

# SYNCHRONIZATION ALGORITHM FOR AN OFDMA SYSTEM

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## ABSTRACT

*This paper investigate the frequency and time recovery for the uplink of an orthogonal frequency-division multiple access (OFDMA) system. We proposed alternating methods for estimating the frequency and timing offsets of a new user entering in the system. The timing estimator is derived from maximum likely hood criterion, where as frequency estimator is based on ad-hock reasoning. Both the techniques rely separation of a fixed pilot symbol. These techniques provide feed forward estimates and allow synchronization in only two OFDM blocks. The performance of the proposed frequency and timing synchronization has been investigated by simulation in a frequency selective fading channel.*

**Keywords:** *Frequency Estimation, Maximum-Likelihood (ML), Orthogonal Frequency-Division Multiple Access (OFDMA), Timing Estimation.*

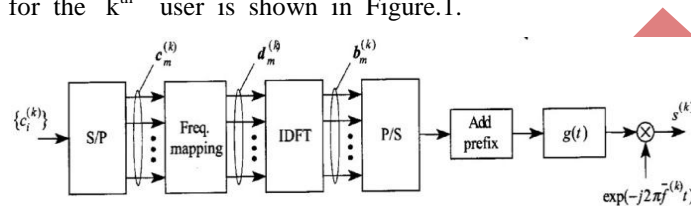
## I INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) is a multiplexing technique in which users simultaneously transmit their own data by modulating an exclusive set of orthogonal subcarriers. OFDMA has gained increased interest in the last few years and has been proposed for the uplink of wireless communication systems [1], [2] and cable TV (CATV) networks [3]. Its main advantage is that separating different users through frequency-division multiple access (FDMA) techniques at the subcarrier level can mitigate multiple-access interference (MAI) within a cell. Also, compared with single-carrier multiple-access systems, OFDMA offers increased robustness to narrowband interference, allows straightforward dynamic channel assignment, and does not need adaptive time-domain equalizers, since channel equalization is performed in the frequency domain through one-tap multipliers [3]. For all this to be true, however, proper frequency and timing synchronization is necessary to maintain orthogonality among the active users.

## II. OFDMA UPLINK SYSTEM

### 2.1 System Description

We consider the uplink of an OFDMA system employing  $N$  subcarriers and accommodating a maximum of  $K$  simultaneously active users. Each user transmits on a set of  $L=N/K$  assigned subcarriers. The block diagram of the transmitter for the  $k^{\text{th}}$  user is shown in Figure.1.



**Fig: 1 Block diagram of the  $k^{\text{th}}$  OFDMA transmitter.**

Extended with the insertion of  $N - L$  zeros to produce  $d_m^{(k)} = [d_m^{(k)}(0), d_m^{(k)}(1), \dots, d_m^{(k)}(N-1)]^T$  and it is fed to an  $N$ -point inverse discrete Fourier transform (IDFT) unit. The components of  $d_m^{(k)}$  are defined as

$$d_m^{(k)}(n) = \begin{cases} c_m^{(k)}(l), & \text{if } n = p_m^{(k)}(l) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$p_m^{(k)}(l)$  is the index of the subcarrier modulated by  $c_m^{(k)}(l)$ . The function associating with  $l^{\text{th}}$  component of  $c_m^{(k)}$  to  $p_m^{(k)}(l)$  is referred to as user specific frequency mapping. Note that the indexes  $p_m^{(k)}(l)$  can be any integer within the interval  $(0, N-1)$ , since we do not assume that the subcarriers of a given user are grouped together.

$$b_m^{(k)}(n) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} c_m^{(k)}(l) e^{j2\pi n p_m^{(k)}(l) / N}, \quad 0 \leq n \leq N-1 \quad (2)$$

And are arranged in vector  $b_m^{(k)}$ . In order to eliminate any interfrequency between any adjacent OFDM symbols, an  $N_G$ -point prefix is appended to  $b_m^{(k)}$  such that  $b_m^{(k)}(l) = b_m^{(k)}(l + N)$  for  $-N_G \leq l \leq -1$ . The resulting vector drives a linear modulator with impulse response  $g(t)$  and signaling interval  $T_s = T/(N+N_G)$ , where  $T$  is the OFDM symbol duration. The transmitted signal reads

$$S^{(k)}(t) = e^{-j2\pi f_c^{(k)}t} \sum_{m=-\infty}^{\infty} \sum_{l=-N_G}^{N-1} b_m^{(k)}(l) \times g(t + \bar{\tau}^{(k)} - lT_s - mT) \quad (3)$$

Where  $m$  counts OFDM blocks and  $l$  counts the time domain samples within a block. As discussed later in Section II-B, the parameters  $\bar{f}^{(k)}$  and  $\bar{\tau}^{(k)}$  serve to compensate for the Doppler shift and transmission delay incurred by  $S^{(k)}(t)$  in passing through the channel. The receiver at the BS is sketched in Figure.2.

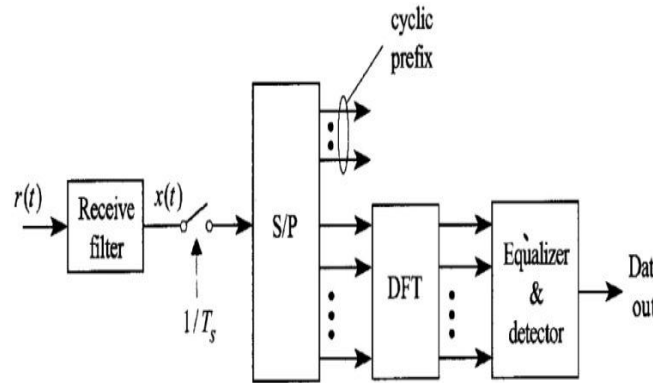


Figure.2 Block diagram of the k<sup>th</sup> OFDMA receiver.

### III. SIGNAL MODEL

Denote  $x_m = [x(mN_T), x(mN_T+1), \dots, x(mN_T+N-1)]^T$  the mth block of samples from the receiver filter (after elimination of the cyclic prefix) and assume that, for any user,  $h^{(k)}(n)$  has a finite support  $0 \leq n \leq P-1$ , with  $P \leq N_G$ . Then, letting and  $h^{(k)} = [h^{(k)}(0), h^{(k)}(1), \dots, h^{(k)}(P-1)]^T$  collecting (2), (6), and (9) yields

$$x_m = e^{j\frac{2\pi v^{(1)} m N_T}{N}} \Gamma(v^{(1)}) D_m(\mu^{(1)}) h^{(1)} + \sum_{k=2}^K B_m^{(k)} h^{(k)} + w_m \quad (4)$$

Where  $\Gamma(v^{(1)})$  is a diagonal matrix

$$\Gamma(v^{(1)}) = \text{diag} \left\{ e^{j\frac{2\pi v^{(1)} n}{N}}, 0 \leq n \leq N-1 \right\} \quad (5)$$

While  $B_m^{(k)}$  and  $D_m(\mu^{(1)})$  are matrices with entries

$$[B_m^{(k)}]_{n,l} = b_m^{(k)}(n-l) \quad 0 \leq n \leq N-1, 0 \leq l \leq P-1 \quad (6)$$

And

$$D_m(\mu^{(1)}) = \begin{cases} b_m^{(k)}(n-l - \mu^{(1)}), & -N_G + \mu^{(1)} \leq n-l \leq N-1 \\ b_{m-1}^{(k)}(n-l - \mu^{(1)} + N_T), & -N_G \leq n-l \leq -N_G - 1 + \mu^{(1)} \end{cases} \quad (13)$$

Also,  $w_m = [w(mN_T), w(mN_T+1), \dots, w(mN_T+N-1)]^T$  is a zero-mean Gaussian vector with covariance matrix  $\sigma^2 I_N$  ( $I_N$  is the identity matrix of order  $N$ ). To separate the user of interest from the others, we compute the DFT of  $x_m$  and select those outputs that correspond to the subcarriers of user #1. This produces the  $L$ -dimensional vector

$$Y_m^{(1)} = F^{(1)} x_m \quad (7)$$

Where  $F^{(1)}$  is an  $L \times N$  matrix with entries

$$[F^{(1)}]_{l,n} = \frac{1}{\sqrt{N}} e^{-j \frac{2\pi n v^{(1)} l}{N}} \quad 0 \leq n \leq N-1, 0 \leq l \leq L-1 \quad (8)$$

Then, substituting (10) into (14) and exploiting the identity  $F^{(1)} B_m^{(k)} = 0_{L \times P}$ , which holds for  $k=2,3,\dots,K$  ( $0_{L \times P}$  is an  $L \times P$  matrix with all-zero entries), yields

$$Y_m^{(1)} = e^{j \frac{2\pi v^{(1)} m N_T}{N}} F^{(1)} \Gamma(v^{(1)}) D_m(\mu^{(1)}) h^{(1)} + \eta_m \quad (9)$$

Where  $\eta_m = F^{(1)} w_m$  is Gaussian with zero mean and covariance matrix  $C_{\eta} = \sigma^2 I_L$ . This equation is a crucial step in our derivation. It says that  $Y_m^{(1)}$  exhibits no interuser interference. Clearly, this relies on the assumption that the already-present users are perfectly synchronized, i.e.,  $v^{(k)} = \mu^{(k)} = 0$  for  $k=2,3,\dots,K$ . The effect of users' imperfect synchronization is addressed later by simulation. To perform frequency and timing estimation, the BS observes  $M$  vectors  $\{Y_m^{(1)}; 1 \leq m \leq M\}$  in succession ( $M$  is a design parameter). The symbols transmitted by the user of interest are Assumed known at the receiver (data-aided operation). Also, for simplicity, we let  $b_m^{(1)} = b_{m-1}^{(1)}$  for  $1 \leq m \leq M$ , i.e., user #1 transmits  $M+1$  identical and known OFDM blocks (pilot symbols). In these circumstances  $D_m(\mu^{(1)})$ , becomes independent of  $m$ . Then, dropping the superscript designating user #1 for simplicity, (16) reduces to

$$Y_m = e^{j \frac{2\pi v m N_T}{N}} F^{(1)} A(v, \mu) h + \eta_m, 1 \leq m \leq M \quad (10)$$

With  $A(v, \mu) = F \Gamma(v) \mathcal{D}(\mu)$  Our task is to exploit vectors  $Y_m$  to estimate  $v$  and  $\mu$ . An ML approach is considered in the next section.

#### IV. SIMULATION RESULTS

The performance of the proposed frequency and timing synchronizers has been investigated by computer simulation in a frequency-selective fading channel. The following assumptions have been made.

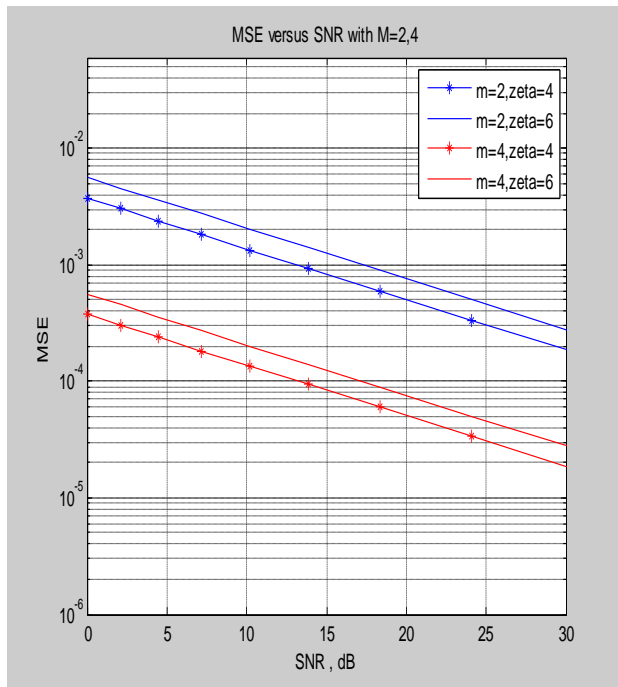


Figure.3 MSE vs SNR for M=2, 4

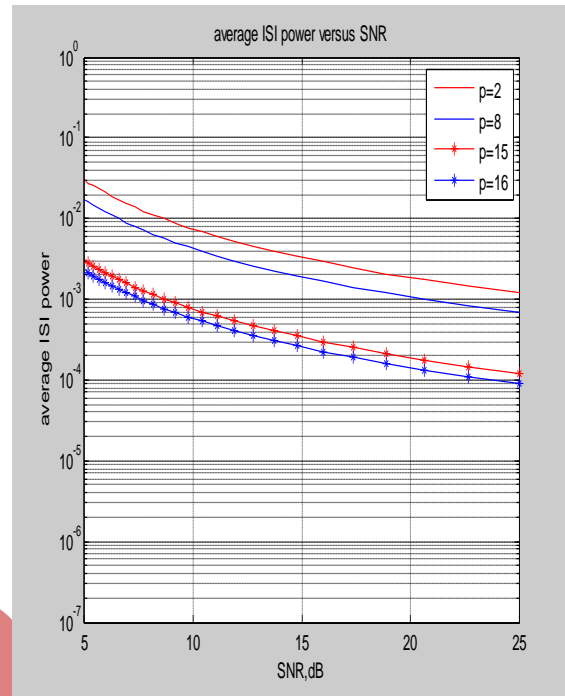


Figure.4 Average ISI power vs SNR

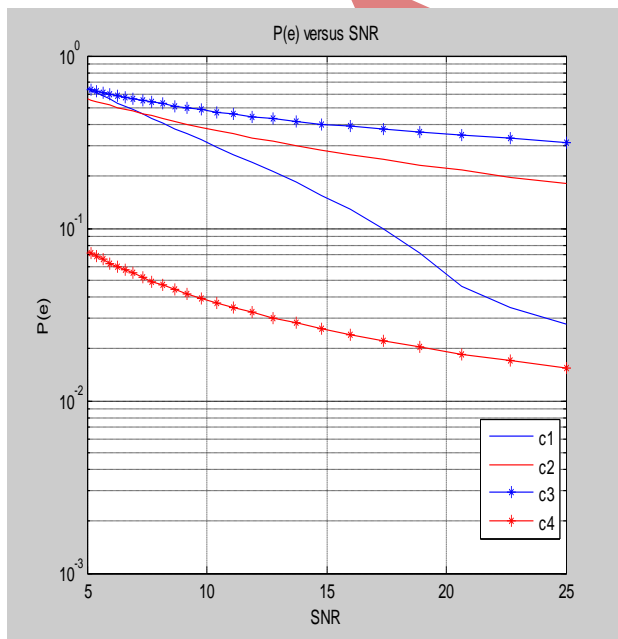


Figure .5 Probability error vs SNR

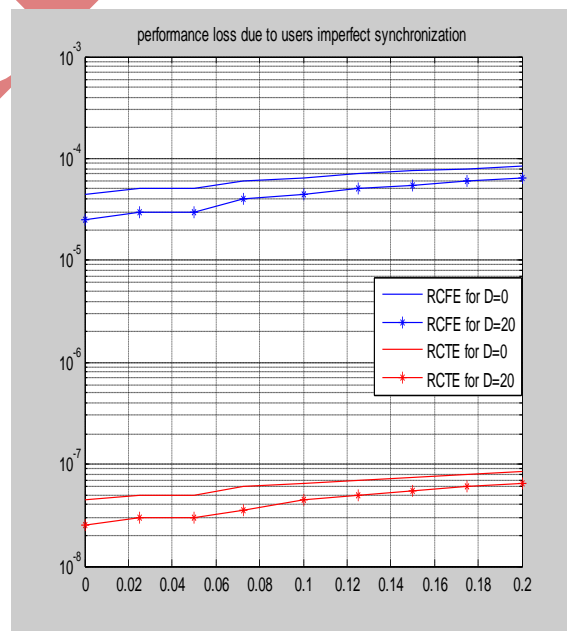


Figure .6 Performance loss due to imperfect synchronization

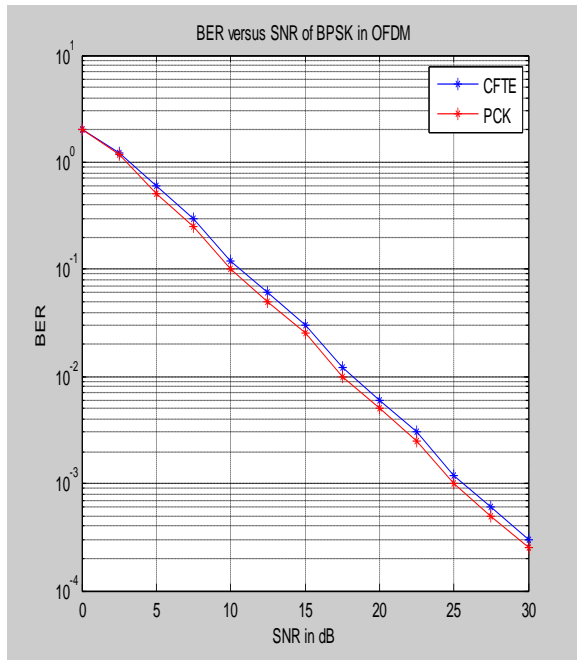


Figure .7 BER vs SNR for Uncoded BPSK

#### 4.1 System Parameters

- 1) The total number of subcarriers is  $N=256$ .
- 2) Each user has  $L=32$  subcarriers uniformly spaced at a distance  $1/LT_s$  from each other.
- 3) The sampling interval is  $T_s=.5\mu s$ .
- 4) The number of observed OFDM blocks is either  $M=2$  or  $M=4$ . The parameter  $Q$  with RCFE is set to  $M/2$ .
- 5) The channel responses  $h^k(n)$  have length  $P=15$ . Their components vary independently and are modeled as complex-valued Gaussian random variables with zero mean and an exponential power delay profile

$$E\{|h^k(n)|^2\} = \sigma_h^2 \times \exp\left(-\frac{2n}{\bar{\mu}}\right) \quad n=0, 1, \dots, 14 \quad (52)$$

A channel snapshot is generated at each simulation run and is kept constant over  $M$  observed blocks. The constant  $\sigma_h^2$  in (52) is chosen such that the signal energy of user #1 per received sample is normalized to unity, i.e.,  $E\{\|D(\mu)h\|^2/N\} = 1$ . Correspondingly, the SNR equals  $\frac{1}{\sigma^2}$ , where  $\sigma^2$  is the variance of the Gaussian noise.

- 6) The cell radius is  $R=2\text{km}$ , so that the maximum propagation delay is  $6.66\mu s$ . Recalling that  $T_s=0.5\mu s$ , this makes the maximum of  $\mu$  equal to 26 and the search in (38) can be limited to  $0 \leq \bar{\mu} \leq 26$
- 7) Application of RCFE-USS would require a cyclic prefix of  $N_G=32$  samples. This is nearly twice the channel duration and may represent an excessive overhead for the system under investigation. For this reason, we have set  $N_G=16$  in the simulations.

8) The maximum frequency offset is  $\nu = 0.3$ . As shown in Section II-B,  $\nu$  is related to the system parameters by the following equation:

$$\nu = \frac{2NT_s f_0 v}{c} \quad (11)$$

Where  $f_0$  is the carrier frequency,  $v$  the mobile speed, and  $c$  is the speed of light. Assuming  $f_0=4\text{GHz}$ , it is seen that  $\nu=0.3$  corresponds to a mobile speed greater than 300 km/h.

## 4.2 System Performance

Figure .3 illustrates the mean-square error (MSE) of the frequency estimates provided by RCFE as a function of SNR. For  $\xi=4,6$  and  $M=2$ , the loss with respect to the bound is approximately 2 dB, and reduces to 1 dB with  $M=4$ . Figure. 4, which shows the average ISI power versus SNR for  $P=2, 8, 15, 16$ . Note that doubling  $M$  increases the performance of RCFE by approximately 3.5 dB. The overall system performance has been computed in terms of uncoded bit-error rate (BER). Figure 5 illustrates  $P(\epsilon)$  as a function of SNR with  $M=2$  with different covariance matrices. Figure 6 shows simulations illustrating the performance loss incurred by RCFE and RCFE in a scenario where the signals of the interfering users are not perfectly synchronized. In particular, frequency errors are modeled as random variables with uniform distribution over  $[-F, F]$ , whereas timing errors take integer values within  $[-D, D]$  with uniform probability. The SNR is 15 dB and the frequency and timing offsets of user #1 are  $\nu=0$  and  $\mu=10$ . It is seen that both the MSE of the frequency estimates and the average ISI power increase with  $F$  and  $D$ . However, the loss with respect to the ideal case  $F=D=0$  (perfect synchronization of the already-present users) is quite tolerable. Figure 7 illustrates the BER for a BPSK system employing the proposed frequency and timing estimators with  $\nu=0$  and  $M=2$ . The curve labeled "PCK" corresponds to perfect channel knowledge ( $\hat{h} = h$ ) and reflects the degradations due to frequency and timing errors. Curve "CFTE" assumes channel, frequency, and timing estimates. Channel estimation is performed by replacing  $\hat{\nu}$  and  $\hat{\mu}$  with their estimates  $\hat{\nu}$  and  $\hat{\mu}$  in (4.19). Theoretical results with ideal synchronization are also indicated for reference. We see that the results with PCK and with ideal synchronization are very close, meaning that the performance loss due to frequency and timing errors is negligible. Imperfect channel estimation is seen to bring about a degradation of 1 dB.

## V. CONCLUSIONS

Synchronization algorithms have been proposed for frequency and timing recovery in the uplink of an OFDMA system. They exploit knowledge of pilot symbols and require a limited computational load. In contrast to other existing methods, they have a fast acquisition and can be used in applications where the subcarriers of different users are interleaved. Their performance has been investigated by theoretical analysis and computer simulations. It has been found that a receiver endowed with these algorithms has virtually the same performance as a perfectly synchronized system.

## REFERENCES

- [1] S. Kaiser and K. Fazel, "A spread-spectrum multicarrier multiple-access system for mobile communications," in *Proc. 1st Int. Workshop on Multicarrier Spread Spectrum*, Apr. 1997, pp. 49–56.
- [2] J. J. van de Beek, P. O. Borjesson, M. L. Boucheret, D. Landstram, J. M. Arenas, P. Odling, C. Ostberg, M. Wahlqvist, and S. K. Wilson, "A time and frequency synchronization scheme for multiuser OFDM," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1900–1914, Nov. 1999.
- [3] H. Sari and G. Karam, "Orthogonal frequency-division multiple access and its application to CATV networks," *Eur. Trans. Telecommun.*, vol. 9, pp. 507–516, Dec. 1998.
- [4] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol. 45, pp. 1613–1621, Dec. 1997.
- [5] J. J. van de Beek, M. Sandell, and P. O. Borjesson, "ML estimation of timing and frequency offset in OFDM systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 1800–1805, July 1997.
- [6] F. Daffara and A. Chouly, "Maximum-likelihood frequency detectors for orthogonal multicarrier systems," in *Proc. Int. Conf. Communications*, June 1993, pp. 766–771.
- [7] P. H. Moose, "A technique for orthogonal frequency-division multiplexing frequency offset correction," *IEEE Trans. Commun.*, vol. 42, pp. 2908–2914, Oct. 1994.
- [8] B. Yang, K. B. Letaief, R. S. Cheng, and Z. Cao, "Timing recovery for OFDM transmission," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 2278–2290, Nov. 2000.