disjoint and can be distant from one another;consequently, to measure the level of consensus of the decision makers for an indeterminate parameter *p*, it is proposed to utilize two indices, which are defined considering the intervals assigned to that parameter in pairs.Urli and Nadeau [22] developed an interactive approach to solve a MOLP problem involving several decision makers and indeterminate parameters. This interactive approach it is based on concordance and discordance indexes to stablish a consensus among the decision makers.

5.1 Concordance Index

For a pair of intervals assigned to a parameter c_{ij}, first the concordance index about $c_{ij}$is defined, which measures the average percentage of overlapping among the pair of intervals which overlap pair wise. Formally; given an indeterminate parameter c_{ij}, M intervals exists which are assigned to it and therefore there are$C_2^M = M!/2!\,(M-2)!$possible pairs of intervals to consider. For each pair of intervals, the percentage of overlapping is determined by the length of the interval common to the two intervals of this pair divided by the length of that of the two intervals which is the shortest. Then, the concordance for the parameter c_{ij}which is denoted by $C\,(c_{ij})$, is defined as follows:

$$: C(c_{ij}) = \sum (\textit{Percentage of overlapping in the pairs of intervals which overlap pair wise}) / C_2^M \qquad (21)$$

Obviously, if there is a complete consensus on a parameter c_{ij}, it will have $C(c_{ij})=1$ and,if there is a total absence of consensus it will have $C(c_{ij})=0$. Therefore, the concordance for c_{ij}will always be between 0 and 1, this value, will be larger as there is more overlapping between the intervals assigned by the decision makers for that parameter.

5.2 Discordance Index

It can be considered that the concordance is a good estimation of the similarity of the intervals assigned by the different decision makers to the indeterminate parameter c_{ij}. But, when some of the intervals assigned to a parameter c_{ij} by the decision makers differ until they eventually are disjoint, the concordance does not express the amplitude of that difference.

So, in addition to the concordance index, it is proposed to compute another index called discordance index. This is an average measure of the relative distance between the intervals in the pairs of intervals which do not overlap pair wise. More precisely, for each pair of intervals which do not overlap (this is, are disjoint), the relative distance between the two intervals of that pair is giving by the distance between the upper bound of the interval most to the left and the lower bound of the one most to the right, that last distance being divided by the one existing between the lower bound of the interval most to the left and the upper bound of the one most to the right. Then the relative discordance to the parameter c_{ij}, denoted$D(c_{ij})$ is defined by:

$$: D(c_{ij}) = \sum (\text{Relative distances between the intervals in the pairs of disjoint intervals}) / C_2^M \quad (22)$$

Obviously, for a given parameter c_{ij}, if there is a complete consensus, it shall have $D(c_{ij}) = 0$ and, if there is a complete absence of concordance it shall have $D(c_{ij}) = 1$ when the intervals are reduced to points. In general, when at least two intervals assigned to c_{ij} by the decision makers are disjoint, it shall have $0 \leq D(c_{ij}) \leq 1$, $D(c_{ij})$ will be larger as the distances between the disjoint intervals are larger.

5.3 Consensus

Two thresholds are calculatedto be compared with the concordance and discordance indexes,c_0 and d_0 respectively, in such way that there is minimum consensus if the following two conditions happen simultaneously:

$$:C(c_{ij}) \geq c_0 \quad (23)$$

$$: D(c_{ij}) \leq d_0 \quad (24)$$

The resulting values from experimental runswere; $c_0 = 0.125$ and $d_0 = 0.125$

VI. APPROACH OF THE PROPOSED MODEL

Actually, a lot of literature exists related to the MCDM, and also, many investigations referring to the Multiobjective Programming Decision Making (MODM).Ultimately, inside the MODM, algorithms have been developed that help to carry out calculations with a great quantity of information such as: Genetic Algorithms, Evolutionary Algorithms, Linguistic Algorithms, Cultural Algorithms, etc.

Vincke [23] mentions that in the MCDM field, three types of problems are considered:decision problems, range problems and classification problems. The goal of the decision makers in each type of problems is different; in the decision problems, the objective is to find the best alternative; in the range problems, the objective is to find the best of all alternatives, which is usually presented as a range from the best to the worst and, in the classification problems the objective is to find which of the alternatives belongs to each class of a set of predetermined classes. This paper is focused on the range and classification problems, since the decision problems are implicit in the other two, even though Zopounidis [24]mentionsthat the classification problems are classically solved with a supervision approach.

In the literature reviewed, the MCDM and MODM analysis, that had been developed, they present models to establish and analyze the weighting in the objective functions and the intervals or ranges of the coefficients in an independent way, this means they do not combine both criteria in one model.The need to combine the weighting and the intervals in the coefficients values in the real decision making in one single model was the motivation to do this investigation, where some decision makers intervene.They have the responsibility to maintain the companies in a competitive level in the global market. These decisions are taken inside the continuous improvement philosophy in several branches of the Industrial Engineering.

In the majority of the studies made, the assignment of weights to the objectives to optimize, are made in a probabilistic way, but, for practical goals of day by day decision making, in most of the cases, it is based in "a priori" information, this means, assigning occurrenceprobabilities of one event, based in historic data without a real analysis of the performances. Similarly, since several objectives exists in a multiobjective programming model, there is a conflict among them, one way to solve this conflict is assigning a weight to them.Nevertheless, these weights could be subjective, having as base the knowledge of the market, the performance of it, the experience, etc., even, it can be weighted according to the hierarchy level inside the organization. In the MCDM mathematic analysis they establish the preferences by different criteria assignment methods, some of them are analyzed in this investigation.

Considering the different MOLP models and the proposed analysis for a minimum consensus scenario among the decision makers mentioned, the proposed model in this paper presentsa heuristic model which combines the optimization by MODM, the objectives and coefficients weighting which are inside of an interval of known values, in which the decision makers have a minimum consensus. The model considers that the parameters are indeterminate as well; the indeterminacy is caused by the fact that the decision makers can assign only one interval of possible values.

Based in the satisficing(see [25]) and optimizing approaches (see [26] and [27]), the model proposed in this paper assumes that a team of *M* decision makers are involved in a decision problem, which can be expressed as a linear program defined by a set of *n* linear objective functions and *k* linear constraints with independent parameter as follow:

$$: Opt. Z_i = f_i(x_1, \ldots x_m) = \sum_{j=1}^{m} p_j c_{ij} x_j ; i=1, 2, \ldots, n \quad (25)$$

Subject to:

$$: \sum_{j=1}^{m} a_{ij} x_j \leq, =, \geq \; b_i ; i=1, 2, \ldots, k \quad (26)$$

$$: x_j \geq 0 ; j=1, 2, \ldots, m \quad (27)$$

Where:

$$: c_{ij} \in \left[c_{ij}^{min}, c_{ij}^{max}\right] ; i=1,2,\ldots, n ; j=1,2,\ldots,m \quad (28)$$

$$: 0 \leq p_j \leq 1 \quad (29)$$

$$: \sum_{j=1}^{m} p_j = 1 \quad (30)$$

Considering Bitran [26] and Inuiguchi and Sakawa [27], in general the equation (25) can be expressed as follows:

$$: Max \{ \Delta x \mid x \in F \} \quad (31)$$

Where $F = \{x \mid x\ Ax \leq b\}$ and Δ is a n x m matrix whose δ_{ij}'s components are mutually independent possibilistic restricted variables. If all possible ranges can be presented by closed intervals $[p_j c_{ij}^{min}, p_j c_{ij}^{max}]$, problem (31) is called a PLMO with weighted intervals. For this particularly case the set Γ becomes as follows:

$$: \eta = \left\{ \Omega = \left(p_j c_{ij}\right) \mid p_j c_{ij}^{min} \leq p_j c_{ij} \leq p_j c_{ij}^{max} \right\} ; i=1,2,\ldots,n, j=1,2,\ldots,m \quad (32)$$

Similar thanBitran [26] mentions, in this model there are two kinds of efficiency solutions sets as well:

$$: NSM = \cap_{\Omega \in \eta} EFM(\Omega) \quad (33)$$

$$: \Pi SM = \cup_{\Omega \in \eta} EFM(\Omega) \quad (34)$$

NSM is an efficiency solution for any $\Omega \in \eta$ and is named a necessary efficient solution. On the other hand ΠSM is a solution for at least one $\Omega \in \eta$ and is named a possible efficient solution. Note that $NSM \subseteq \Pi SM$, i.e, a necessary efficient solution is a possible efficient solution.

Let $P(x)$ a set of n by m matrices in which x is efficient, i.e.,

$$:P(x) = \{ \Omega \mid \nexists x^* \in F \text{ such that } \Omega x^* \geq \Omega x \text{ and } \Omega x^* \neq \Omega x \quad (35)$$

A solutionx belongs to the set of necessary efficient solutions if it is an efficient solution and if the matrix of weighted intervals is contained within the matrix of cost coefficients intervals of the variables in the feasible solutions space; expressed in the following way:

$$:x \in NSM \Leftrightarrow N_\eta (P(x)) = 1 \Leftrightarrow \eta \subseteq P(x) \in F \quad (36)$$

A solution$x$ belongs to the set of possible efficient solutions if it is a possible solution and if the intersection between the matrix of weighted intervals of cost coefficients intervals of the variables and the matrix of intervals is a not empty set; expressed as:

$$: x \in \Pi SM \Leftrightarrow \Pi_\eta (P(x)) = 1 \Leftrightarrow \eta \cap P(x) \neq 0 \quad (37)$$

In addition, let $Q(x)$ be a set of n row vectors in which x is a solution that maximizes a linear objective function $\mathbf{cy}$ subject to $\mathbf{y} \in F$, this means:

$$:Q(x) = \{\mathbf{c} \mid \mathbf{cx} = max_{y \in F} \mathbf{cy}, x \in F\} \quad (38)$$

Let $RNM(\eta)$ be the value of the cost coefficients of the variables defined as weighted intervals to obtain an efficient solution and it is defined by the intersection of the weighted cost coefficients and the intervals of the weighted cost coefficients of the objective functions variables and, it is:

$$:RNM(\eta) = \{\mathbf{c} \mid \forall \Omega \in \eta, \exists \mathbf{z} > 0 \mid \mathbf{c} = \mathbf{z}\Omega\} = \cap_{\Omega \in \eta} RM(\Omega) \quad (39)$$

Let $R\Pi M(\eta)$ be the value of the cost coefficients of the variables defined as weighted intervals to obtain a possible solution and it is defined by the union of the weighted cost coefficients and the intervals of the weighted cost coefficients of the objective functions variables and, it is:

$$:R\Pi M(\eta) = \{\mathbf{c} \mid \exists \mathbf{z} > 0 \text{ and } \exists \Omega \in \eta \mid \mathbf{c} = \mathbf{z}\Omega\} = \cup_{\Omega \in \eta} RM(\Omega) \quad (40)$$

Where $RM(\Omega)$ is the value of the coefficients to obtain a feasible solution expressed as follows:

$$: RM(\Omega) = \{\mathbf{c} \mid \exists \mathbf{z} > 0 \mid \mathbf{c} = \mathbf{z}\Omega\} \quad (41)$$

If the intersection between an efficient solution and a set of n rows vectors in which the x is a solution that maximizes a linear objective function and it is not an empty set, it is said that the solution x is a necessary efficient solution, defined by:

$$:RNM(\eta) \cap Q(x) \neq \varnothing \Rightarrow x \in NSM \quad (42)$$

$$:x \in \Pi SM \Leftrightarrow R\Pi M(\eta) \cap Q(x) \neq \varnothing \quad (43)$$

Also, when $RNM(\eta)$ is a no empty set we have:

$$: x \in NSM \Leftrightarrow RNM(\eta) \cap Q(x) \neq \varnothing \quad (44)$$

The statements above can be proved as follows; the right side of statement $\Rightarrow$ (if) in (42) is the necessary efficiency definition, for the statement $\Leftrightarrow$ (only if) in (44) that it is the part to prove, it is assume that $RNM(\eta) \cap Q(x) \neq \varnothing$ and at least one $\Omega \in \eta$exists that $RM(\Omega) \cap Q(x) \neq \varnothing$, e.g. $x \notin EFM(\Omega)$, this leads to $x \notin NSM$, then $RNM(\eta) \cap Q(x) \neq \varnothing \Leftarrow x \in NSM$■

VII. RESULTS AND DISCUSSION

The following it is an application of the proposed model by Rodríguez-Morachis [28], using two of eleven decision variables from one validation ran. There are four decision makers involved in the process: operations, manufacturing, materials and customer services managers. The model has threeoriginal objective functions; maximize sales price, maximize net margin and minimize operation cost, however, considering the satisficing approach, these three objective functions are converted into six, due to the upper and lower limits of the decision variables coefficients.Table 1 shows the corporative and adjusted intervals coming out from improvements in the manufacturing process, which are the ones that will be used as variable coefficients in the objective functions, in the table 2 are included the concordance and discordance indexes showing minimum consensus since$C(c_{ij})$≥0.125 and $D(c_{ij})$ ≤0.125, for all i and j.

Based in the model proposed by Aldian and Taylor [17], table 3 shows the weight assigned by the decision makers for each decision variable.

Table 1 Intervals of the Function Objectives Coefficients

	X_1		X_2	
	$C(c_{ij})$	$D(c_{ij})$	$C(c_{ij})$	$D(c_{ij})$
Net margin	0.3888	0.0555	0.2500	0.1111
Sales price	0.1666	0.0555	0.3333	0.1111
Operation cost	0.1666	0.1106	0.3333	0.0873

Table 2 Concordance and Discordance Inexes

Corporative Intervals for the brakeshoes case						
Product	Sales price ($)		Net margin ($)		Operation cost ($)	
	Min	Max	Min	Max	Min	Max
X_1	1.6794	1.8179	0.3072	0.6252	1.1927	1.3730
X_2	1.9531	2.1142	0.3572	0.7271	1.3871	1.5967
Adjusted Intervals						
X_1	1.70	1.85	0.40	0.70	1.2	1.3506
X_2	2.00	2.15	0.45	0.75	1.4	1.5908

Table 3 Weights per model

Product	Weight
X_1	0.5944
X_2	0.4051

Taking into account the weights and intervals showed in tables 1 and 3, the objective functions are as follows:

$Max\ Z_1 = 0.5944(0.4,0.7)\ X_1 + 0.4051\ (0.45,0.75)X_2$ Net margin (45)

$Max\ Z_2 = 0.5944(1.7,1.85)\ X_1 + 0.4051(2,2.15)X_2$ Sales price (46)

$MinZ_3 = 0.5944(1.2,1.3506)X_1 + 0.4051(1.4,1.5908)X_2$ Operation cost (47)

These 3 objective functionsbecomes to:

$Max\ F_1 = 0.4161\ \ X_1 + 0.3038\ \ X_2$ Net margin upper limit (48)

$Max\ F_2 = 0.2378\ \ X_1 + 1823\ \ X_2$ Net margin lower limit (49)

$Max\ F_3 = 1.097\ \ X_1 + 0.871\ \ X_2$ Sales price upper limit (50)

$Max\ F_4 = 1.01\ \ \ X_1 + 0.8102\ \ X_2$ Sales price lower limit (51)

$Min\ F_5 = 0.8028\ \ X_1 + 0.644\ \ X_2$ Operation cost upper limit (52)

$Min\ F_6 = 0.7133\ \ X_1 + 0.5671\ X_2$ Operation cost lower limit (53)

Subject to:

$X_1 \geq 1500$	Weekly requirements	(54)
$X_2 \geq 1300$	Weekly requirements	(55)
$0.0003436\ X_1 + 0.0003436\ X_2 \leq 15.18$	Support stamping	(56)
$0.000418 X_1 + 0.000418 X_2 \leq 15.18$	Circle stamping	(57)
$0.00159\ X_1 + 0.00187\ X_2 \leq 15.18$	Welding	(58)
$0.00112\ X_1 + 0.00194\ X_2 \leq 15.18$	Forming	(59)
$0.000719\ X_1 + 0.0.00242\ X_2 \leq 15.18$	Flattened	(60)
$0.00276\ X_2 \leq 15.18$	Flushing	(61)
$0.00216\ X_1 \leq 15.18$	Drilling	(62)
$X_1, X_2 \geq 0$		(63)

Solving the problem above the MOLP modified simplex method, the following two solutions were obtained; the first solution is X_1 = 7000, X_2 = 2166; the second one is X_1 = 1500, X_2 = 1300. Fig. 1 shows the feasible region and the twomentioned solutions, marked with a blue dot. Also, in this figure it can be observed that both solutions are necessary efficient solutions (Pareto optimal) which showsthat theproposed MOLP is validated, showing the detail just for the solution 1. After cost-benefit and dominance analyses, the best solution found out of the two is solution 1.

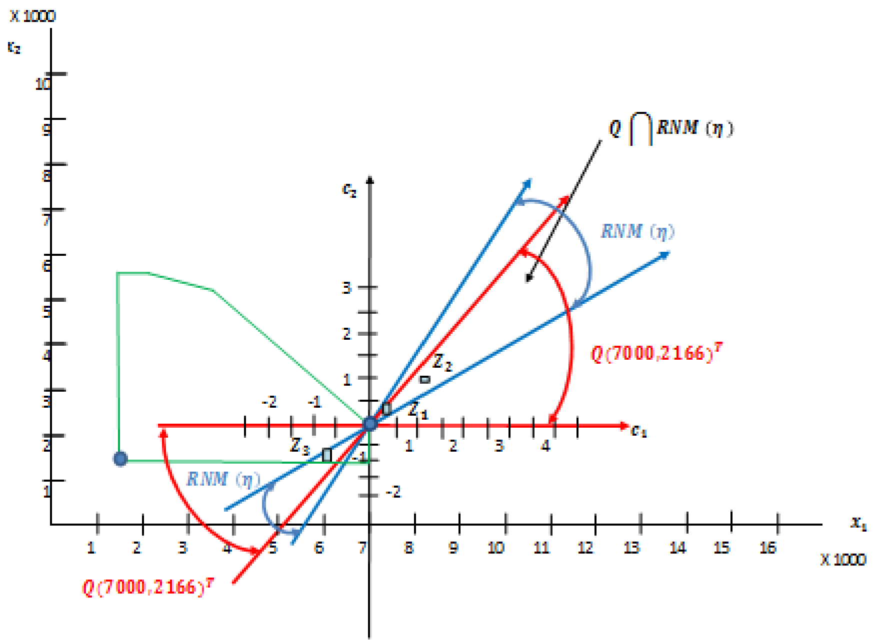

Figure 1 Feasible region and detail for one of the two necessary efficient solutions

VIII. CONCLUSIONS

It is shown that the proposed MOLP model provides necessary efficient solutions (or Pareto optimal), in such way that its application in the industrial real world helps the people responsible to make decisions. This model was applied in several industrial processes providing less solutions that the model without weights. One of the advantages of this proposed model is that it gives similar importance to the products reducing potential hierarchical decisions. Even when the business of the companies change every day due tocontinuous improvements that are reflected in cost reduction, flexibility of their processes and so on, the proposed MOLP model considers thesechanges if a new model is solved every week,absorbing the mentioned changes in the coefficients of the decision variables at the moment that its results are loaded in the ERP to do the weekly production planning. As a consequence of the implementation of this model, all products were shipped on time to the customers.

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