Reciprocal Degree Distance of Some Planar Graphs

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Abstract

The Reciprocal degree distance(RDD), defined for a connected graph G is vertex-degree-weighted sum of the reciprocal distance, that is,

$$RDD(G) = \sum_{u,v \in V(G)} \frac{(deg_G(u) + deg_G(v))}{d_G(u,v)}.$$

The reciprocal degree distance is a weight version of the Harary index, just as the degree distance is a weight version of the Wiener index. In this paper, we present formula for the reciprocal degree distance of multi-star graph, planar graph, and some new operation $\hat{e}$.

Keywords : Degree distance, Planar graph, Tree graph, Regular graph, Star graph, Reciprocal degree distance.

1 Introduction

A topological index is a numerical descriptor of a molecule, based on a certain topological feature of the corresponding molecular graph. A representation of an object, giving information only about the number of elements composing it and their connectivity is named as topological representation of an object. A topological representation of a molecule is called molecular graph. A molecular graph is a collection of vertices representing the atoms in the molecule and set of edges representing the covalent bonds. The advantage of topological indices is that they may be used directly as simple numerical description in comparison with physical, chemical or biological parameters of molecules in Quantitative Structure Property Relationships (QSPR) and in Quantitative Structure Activity Relationships (QSAR). One of the most widely known topological descriptor is the Wiener index named after chemist Harold Wiener. The Wiener index of a graph is defined as the sum of distances between all pairs of vertices in a connected graph. The degree distance[2] of a connected graph $G = (V, E)$ is defined as, $DD(G) = \sum_{u,v \in V(G)} (deg_G(u)+deg_G(v))d_G(u,v)$ and the reciprocal degree distance[3] of G is defined as, $RDD(G) = \sum_{u,v \in V(G)} \frac{(deg_G(u) + deg_G(v))}{d_G(u,v)}$, where $deg_G(u)$ is the degree of the vertex u in G and $d_G(u,v)$ is the shortest distance between u and v in G.

In[3] the reciprocal degree distance of some graphs are obtained. In this paper we compute the Reciprocal degree distance of some planar graphs and tree graphs.

2 Multi-Star Graph $K_{1,\underbrace{n,n,\ldots,n}_{m-times}}$

Starting from the star graph $K_{1,n}$ with vertices $\{v_0, v_1, v_2, \ldots, v_n\}$, introduce an edge to each of the pendent vertices $v_1, v_2, \ldots, v_n$ to get the resulting graph $K_{1,n,n}$ with vertices $\{v_0, v_1, \ldots, v_n, v_{(n+1)}, \ldots, v_{2n}\}$, again introduce an edge to each of the pendent vertices $v_{(n+1)}, \ldots, v_{2n}$, to get the graph $K_{1,n,n,n}$. Repeating this $(m-1)$ times we get a graph $K_{1,\underbrace{n,n,\ldots,n}_{m-times}}$ with $(mn+1)$ vertices $\{v_0, v_1, v_2, \ldots, v_n, v_{(n+1)}, \ldots, v_{(2n)}, v_{(2n+1)}, \ldots, v_{3n}, \ldots, v_{((m-1)n+1)}, \ldots, v_{mn}\}$ and mn edges[5], as shown in Fig 2.1

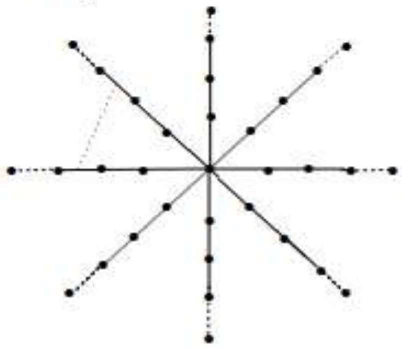

Fig 2.1

Theorem 2.1. *Reciprocal degree distance of the Multi-star graph $K_{1,n,n,\ldots,n}$ is*

$$RDD\left(K_{1,\underbrace{n,n,\ldots,n}_{m-times}}\right) = (2n^2+4n)\sum_{k=1}^{m-1}\frac{1}{k} + \frac{3n^2+n}{2m} + 3n(n-1)\sum_{k=m+1}^{(2m-1)}\frac{1}{k} + 4n\sum_{k=1}^{(m-2)}\frac{m-k-1}{k} + 4n(n-1)\left[\sum_{k=3}^{m}\frac{1}{k} + \sum_{k=5}^{m+1}\frac{1}{k} + \ldots + \sum_{k=2m-1}^{2m}\frac{1}{k} + \frac{1}{2m-3}\right].$$

Proof. Let $\{v_0, v_1, v_2, \ldots, v_n, v_{(n+1)}, \ldots, v_{2n}, \ldots, v_{((m-1)n+1)}, \ldots, v_{mn}\}$ be the set of $(mn+1)$ vertices of the multi-star graph $K_{1,\underbrace{n,n,\ldots,n}_{m-times}}$. Then,

$$RDD\left(K_{1,\underbrace{n,n,\ldots,n}_{m-times}}\right)$$
$$=\left\{n(n+2)\left[1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{(m-1)}\right]+\frac{n(n+1)}{m}\right\}$$
$$+\left\{4\left[(n-1)+(n-2)+\ldots+1\right]\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2(m-1)}\right]\right\}$$
$$+\left\{4n(n-1)\left[\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{m}\right]+4n(n-1)\left[\frac{1}{5}+\frac{1}{6}+\ldots+\frac{1}{(m+1)}\right]\right.$$
$$\left.+\ldots+4n(n-1)\left[\frac{1}{(2m-3)}\right]\right\}$$

$$+\left\{3n(n-1)\left[\frac{1}{(m+1)}+\frac{1}{(m+2)}+\ldots+\frac{1}{2m-1}\right]\right\}$$
$$+\left\{4n\left[1+\frac{1}{2}+\ldots+\frac{1}{(m-2)}\right]+4n\left[1+\frac{1}{2}+\ldots+\frac{1}{(m-3)}\right]+\ldots+4n\left[1\right]\right\}$$
$$+\left\{3n\left[\frac{1}{(m-1)}+\frac{1}{(m-2)}+\frac{1}{(m-3)}+\ldots+\frac{1}{2}+1\right]\right\}$$
$$+\left\{\frac{2}{2m}\left[(n-1)+(n-2)+\ldots+1\right]\right\}$$
$$=(2n^2+4n)\sum_{k=1}^{m-1}\frac{1}{k}+\frac{3n^2+n}{2m}+3n(n-1)\sum_{k=m+1}^{(2m-1)}\frac{1}{k}+4n\sum_{k=1}^{(m-2)}\frac{m-k-1}{k}$$
$$+4n(n-1)\left[\sum_{k=3}^{m}\frac{1}{k}+\sum_{k=5}^{m+1}\frac{1}{k}+\ldots+\sum_{k=2m-1}^{2m}\frac{1}{k}+\frac{1}{2m-3}\right].$$

Corollary 2.2.

$$RDD\left(K_{1,\underbrace{n,n,\ldots,n}_{m-times}}\right)$$
$$=(2n^2+4n)\sum_{k=1}^{m-1}\frac{1}{k}+\frac{3n^2+n}{2m}+3n(n-1)\sum_{k=m+1}^{(2m-1)}\frac{1}{k}+4n\sum_{k=1}^{(m-2)}\frac{m-k-1}{k}$$
$$+4n(n-1)\begin{cases}\left\{\sum_{k=1}^{\frac{(m-3)}{2}}k\left(\frac{1}{2k+1}+\frac{1}{2k+2}\right)+\left\lfloor\frac{m}{2}\right\rfloor\left(\frac{1}{m}\right)\right. \\ \left.\quad+\sum_{k=(\frac{m+1}{2})}^{[(\frac{m+1}{2})+(\frac{m-5}{2})]}(m-k-1)\left(\frac{1}{2k}+\frac{1}{2k+1}\right)\right\}, & \textit{if } m \textit{ is odd} \\ \left\{\sum_{k=1}^{(\frac{m}{2}-1)}k\left(\frac{1}{2k+1}+\frac{1}{2k+2}\right)+\left(\frac{m}{2}-1\right)\left(\frac{1}{m+1}\right)\right. \\ \left.\quad+\sum_{k=(\frac{m}{2}+1)}^{[(\frac{m}{2}+1)+(\frac{m}{2}-3)]}(m-k+1)\left(\frac{1}{2k}+\frac{1}{2k+1}\right)\right\}, & \textit{if } m \textit{ is even.}\end{cases}$$

3 Planar graph

Pl_n, $n \geq 3$ is a graph obtained by the join of P_{n-2} and P_2.

Theorem 3.1. *Reciprocal degree distance of Pl_n $(n \geq 5)$ graph is* $RDD(Pl_n) = 4n^2 - 3n - 16$.

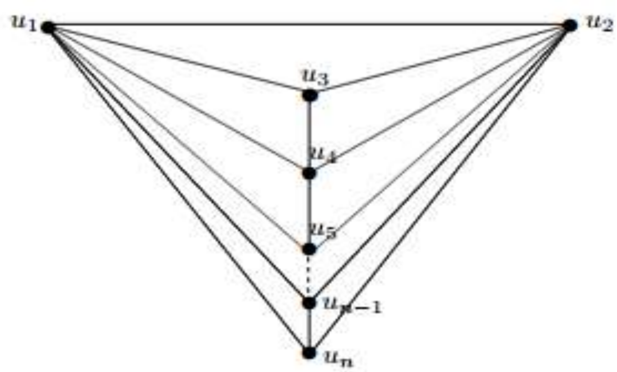

Fig 3.1

Proof. Let $G = Pl_n$, $(n \geq 5)$ be a graph and $V(G) = \{u_1, u_2, u_3, ..., u_n\}$. Then the Reciprocal degree distance of G is

$$RDD(G) = \sum_{i<j} \frac{[deg(u_i) + deg(u_j)]}{d(u_i, u_j)}$$

Note that, $deg_G(u_1) = deg_G(u_2) = n - 1$, $deg_G(u_3) = deg_G(u_n) = 3$,
$deg_G(u_4) = \ldots = deg_G(u_{n-1}) = 4$, $d_G(u_1, u_j) = 1$, $j = 2, 3, \ldots, n$
$d_G(u_2, u_j) = 1$, $j = 1, 3, 4, \ldots, n$,
$d_G(u_j, u_{j+1}) = 1$, $j = 3, \ldots, n-1$
$d_G(u_i, u_j) = 2$, $i = 3, 4, \ldots (n-2)$, $(i+2) \leq j \leq n$.
Hence,

$$\begin{aligned}
RDD(G) =&[n(n-1)+2(3)+4(n-4)]+[(n-2)(n-1)+2(3)+4(n-4)] \\
&+\left\{3\left[1+\underbrace{\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}}_{(n-4)-times}\right]+4\left[1+\underbrace{\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}}_{(n-5)-times}\right]+\frac{3}{2}\right\} \\
&+\left\{4\left[1+\underbrace{\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}}_{(n-5)-times}\right]+4\left[1+\underbrace{\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}}_{(n-6)-times}\right]+\frac{3}{2}\right\} \\
&+\left\{4\left[1+\underbrace{\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}}_{(n-6)-times}\right]+4\left[1+\underbrace{\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}}_{(n-7)-times}\right]+\frac{3}{2}\right\} \\
&+\cdots+\left\{4\left[1+\frac{1}{2}+\frac{1}{2}\right]+4\left[1+\frac{1}{2}\right]+\frac{3}{2}\right\} \\
&+\left\{4\left[1+\frac{1}{2}\right]+4[1]+\frac{3}{2}\right\}+4(1)+3(1) \\
=&4n^2-3n-16.
\end{aligned}$$

4 New graph operation $\hat{e}$

Definition 4.1. *$G_1\hat{e}G_2$ is a connected graph obtained from G_1 and G_2 by introducing an edge between an arbitrary vertex of G_1 and an arbitrary vertex of G_2[1]. If G_1 is a graph with p_1 vertices and q_1 edges and G_2 is a graph with p_2 vertices and q_2 edges then $G_1\hat{e}G_2$ will have (p_1+p_2) vertices and (q_1+q_2+1) edges. If $G_1 = C_m$ and $G_2 = C_n$ interesting graph structure $G = C_m\hat{e}C_n$ is obtained.*

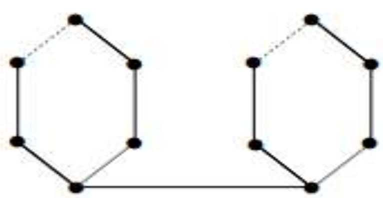

Fig 4.1

Lemma 4.2. *The Reciprocal degree distance of cycle is*

$$RDD(C_n) = \begin{cases} 4n\left[1+\frac{1}{2}+\cdots+\frac{1}{(\frac{n}{2}-1)}\right]+4, & \text{if } n \text{ is even} \\ 4n\left[1+\frac{1}{2}+\cdots+\frac{1}{(\frac{n-1}{2})}\right], & \text{if } n \text{ is odd.} \end{cases}$$

Proof. We consider the following cases.

Case(i): n is even.

$$\begin{aligned}RDD(C_n) =& n\left(\frac{2+2}{1}\right)+n\left(\frac{2+2}{2}\right)+n\left(\frac{2+2}{3}\right)\\&+\ldots+n\left(\frac{2+2}{\frac{n}{2}-1}\right)+\left(\frac{n}{2}\right)\left(\frac{2+2}{\frac{n}{2}}\right)\\=&4n\left[1+\frac{1}{2}+\cdots+\frac{1}{(\frac{n}{2}-1)}\right]+4\end{aligned}$$

Case(ii): n is odd.

$$\begin{aligned}RDD(C_n) =& n\left(\frac{2+2}{1}\right)+n\left(\frac{2+2}{2}\right)+n\left(\frac{2+2}{3}\right)+\ldots+n\left(\frac{2+2}{\frac{n-1}{2}}\right)\\=&4n\left[1+\frac{1}{2}+\cdots+\frac{1}{(\frac{n-1}{2})}\right]\end{aligned}$$

□

Remark 4.3. *Let G be a graph obtained by adjoining a vertex to a cycle C_n then*

$$RDD(G) - \sum_{\substack{u,v\in V(G),\\ v\notin(C_n)}} \frac{deg_G(u)+deg_G(v)}{d_G(u,v)}$$

$$= \begin{cases} (4n+2)\left[1+\frac{1}{2}+\cdots+\frac{1}{(\frac{n}{2}-1)}\right]+(4+\frac{2}{n}), & \textit{if } n \textit{ is even}\\ (4n+2)\left[1+\frac{1}{2}+\cdots+\frac{1}{(\frac{n-1}{2})}\right], & \textit{if } n \textit{ is odd.}\end{cases}$$

Theorem 4.4. *The Reciprocal degree distance of the graph $G = C_m \hat{e} C_n$ is*

$$RDD(G) = \begin{cases} \Big\{(4n+2)\sum_{k=1}^{(\frac{n}{2}-1)} \frac{1}{k} + (4m+2)\sum_{k=1}^{(\frac{m-1}{2})} \frac{1}{k} + \frac{2}{n} \\ +8\sum_{k=2}^{(\frac{m+1}{2})} \frac{1}{(k+\frac{n}{2})} + 10\left[1 + \sum_{k=2}^{(\frac{m+1}{2})} \frac{1}{k} + \sum_{k=2}^{(\frac{n}{2})} \frac{1}{k} + \frac{1}{(2+n)}\right] \\ +16\sum_{k=1}^{(\frac{m-3}{2})} k\left[\frac{1}{k+2} + \frac{1}{[\frac{m+1}{2}-k]+\frac{n}{2}}\right] \\ +16\left(\frac{m+1}{2}-1\right)\left[\frac{1}{\frac{m+1}{2}+1} + \frac{1}{\frac{m+1}{2}+2} + \cdots + \frac{1}{2+(\frac{n}{2}-1)}\right]\Big\}, & \textit{if } n\textit{-even and } m\textit{-odd} \\ \Big\{(4m+2)\sum_{k=1}^{(\frac{m-1}{2})} \frac{1}{k} + (4n+2)\sum_{k=1}^{(\frac{n-1}{2})} \frac{1}{k} + 6 \\ +10\left[\sum_{k=2}^{\frac{m+1}{2}} \frac{1}{k} + \sum_{k=2}^{\frac{m+1}{2}} \frac{1}{k}\right] \\ +16\sum_{k=1}^{\frac{n-3}{2}} k\left[\frac{1}{k+2} + \frac{1}{[\frac{m+1}{2}-k]+[\frac{n+1}{2}]}\right] \\ +16(\frac{n+1}{2}-1)\left[\frac{1}{\frac{n+1}{2}+1} + \cdots + \frac{1}{\frac{m+1}{2}+1}\right]\Big\}, & \textit{if } m,n \textit{ are odd;} \\ \Big\{14 + (4m+2)\sum_{k=1}^{(\frac{m}{2}-1)} \frac{1}{k} + (4n+2)\sum_{k=1}^{(\frac{n}{2}-1)} \frac{1}{k} \\ +2\left[\frac{1}{m}+\frac{1}{n}\right] + 10\left[\sum_{k=2}^{\frac{m}{2}} \frac{1}{k} + \sum_{k=2}^{\frac{n}{2}} \frac{1}{k} + \left(\frac{1}{m+2} + \frac{1}{n+2}\right)\right] \\ +8\left[\frac{1}{m+n+2} + \sum_{k=1}^{(\frac{n}{2}-1)} \frac{1}{(\frac{m}{2}+1)+k} + \sum_{k=2}^{\frac{m}{2}} \frac{1}{k+\frac{n}{2}}\right] \\ +16\sum_{k=1}^{(\frac{n}{2}-2)} k\left[\frac{1}{k+2} + \frac{1}{[\frac{m}{2}-k]+[\frac{n}{2}]}\right] \\ +16(\frac{n}{2}-1)\left[\frac{1}{(\frac{n}{2}+1)} + \cdots + \frac{1}{(\frac{m}{2}+1)}\right]\Big\}, & \textit{if } m,n \textit{ are even.} \end{cases}$$

Proof. Let $\{w_1, w_2, \ldots, w_m\}$ and $\{v_1, v_2, \ldots, v_n\}$ be vertex sets of C_m and C_n, respectively. By the operation defined above, we get the resulting graph $G = C_m \hat{e} C_n$.

$$\begin{aligned} RDD(G) &= \sum_{u_i,u_j \in V(G)} \frac{deg_G(u_i) + deg_G(u_j)}{d_G(u_i,u_j)} \\ &= \sum_{w_i,w_j \in V(C_m)} \frac{deg_G(w_i) + deg_G(w_j)}{d_G(w_i,w_j)} + \sum_{v_i,v_j \in V(C_n)} \frac{deg_G(v_i) + deg_G(v_j)}{d_G(v_i,v_j)} \\ &+ \sum_{\substack{v_i \in V(C_n), \\ w_j \in V(C_m)}} \frac{deg_G(v_i) + deg_G(w_j)}{d_G(v_i,w_j)}. \end{aligned} \tag{4.1}$$

Case(i): m is odd and n is even.

$$\begin{aligned} RDD(G) =&(4m+2)\left[1 + \frac{1}{2} + \cdots + \frac{1}{(\frac{m-1}{2})}\right] + (4n+2)\left[1 + \frac{1}{2} + \cdots + \frac{1}{(\frac{n}{2}-1)}\right] \\ &+ \left(4 + \frac{2}{n}\right) + \sum_{\substack{v_i \in V(C_n), \\ w_j \in V(C_m)}} \frac{deg_G(v_i) + deg_G(w_j)}{d_G(v_i,w_j)}. \end{aligned} \tag{4.2}$$

Here, $d_G(v_i, w_j)$ is the distance between the vertices of C_n to those of C_m in G, as given in Table 4.2. We calculate $\sum_{\substack{v_i \in V(C_n), \\ w_j \in V(C_m)}} \frac{deg_G(v_i) + deg_G(w_j)}{d_G(v_i,w_j)}$ using the following procedure.

From each of the 2^{nd} row to m^{th} rows, add reciprocals of elements of 2^{nd} and n^{th} columns, 3^{rd} and $(n-1)^{th}$ columns,..., $(\frac{n}{2})^{th}$ and $(\frac{n}{2}+2)^{th}$ columns, and multiplying by the sum of degrees of corresponding vertices we get,

$$\begin{aligned} &\Big\{(4)(4)\left[\frac{1}{2+1} + \frac{1}{3+1} + \cdots + \frac{1}{(\frac{m+1}{2}-1)+1} + \frac{1}{(\frac{m+1}{2}+1)}\right] \\ &+(4)(4)\left[\frac{1}{2+2} + \frac{1}{3+2} + \cdots + \frac{1}{(\frac{m+1}{2}-1)+2} + \frac{1}{(\frac{m+1}{2}+2)}\right] + \cdots + \\ &(4)(4)\left[\frac{1}{2+(\frac{n}{2}-1)} + \frac{1}{3+(\frac{n}{2}-1)} + \cdots + \frac{1}{(\frac{m+1}{2}-1)+(\frac{n}{2}-1)} \right. \\ &\left. + \frac{1}{(\frac{m+1}{2})+(\frac{n}{2}-1)}\right]\Big\}. \end{aligned} \tag{4.3}$$

From each of the 2^{nd} row to m^{th} row, add reciprocals of elements of 1^{st} column, and multiplying by the sum of degrees of corresponding vertices we get,

$$\left\{(3+2)(2)\left[\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{(\frac{m+1}{2}-1)}+\frac{1}{(\frac{m+1}{2})}\right]\right\}. \qquad (4.4)$$

From each of the 2^{nd} to n^{th} columns, add reciprocals of elements of 1^{st} row, and multiplying by the sum of degrees of corresponding vertices we get,

$$\left\{(3+2)(2)\left[\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1+(\frac{n}{2}-1)}\right]+(3+2)\frac{1}{(1+\frac{n}{2})}\right\}. \qquad (4.5)$$

For the vertices w_1 and v_1 we get,

$$\frac{(3+3)}{1}. \qquad (4.6)$$

Using (4.3), (4.4), (4.5) and (4.6) in (4.2) we get,

$$\begin{aligned}RDD(G) =&(4n+2)\sum_{k=1}^{(\frac{n}{2}-1)}\frac{1}{k}+(4m+2)\sum_{k=1}^{(\frac{m-1}{2})}\frac{1}{k}+\frac{2}{n}\\&+8\sum_{k=2}^{(\frac{m+1}{2})}\frac{1}{(k+\frac{n}{2})}+10\left[1+\sum_{k=2}^{(\frac{m+1}{2})}\frac{1}{k}+\sum_{k=2}^{(\frac{n}{2})}\frac{1}{k}+\frac{1}{(2+n)}\right]\\&+16\sum_{k=1}^{(\frac{m-3}{2})}k\left[\frac{1}{k+2}+\frac{1}{[\frac{m+1}{2}-k]+(\frac{n}{2})}\right]\\&+16\left(\frac{m+1}{2}-1\right)\left[\frac{1}{\frac{m+1}{2}+1}+\frac{1}{\frac{m+1}{2}+2}+\cdots+\frac{1}{2+(\frac{n}{2}-1)}\right].\end{aligned}$$

Case(ii): m is even and n is odd.
By interchanging the roles of m and n in $Case(i)$, we obtain,

$$\begin{aligned}RDD(G) =&(4m+2)\sum_{k=1}^{(\frac{m}{2}-1)}\frac{1}{k}+(4n+2)\sum_{k=1}^{(\frac{n-1}{2})}\frac{1}{k}+\frac{2}{m}+8\sum_{k=2}^{(\frac{n+1}{2})}\frac{1}{(k+\frac{m}{2})}\\&+10\left[1+\sum_{k=2}^{(\frac{n+1}{2})}\frac{1}{k}+\sum_{k=2}^{(\frac{m}{2})}\frac{1}{k}+\frac{1}{(2+m)}\right]\\&+16\sum_{k=1}^{(\frac{n-3}{2})}k\left[\frac{1}{k+2}+\frac{1}{[\frac{n+1}{2}-k]+(\frac{m}{2})}\right]\\&+16(\frac{n+1}{2}-1)\left[\frac{1}{\frac{n+1}{2}+1}+\frac{1}{\frac{n+1}{2}+2}+\cdots+\frac{1}{2+(\frac{m}{2}-1)}\right].\end{aligned}$$

Case(iii): m and n are odd.

$$RDD(G) = (4m+2)\sum_{k=1}^{(\frac{m-1}{2})}\frac{1}{k} + (4n+2)\sum_{k=1}^{(\frac{n-1}{2})}\frac{1}{k} + \sum_{\substack{v_i\in V(C_n),\\ w_j\in V(C_m)}}\frac{deg_G(v_i)+deg_G(w_j)}{d_G(v_i,w_j)}.$$

By constructing the distance matrix D and using the similar procedure (as in $Case(i)$) to evaluate $\sum_{\substack{v_i\in V(C_n),\\ w_j\in V(C_m)}}\frac{deg_G(v_i)+deg_G(w_j)}{d_G(v_i,w_j)}$, we obtain,

$$\begin{aligned} RDD(G) =& (4m+2)\sum_{k=1}^{(\frac{m-1}{2})}\frac{1}{k} + (4n+2)\sum_{k=1}^{(\frac{n-1}{2})}\frac{1}{k} + 6 \\ &+ 10\left[\sum_{k=2}^{\frac{m+1}{2}}\frac{1}{k} + \sum_{k=2}^{\frac{m+1}{2}}\frac{1}{k}\right] \\ &+ 16\sum_{k=1}^{\frac{n-3}{2}} k\left[\frac{1}{k+2} + \frac{1}{[\frac{m+1}{2}-k]+[\frac{n+1}{2}]}\right] \\ &+ 16(\frac{n+1}{2}-1)\left[\frac{1}{\frac{n+1}{2}+1} + \cdots + \frac{1}{\frac{m+1}{2}+1}\right]. \end{aligned}$$

Case(iv): m and n are even.

$$\begin{aligned} RDD(G) =& (4m+2)\sum_{k=1}^{(\frac{m}{2}-1)}\frac{1}{k} + \left[4+\frac{2}{m}\right] + (4n+2)\sum_{k=1}^{(\frac{n}{2}-1)}\frac{1}{k} + \left[4+\frac{2}{n}\right] \\ &+ \sum_{\substack{v_i\in V(C_n),\\ w_j\in V(C_m)}}\frac{deg_G(v_i)+deg_G(w_j)}{d_G(v_i,w_j)}. \end{aligned}$$

Consider,

$$\begin{aligned} &\sum_{\substack{v_i\in V(C_n),\\ w_j\in V(C_m)}}\frac{deg_G(v_i)+deg_G(w_j)}{d_G(v_i,w_j)} \\ =& 6 + 10\left[\sum_{k=2}^{\frac{m}{2}}\frac{1}{k} + \sum_{k=2}^{\frac{n}{2}}\frac{1}{k} + \left(\frac{1}{m+2}+\frac{1}{n+2}\right)\right] \\ &+ 8\left[\frac{1}{m+n+2} + \sum_{k=1}^{(\frac{n}{2}-1)}\frac{1}{(\frac{m}{2}+1)+k} + \sum_{k=2}^{\frac{m}{2}}\frac{1}{k+\frac{n}{2}}\right] \\ &+ 16\sum_{k=1}^{(\frac{n}{2}-2)} k\left[\frac{1}{k+2} + \frac{1}{[\frac{m}{2}-(k-1)]+[\frac{n}{2}-1]}\right] \\ &+ 16(\frac{n}{2}-1)\left[\frac{1}{(\frac{n}{2}+1)} + \cdots + \frac{1}{(\frac{m}{2}+1)}\right]. \end{aligned}$$

Hence,

$$\begin{aligned} RDD(G) = & 14 + (4m+2)\sum_{k=1}^{(\frac{m}{2}-1)} \frac{1}{k} + (4n+2)\sum_{k=1}^{(\frac{n}{2}-1)} \frac{1}{k} \\ & + 2\left[\frac{1}{m} + \frac{1}{n}\right] + 10\left[\sum_{k=2}^{\frac{m}{2}} \frac{1}{k} + \sum_{k=2}^{\frac{n}{2}} \frac{1}{k} + \left(\frac{1}{m+2} + \frac{1}{n+2}\right)\right] \\ & + 8\left[\frac{1}{m+n+2} + \sum_{k=1}^{(\frac{n}{2}-1)} \frac{1}{(\frac{m}{2}+1)+k} + \sum_{k=2}^{\frac{m}{2}} \frac{1}{k+\frac{n}{2}}\right] \\ & + 16\sum_{k=1}^{(\frac{n}{2}-2)} k\left[\frac{1}{k+2} + \frac{1}{[\frac{m}{2}-k]+[\frac{n}{2}]}\right] \\ & + 16(\frac{n}{2}-1)\left[\frac{1}{(\frac{n}{2}+1)} + \cdots + \frac{1}{(\frac{m}{2}+1)}\right]. \end{aligned}$$

Table 4.2

	v_1	v_2	$\cdots$	$v_{\frac{n}{2}}$	$v_{(\frac{n}{2}+1)}$	$\cdots$	v_{n-1}	v_n
w_1	1	$(1+1)$	$\cdots$	$[1+(\frac{n}{2}-1)]$	$[1+\frac{n}{2}]$	$\cdots$	$(1+2)$	$(1+1)$
w_2	2	$(2+1)$	$\cdots$	$[2+(\frac{n}{2}-1)]$	$[2+\frac{n}{2}]$	$\cdots$	$(2+2)$	$(2+1)$
w_3	3	$(3+1)$	$\cdots$	$[3+(\frac{n}{2}-1)]$	$[3+\frac{n}{2}]$	$\cdots$	$(3+2)$	$(3+1)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
$w_{\frac{m+1}{2}-1}$	$(\frac{m+1}{2}-1)$	$[(\frac{m+1}{2}-1)+1]$	$\cdots$	$[(\frac{m+1}{2}-1)+(\frac{n}{2}-1)]$	$[(\frac{m+1}{2}-1)+\frac{n}{2}]$	$\cdots$	$[(\frac{m+1}{2}-1)+2]$	$[(\frac{m+1}{2}-1)+1]$
$w_{\frac{m+1}{2}}$	$(\frac{m+1}{2})$	$[(\frac{m+1}{2})+1]$	$\cdots$	$[(\frac{m+1}{2})+(\frac{n}{2}-1)]$	$[(\frac{m+1}{2})+\frac{n}{2}]$	$\cdots$	$[\frac{m+1}{2}+2]$	$[\frac{m+1}{2}+1]$
$w_{\frac{m+1}{2}+1}$	$(\frac{m+1}{2})$	$[(\frac{m+1}{2})+1]$	$\cdots$	$[(\frac{m+1}{2})+(\frac{n}{2}-1)]$	$[(\frac{m+1}{2})+\frac{n}{2}]$	$\cdots$	$[\frac{m+1}{2}+2]$	$[\frac{m+1}{2}+1]$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
w_m	2	$(2+1)$	$\cdots$	$[2+(\frac{n}{2}-1)]$	$[2+\frac{n}{2}]$	$\cdots$	$(2+2)$	$(2+1)$

Distance matrix : distance between the vertices w_i in C_m and v_j in C_n in the graph $C_m \hat{e} C_n$, when n is even, m is odd.

□

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