

COMPARISON OF SELF CANCELLATION AND EXTENDED KALMEN FILTER METHODS FOR REDUCING ICI IN OFDM

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) is emerging as preferred modulation scheme in modern high data rate wireless communication systems. A well known problem of OFDM system is sensitivity to frequency offset between the transmitted and received signals, which may be caused by Doppler shift in the channel or by the difference between the transmitter and receiver local oscillator frequencies. This carrier frequency offset causes loss of orthogonality between sub carriers and the signal transmitted on each carrier are not independent of each other, leading to inter carrier interference (ICI)[1]-[4].

In this paper, the effects of ICI have been analyzed and two solutions to combat ICI have been presented. The first method is a self cancellation scheme, in which redundant data is transmitted onto adjacent subcarriers such that the ICI between adjacent sub carriers cancels out at the receiver. The other technique the Extended Kalman Filter (EKF) method statistically estimate the frequency offset and correct the offset using the estimated value at the receiver.

Keywords: Extended Kalman Filter (EKF), OFDM, ICI, AWGN, BER

1. INTRODUCTION

OFDM is a combination of modulation and multiplexing. Multiplexing generally refers to Independent signals, those produced by different sources. In OFDM the signal itself is first split into independent channels, modulated by data and then re-multiplexed to create the OFDM carrier. The sub-carriers should be orthogonal to each other to improve spectral efficiency. At the receiver side it is easy to recover data in each sub-carrier as long as carriers are orthogonal to each other. As more and more carriers are added, the bandwidth approaches $(N+1)/N$ Bits per Hz. Larger number of carriers gives better spectral efficiency. The main concept in OFDM is Orthogonality of the sub-carriers. A typical discrete-time baseband OFDM transceiver system is shown in Figure 1.1. First, a serial-to-parallel (S/P) converter groups the stream of input bits from the source encoder into groups of $\log_2 M$ bits, where M is the alphabet of size of the digital modulation scheme employed on each sub-carrier.

A total of N such symbols, X_m , are created. Then, the N symbols are mapped to bins of an inverse fast Fourier transform (IFFT). These IFFT bins correspond to the orthogonal sub-carriers in the OFDM Symbol. Therefore, the OFDM symbol can be expressed as

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{j \frac{2\pi n m}{N}} \quad 0 \leq n \leq N-1 \quad (1.1)$$

here are the baseband data on each sub-carrier The digital-to-analog (D/A) converter then creates an analog time-domain signal which is transmitted through the channel.

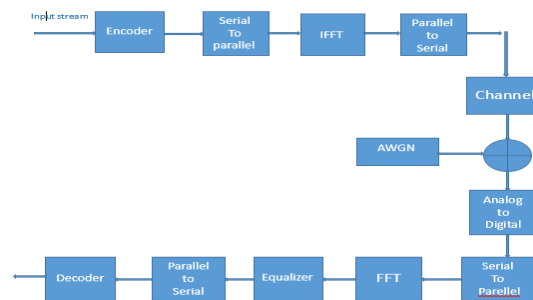


Fig 1.1: Baseband OFDM Transceiver System.

At the receiver, the signal is converted back to a discrete N point sequence $y(n)$, corresponding to each sub-carrier. This discrete signal is demodulated using an N-point fast Fourier transform (FFT) operation at the receiver. The demodulated symbol stream is given by:

$$y(m) = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j \frac{2\pi n m}{N}} + W(m) \quad 0 \leq m \leq N-1 \quad (1.2)$$

Where $W(m)$ corresponds to the FFT of the samples of $w(t)$, which is the Additive White Gaussian Noise (AWGN) introduced in the channel.

II ICI SELF CANCELLATION SCHEME

2.1 ICI Cancelling Modulation

The ICI coefficient gradually changed with respect to the sub-carrier index k and difference between $S(l-k)$ and $S(l+1-k)$ is very small. Therefore, if a data pair $(a, -a)$ is modulated onto two adjacent subcarriers $(l, l+1)$, where 'a' complex data, then the ICI signals generated by the subcarrier l will be cancelled out significantly by the ICI generated by subcarrier $l+1$. Assume the transmitted symbols are constrained so that

$$X(1) = -X(0), X(3) = -X(2), \dots, X(N-1) = -X(N-2),$$

The received signal on subcarrier k become

$$Y(k) = \sum_{l=0}^{N-1} X(l) [S(l-k) - S(l+1-k)] + n_k \quad (2.1)$$

And on subcarrier $k+1$ is

$$Y(k+1) = \sum_{l=0}^{N-1} X(l) [S(l-k-1) - S(l-k)] + n_{k+1} \quad (2.2)$$

Then denote new ICI coefficient as

$$S(l-k) - S(l+1-k) = S'(l-k) \quad (2.3)$$

Clearly from above concept $|S'(l-k)| \ll |S(l-k)|$ and there is reduction in ICI. In addition, the summation in (2.1) only takes even values; the total number of the interference signals is reduced to half compared with that in equation (2.2). Consequently, the ICI signals in equation (2.1) are much smaller than those in equation (2.2)

2.2 ICI Cancelling Demodulation

In considering a further reduction of ICI, a so called ICI cancelling demodulation scheme is analyzed. The demodulation is suggested to work in such a way that each signal at the $k+1^{\text{th}}$ subcarrier (now k denotes even number) is multiplied by “-1” and then summed with the one at the k^{th} subcarrier. Then the resultant data sequence is used for making symbol decision. It can be represented as follows

$$Y''(k) = Y(k) - Y(k+1) = \sum_{l=0}^{N-1} X(l) [-S(l-k-1) + 2S(l-k) - S(l+1-k)] + n_k - n_{k+1}$$

Then corresponding ICI coefficient becomes

$$S''(l-k) = -S(l-k-1) + 2S(l-k) - S(l+1-k) \quad (2.4)$$

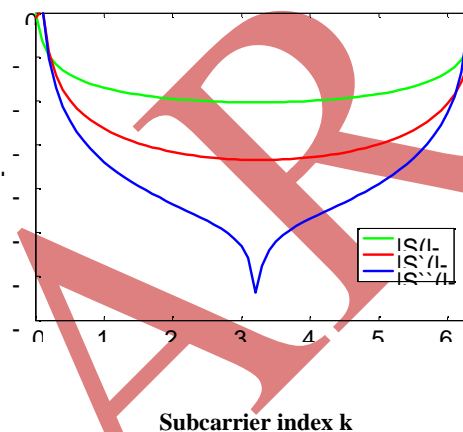


Fig 2.1: Comparison among $|S(l-k)|, |S'(l-k)|, |S''(l-k)|$ for $N=64, \epsilon = 0.4$

From Fig 2.1 Thus, the ICI signals become smaller when applying ICI cancelling modulation. On the other hand, the ICI cancelling demodulation can further reduce the residual ICI in the received signals. This combined ICI cancelling modulation and demodulation method is called the ICI self-cancellation scheme. Until now, three types of ICI coefficients are obtained: 1) $S(l-k)$ for standard OFDM signal. 2) $S'(l-k)$ For ICI cancellation modulation. 3) $S''(l-k)$ For combined ICI cancelling modulation and demodulation. Using ICI coefficient given by equation (3.5), the theoretical CIR of the ICI self cancellation scheme can be given as

$$CIR = \frac{|-S(-1) + 2S(0) - S(1)|^2}{\sum_{l=2,4,\dots}^{N-1} |-S(l-1) + 2S(l) - S(l+1)|^2}$$

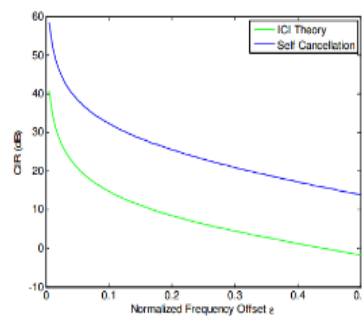


Fig 3.2: CIR versus ϵ for standard OFDM system

Fig.2.2 shows the theoretical CIR curve with simulation results. As a reference, the CIR of a standard OFDM system using equation (2.5) is also shown. Such an ICI cancellation scheme gives more than 15-dB CIR improvement in the range $0 < \epsilon < 0.5$. Especially for small to medium frequency $0 < \epsilon < 0.2$ offsets in the range, the CIR improvement can reach 17 dB. But Due to the repetition coding, the bandwidth efficiency of the ICI self-cancellation scheme is reduced by half.

III EXTENDED KALMAN FILTERING

Kalman filters are common in communications and signal processing literature. The Kalman filter is a remarkably versatile and powerful recursive estimation algorithm that has found various applications in communications, such as adaptive equalization of telephone channels, adaptive equalization of fading dispersive channels, and adaptive antenna arrays. As a recursive filter, it is particularly applicable to non-stationary processes such as signals transmitted in a time-variant radio channel. In estimating non-stationary processes, the Kalman filter computes estimates of its own performance as part of the recursion and use this information to update the estimate at each step. Therefore, the estimation procedure is adjusted to the time-variant statistical characteristics of the random process.

3.1 Problem Formulation

A state-space model of the discrete Kalman filter is defined as

$$Z(n) = a(n)d(n) + v(n) \quad (3.1)$$

In this model, the observation $Z(n)$ has a linear relationship with the desired value $d(n)$. By using the discrete Kalman filter, $d(n)$ can be recursively estimated based on the observation of $z(n)$ and the updated estimation in each recursion is optimum in the minimum mean square sense.

As illustrated in frequency offset model, the received symbols are

$$y(n) = x(n)e^{j\frac{2\pi n\epsilon(n)}{N}} + w(n) \quad (3.2)$$

It is obvious that the observation is in a nonlinear relationship with the desired $\epsilon(n)$ value i.e.

$$y(n) = f(\epsilon(n)) + w(n) \quad (3.3)$$

$$\text{Where } f(\epsilon(n)) = x(n)e^{j\frac{2\pi n\epsilon(n)}{N}} \quad (3.4)$$

In order to estimate $\epsilon(n)$ efficiently in computation, we build an approximate linear relationship using the first-order Taylor's expansion:

$$y(n) = f(\hat{\epsilon}(n-1)) + f'(\hat{\epsilon}(n-1))[\epsilon(n) - \hat{\epsilon}(n-1)] + w(n)$$

Where $\hat{\epsilon}(n-1)$ is estimation of $\epsilon(n-1)$

$$f'(\hat{\epsilon}(n-1)) = \frac{\partial f(\epsilon(n))}{\partial \epsilon(n)} \Big|_{\epsilon(n)=\hat{\epsilon}(n-1)} = \frac{2\pi n'}{N} e^{j \frac{2\pi n' \hat{\epsilon}(n-1)}{N}}$$

Define

$$z(n) = y(n) - f(\hat{\epsilon}(n-1)) \quad (3.7)$$

$$d(n) = \epsilon(n) - \hat{\epsilon}(n-1) \quad (3.8)$$

And the following relationship:

$$z(n) = f'(\hat{\epsilon}(n-1))d(n) + w(n) \quad (3.9)$$

Which has the same form as (3.1), i.e. $z(n)$ is linearly related to $d(n)$. Hence the normalized frequency offset $\epsilon(n)$ can be estimated in a recursive procedure similar to the discrete Kalman filter. As linear approximation is involved in the derivation, the filter is called the extended Kalman filter (EKF).

The EKF provides a trajectory of estimation for $\epsilon(n)$. The error in each update decreases and the estimate becomes closer to the ideal value during iterations. It is noted that the actual error in each recursion between $\epsilon(n)$ and $\hat{\epsilon}(n)$ does not strictly obey (3.9). Thus there is no guarantee of optimal MMSE estimates in the EKF scheme. However it has been proven that EKF is a very useful method of obtaining good estimates of the system state. Hence this has motivated to explore the performance of EKF in ICI cancellation in an OFDM system.

3.2 Assumptions

In the following estimation using the EKF, it is assumed that the channel is slowly time varying so that the time-variant channel impulse response can be approximated to be quasistatic during the transmission of one OFDM frame. Hence the frequency offset is considered to be constant during a frame. The preamble preceding each frame can thus be utilized as a training sequence for estimation of the frequency offset imposed on the symbols in this frame. Furthermore, In order to estimate frequency offset the channel is assumed to be flat fading and ideal channel estimation is available at the receiver. Therefore in our derivation and simulation, the one-tap equalization is temporarily suppressed.

3.3 ICI Cancellation

There are two stages in the EKF scheme to mitigate the ICI effect: the offset estimation scheme and the offset correction scheme.

3.3.1 Offset Estimation Scheme

To estimate the quantity $\epsilon(n)$ using an EKF in each OFDM frame, the state equation is built as

$$\epsilon(n) = \epsilon(n-1) \quad (3.10)$$

i.e., in this case we are estimating an unknown constant ϵ . This constant is distorted by a non-stationary process $w(n)$, an observation of which is the preamble symbols preceding the data symbols in the frame. The observation equation is

$$y(n) = x(n) e^{j \frac{2\pi n' \epsilon(n)}{N}} + w(n) \quad (3.11)$$

Where $y(n)$ denotes the received preamble symbols distorted in the channel, $w(n)$ the AWGN, and $x(n)$ the IFFT of the preambles $X(k)$ that are transmitted, which are known at the receiver. Assume there are N_p preambles preceding the data symbols in each frame are used as a training sequence and the variance σ^2 of the AWGN $w(n)$ is stationary. The computation procedure is described as follows.

1. Initialize the estimate $\hat{\epsilon}(0)$ and corresponding state error $P(0)$.
2. Compute the $H(n)$, the derivative of $y(n)$ with respect to $\epsilon(n)$ at $\hat{\epsilon}(n-1)$, the estimate obtained in the previous iteration.
3. Compute the time-varying Kalman gain $K(n)$ using the error variance $P(n-1)$, $H(n)$ and σ^2
4. Compute the estimate $\hat{y}(n-1)$, using $x(n)$ and $\hat{\epsilon}(n-1)$, i.e. based on the observations up to $n-1$, compute the error between the true observation $y(n)$ and \hat{y} .
5. Update the estimate $\hat{\epsilon}(n)$ by adding $k(n)$ -weighted error between the observation $y(n)$ and \hat{y} to the previous estimation $\hat{\epsilon}(n-1)$.
6. Compute the state error $P(n)$ with the Kalman gain $k(n)$, $H(n)$ and the previous error $P(n-1)$.
7. If n is less than N_p , increment n by 1 and go to step 2; otherwise stop.

The pseudo code of computation is summarized below.

Initialize $P(0), \hat{\epsilon}(0)$.

For $n=1, 2, \dots, N_p$ compute

$$H(n) = \frac{\partial y(x)}{\partial x_{\epsilon(n)}} = \frac{j2\pi n}{N} e^{j\frac{2\pi n \epsilon(n-1)}{N}} x(n)$$

$$k(n) = P(n-1)H^*(n)[P(n-1) + \sigma^2]^{-1}$$

$$\hat{\epsilon}(n) = \hat{\epsilon}(n-1) + K(n) \left[y(n) - x(n) e^{j\frac{2\pi n \epsilon(n-1)}{N}} \right]$$

$$P(n) = [1 - k(n)H(n)]P(n-1)$$

Through the recursive iteration procedure described above, an estimate of the frequency offset $\hat{\epsilon}$ can be obtained.

3.3.2 Offset Correction Scheme

The ICI distortion in the data symbols that follow the training sequence can then be mitigated by multiplying the received data symbols with a complex conjugate of the estimated frequency offset and applying FFT, i.e.

$$\hat{x}(n) = \text{FFT} \left\{ y(n) e^{-j\frac{2\pi n \hat{\epsilon}(n)}{N}} \right\} \quad (3.12)$$

IV SIMULATION RESULTS

In order to compare these two different cancellation schemes, BER curves were used to evaluate the performance of each scheme. For the simulations MATLAB was employed with its Communications Toolbox. The OFDM transceiver system was implemented. Simulations for cases of normalized frequency offsets equal to 0.05, 0.15, and 0.30 are used.

Table 3.1: Simulation conditions for Standard OFDM and ICI self cancellation methods

Parameter	Specifications
FFT Size	64
Number of Carriers in OFDM symbol	52
Doppler Shift	0,0.15,0.3
Guard Length	12
Signal Constellation	2 QAM, 4 QAM
OFDM symbols for one loop	1000

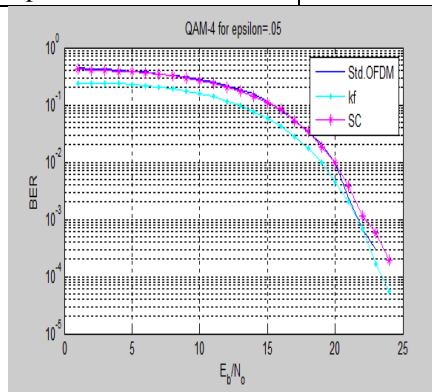


Fig 3.4 BER Performance with $\epsilon=0.05$

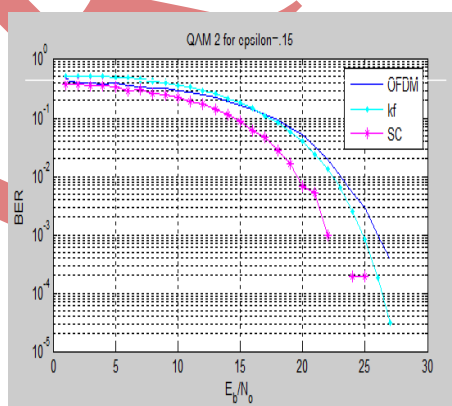
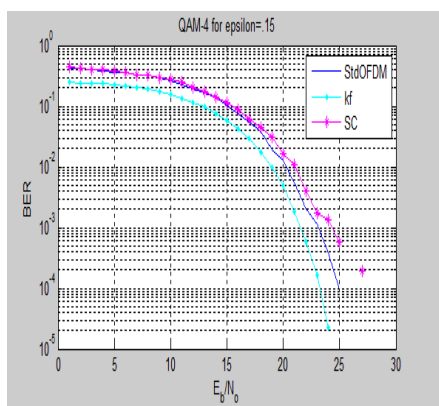


Fig 3.5 BER Performance with $\epsilon=0.15$

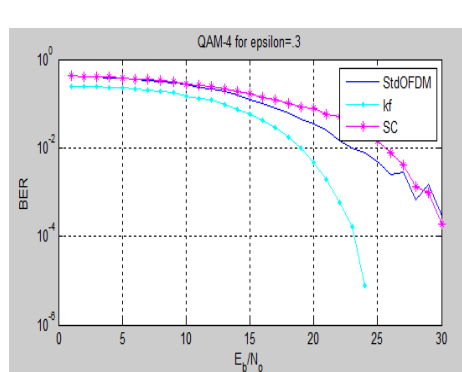
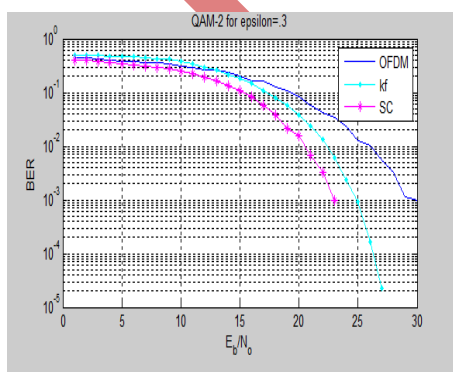


Fig 3.6 BER Performance with $\epsilon=0.30$

It is observed in the figures that each method has its own advantages. In the presence of small frequency offset and binary alphabet size, self cancellation gives the best results. However, for larger alphabet sizes and larger frequency offset such as 4-QAM and frequency offset of 0.30, self cancellation does not offer much increase in performance. The Kalman filter method indicates that for very small frequency offset, it does not perform very well, as it hardly improves BER. However, for high frequency offset the Kalman filter does perform extremely well. It gives a significant boost to performance. Tables summarize required values of SNR for BER specified at 10^{-2} . Significant gains in performance can be achieved using the ML and EKF methods for a large frequency offset.

Method	ϵ =0.05	Gain (dB)	ϵ =0.15	Gain (dB)	ϵ =0.30	Gain (dB)
STD OFDM	20		23		26	
SC	20	0	18	1	20.5	13
EKF	17	3	22	6	23	20.5

IV CONCLUSION

In this paper, two methods were explored for mitigation of the ICI. The ICI self cancellation (SC) and the extended Kalman filtering (EKF) method and Total ICI cancellation schemes are proposed. The choice of which method to employ depends on the specific application. For example, self cancellation does not require very complex hardware or software for implementation. For small alphabet sizes (BPSK) and for low frequency offset values, The SC scheme delivers good performance in terms of BER. However, for higher order modulation schemes, the EKF perform better. The self-cancellation technique does not completely cancel the ICI from adjacent sub-carriers. However, it is not bandwidth efficient as there is a redundancy of 2 for each carrier.

On the other hand, the EKF method does not reduce bandwidth efficiency as the frequency offset can be estimated from the preamble of the data sequence in each OFDM frame. However, it is more complex implementation method compared to SC method. In addition, this method requires a training sequence to be sent before the data symbols for estimation of the frequency offset. It can be adopted for the receiver design for IEEE 802.11a because this standard specifies preambles for every OFDM frame. The preambles are used as the training sequence in estimating the frequency offset.

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