ANALYSIS OF ANTI-WINDUP TECHNIQUES

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ABSTRACT

Mathematical modelling of the water tank system has been performed. Water level of the water tank is control by PID controller, use of the PID controller with actuator saturation causes Integrator windup, due to which performance of the linear system degrades i.e. overshoot and settling time of the system increases. Anti-windup technique is used to tackle the problem of performance degradation, different anti-windup techniques are explained here and the effect these techniques on the linear system performance is tabulated.

Keyword: Anti-Windup, Integrator Windup, Overshoot, Risetime, Settling Time.

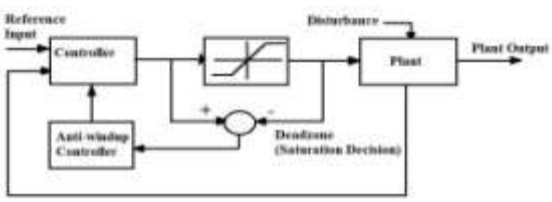
I INTRODUCTION

Mathematical modelling of the water tank system has been done with the help of Bernoulli's equation and some assumption. An anti-windup compensator consists of a nominal (most often linear) controller added with anti-windup compensation. An important property of anti-windup compensation is that it leaves the loop unaffected as long as saturation does not occur. Consequently, the control action provided by the anti-windup compensator is identical to that of the nominal controller, as long as the control signals operate within the saturation limits. The design can be split into two parts where the first part concerns the linear controller and the second the anti-windup compensation. The figure 1 represents the Basic anti-windup scheme[1][2].

II ACTUATOR

The Figure 2 is the actuator, If u' is the output of actuator (limiter), u_{max} is the maximum output value of the actuator and u_{min} is the minimum output value of the actuator and 'u' is the input value of the actuator then

$$u' = \begin{cases} umax, & \text{if } u \geq umax \\ u, & \text{if } umin \leq u \leq umax \\ umin, & \text{if } u \leq umin \end{cases}$$



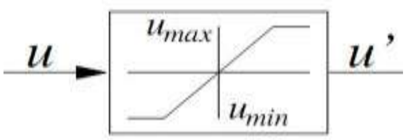


Fig. 1The Basic Anti-Windup scheme **Fig. 2 Actuator**

III FLUID FLOW MODELING

A fluid flow system is shown in figure 3. The reservoir (or tank) contains water that vacates through an output port. To obtain mathematical model for the fluid flow tank system, we make some key assumptions. We assume that the water in the tank is incompressible and that the flow is irrotational, inviscid, and steady. To obtain the mathematical model of the water tank system we apply some basic principle i.e. conservation of mass. The mass of water in the tank at any given time is [3]

Table:1

ρ (kg/m³)	g (m/s²)	A₁ (m²)	A2 (m2)	H* (m)	Q* (kg/s)
1000	9.8	π/4	π/400	1	34.77

$$m=\rho A_1 H, \quad (1)$$

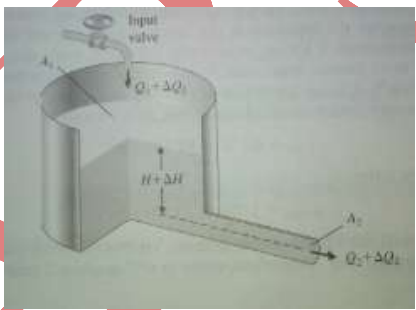


Fig.3 Fluid Flow System

Where A_1 is the area of the tank, m is mass of the water in the tank, ρ is the water density, and H is the height of the water in the reservoir (tank). The constants for the reservoir (tank) system are given in Table 1[3].

Taking the time derivative of Equation (1) we get

$$m' = \rho A_1 H' \quad (2)$$

The change in mass of water in the reservoir is equal to the mass that enters the tank minus the mass that leaves the tank,

$$m' = \rho A_1 H' = Q_1 - \rho A_2 v_2 \quad (3)$$

Where Q_1 is the steady- state input mass flow rate, v_2 is the exit velocity, and A_2 is the output port area. The exit velocity, v_2 is a function of the water height. From Bernoulli's equation [4] we have

$$\frac{1}{2}\rho {v_1}^2 + P_1 + \rho g H = \frac{1}{2}\rho {v_2}^2 + P_2 \quad (4)$$

Where v_1 is the velocity of water at the input port of the water tank, and P_1 and P_2 are the atmospheric pressures at the input and output, respectively. But P_1 and P_2 are equal atmospheric pressure and A_2 is sufficiently small ($A_2 = A_1/100$) so the water flow out slowly and the velocity v_1 is negligible. Thus Bernoulli's equation reduces to

$$v_2 = \sqrt{2gH} \quad (5)$$

In equation (2) substituting equation (5) and solving for height H we get

$$H = -\left[\frac{A_2}{A_1}\sqrt{2g}\right]\sqrt{H} + \frac{1}{\rho A_1}Q_1 \quad (6)$$

Using Equations (5), we get the mass flow rate at the output port

$$Q_2 = \rho A_2 v_2 = \left(\rho\sqrt{2gA_2}\right)\sqrt{H} (7)$$

Assuming that

$$k_1 := -\frac{A_2\sqrt{2g}}{A_1}, \quad k_2 := -\frac{1}{\rho A_1}, \quad k_3 := \rho\sqrt{2gA_2}.$$

Then, we get

$$H = k_1\sqrt{H} + k_2 Q_1$$

$$Q_2 = k_3\sqrt{H} (8)$$

The model in Equation (8) can be represented in the functional form as below.

$H = f(H, Q_1)$,

$$Q_2 = h(H, Q_1), \quad (9)$$

Where

$$f(H,Q_1) = k_1\sqrt{H} + k_2 Q_1 \text{ and } h(H,Q_1) = k_3\sqrt{H}$$

Using Taylor series expansions A set of linearized equations describing the height of the water in the reservoir is obtained about an equilibrium flow condition. When the tank system is in equilibrium, we have H'=0. We can define Q* and H* as the equilibrium input mass flow rate and water level, respectively. The relationship between Q* and H* is given by

$$Q^* = \frac{k_1}{k_2}\sqrt{H^*} = \rho\sqrt{2gA_2}\sqrt{H^*} \quad (10)$$

This condition occurs when just enough water enters the tank in A_1 to make up for the amount leaving A_2. We can the water level and input mass flow rate as

$$H = H^* + \Delta H,$$

$$Q_1 = Q^* + \Delta Q_1, \quad (11)$$

Where ΔH and ΔQ_1 are small deviations from the equilibrium (steady-state) values. The Taylor series expansion about the equilibrium conditions is given by

$$H = f(H, Q_1) = f(H^*, Q^*) + \left.\frac{\partial f}{\partial H}\right|_{\substack{H=H^* \\ Q_1=Q^*}}(H - H^*) + \left.\frac{\partial f}{\partial f}\right|_{\substack{H=H^* \\ Q_1=Q^*}}(Q_1 - Q^*) + \ldots\ldots \quad (12)$$

Where

$$\left.\frac{\partial f}{\partial H}\right|_{\substack{H=H^*\\Q_1=Q^*}} = \left.\frac{\partial(k_1\sqrt{H}+k_2Q_1)}{\partial H}\right|_{\substack{H=H^*\\Q_1=Q^*}} = \frac{k_1}{2\sqrt{H^*}}$$

and $\left.\frac{\partial f}{\partial Q_1}\right|_{\substack{H=H^*\\Q_1=Q^*}} = \left.\frac{\partial(k_1\sqrt{H}+k_2Q_1)}{\partial Q_1}\right|_{\substack{H=H^*\\Q_1=Q^*}} = k_2$

Using equation (10) we get

$$\sqrt{H^*} = \frac{Q^*}{\rho\sqrt{2g}A_2}$$

So that

$$\left.\frac{\partial f}{\partial H}\right|_{\substack{H=H^*\\Q_1=Q^*}} = -\frac{A_2^2 g\rho}{A_1 Q^*}$$

If follows from Equation (2.10) that

H'=ΔH',

Since H* is constant. Also, the term f(H*, Q*) is identically zero, by definition of the equilibrium condition. In the Taylor series expansion neglecting the higher order terms we get

$$\Delta H = -\frac{A_2{}^2}{A_1}\frac{g\rho}{Q^*}\Delta H + \frac{1}{\rho A_1}\Delta Q_1 \quad (13)$$

Equation (13) is a linear equation describing the change in water level ΔH form the steady state due to a change from the nominal input mass flow rate ΔQ_1. By taking the Laplace transform (with zero initial conditions) of the equation (13) We can get the relationship between change in the water level in the tank,ΔH(s), due to change in input mass flow rate into the tank $\Delta Q_1(s)$

$$\frac{\Delta H}{\Delta Q_1} = \frac{k_2}{s+\tau} \quad (14)$$

Where $\tau = \frac{A_2{}^2}{A_1}\frac{g\rho}{Q^*}$

Putting the value of k_2 and τ in equation (14) with the help of table 1we get the transfer function of the water tank system, i.e.

$$\frac{\Delta H}{\Delta Q_1} = \frac{1.273\times10^{-3}}{s+0.02212} \quad (15)$$

IV FINDING OF FOPTD SYSTEM

There are various methods to find the FOPTD system , here we have used Two Point method to find FOPTD system. Two Point method is explained in [5], by using Two point method the FOPTD system for water tank system is obtained which is given below

.

$$\frac{\Delta H}{\Delta Q_1} = \frac{0.0575}{1+45.03} e^{-0.32s} \quad (16)$$

V EFFECT OF ACTUATOR SATURATION IN LINEAR CONTROL SYSTEM

Due to actuator saturation[6] settling time and overshoot of the linear control system increases. Simulink model response of the water tank system is represented in fig 4 and fig 5 respectively.

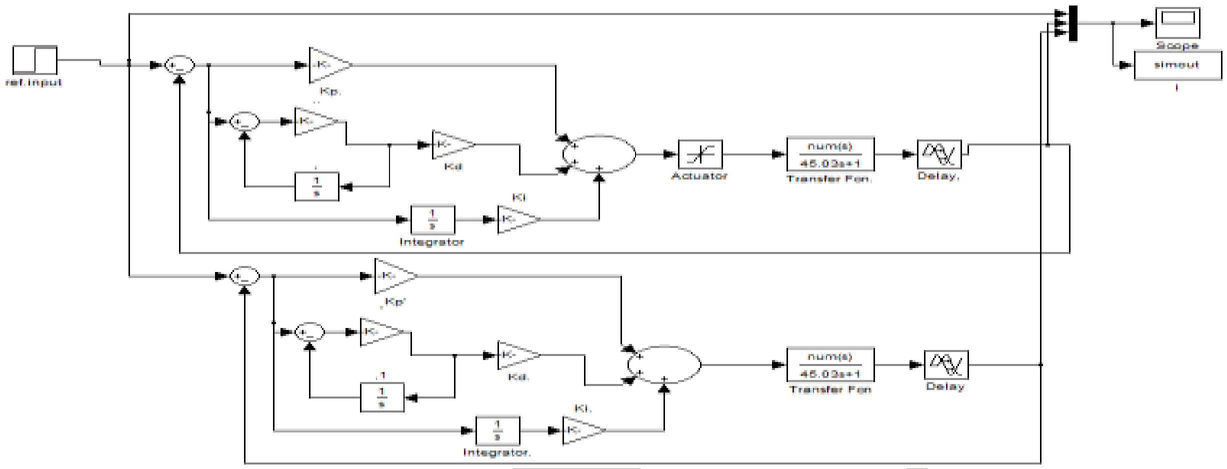


Fig.4 Simulink Model

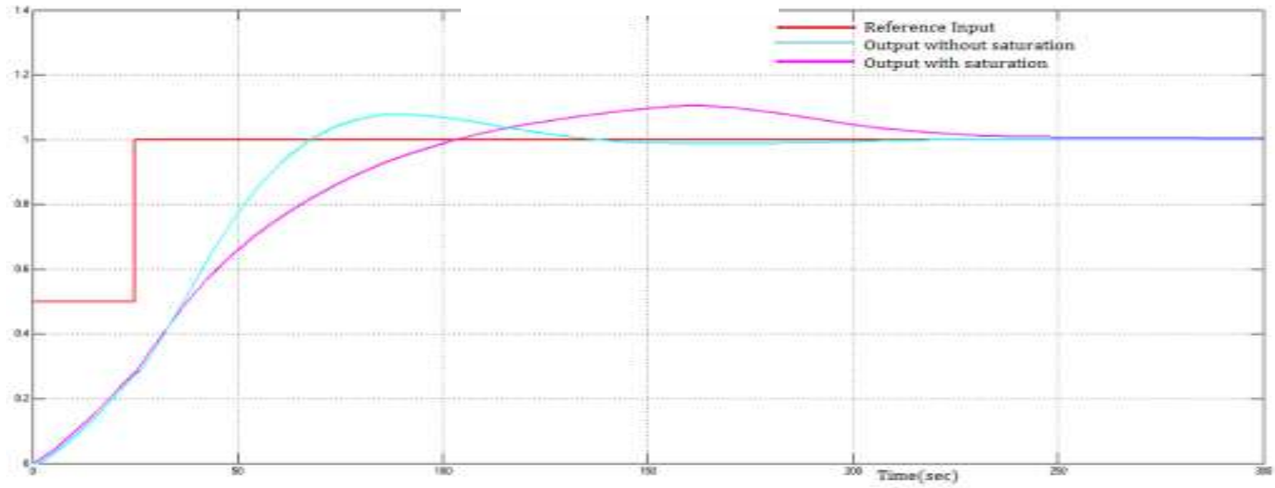


Fig.5 Response

VI ANTI-WINDUP TECHNIQUES

There are various anti-windup technique to tackle the problem of integrator windup. Some of the techniques are explained below.

6.1. Conditional Integration (CI)

A classical effective methodology is the so called conditional integration. It consists of switching off the integration (in other words, the error to be integrated is set to zero) [7] when a certain condition is verified. For this reason this method is also known as integral clamping. The integral term make equal to zero if output of the controller is not equal to the input to the plant. Simulationand response of the water tank system with conditional Integration is represented in figure 6 and figure 7

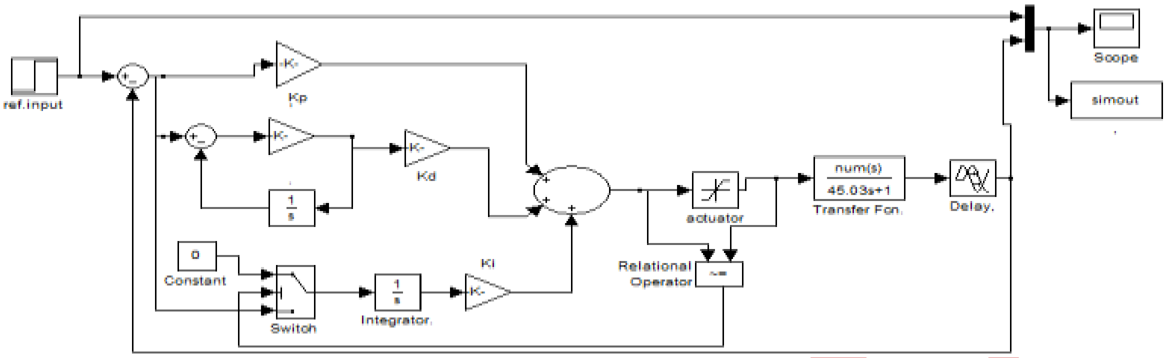


Fig.7 Simulink Model

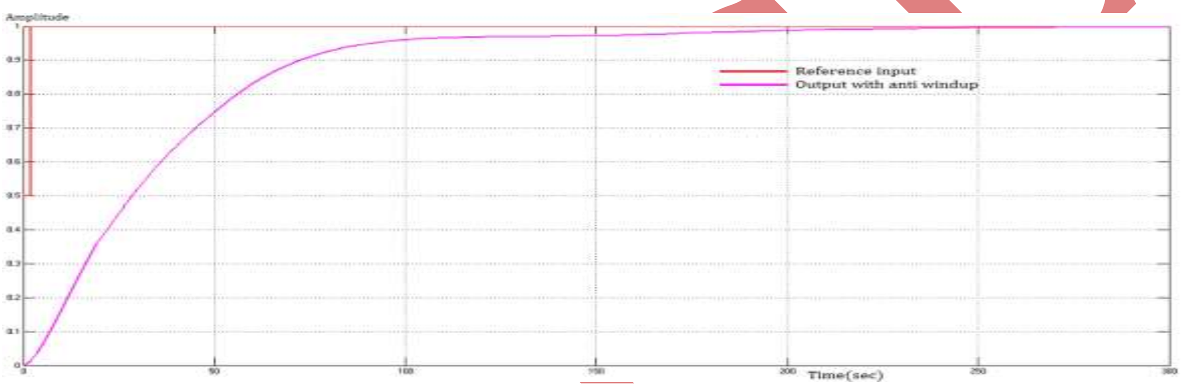


Fig.8 Response

6.2. Back-Calculation (BC1)

An alternative approach to conditional integration is developed which is known as the back-calculation (BC1) anti-windup scheme [8]. In this scheme recoumputing of the integral term phenomena is used when the controller output gets saturate. In particular, the integral value is reduced or increased depending on the controller output signal.

$$e_i = \frac{Kp}{Ti} e + \frac{1}{Tt}(u - u')$$

Where K_p- Proportional gain, Ti-Integral time constant, Tt- Tracking time constant, u and u' are the controller output and process input respectively, e_i is the integral term. The value of tracking time(T_t) constant determines the rate at which the integral term is reset. Tuning rules for the tracking time constant have been proposed. In (Astr¨om and Hagglund, 1995) it is suggested that

$$T_t = \sqrt{(Ti*Td)}$$

Simulink model and response of the water tank system is shown in figure 8 and figure 9

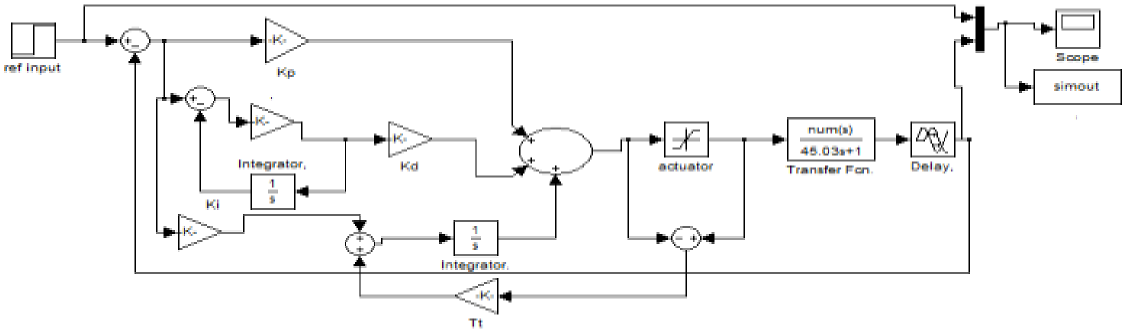


Fig.8 Simulink Model

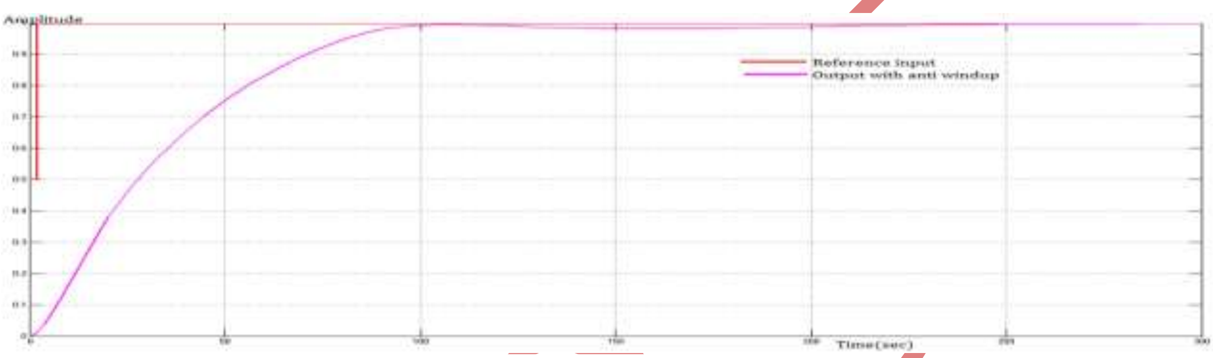


Fig.9 Response

6.3 Back-Calculation (BC2)

In Back-Calculation (BC1) the tracking time T_t depends on both the value of T_i and T_d, hence Back-Calculation (BC1) method is useful only for PID controller but if we are using PI controller then this method is not suitable because in PI controller Derivative time constant T_d is zero. To solve this problem a new anti-windup technique is developed in which tracking time constant T_t does not depends on the value of T_d ,it depends on only integral time constant.

$$T_t = T_i$$

Simulink model and response of the water tank system using Back-calculation (BC2) anti-windup scheme is shown in the fallowing figure 10 and 11

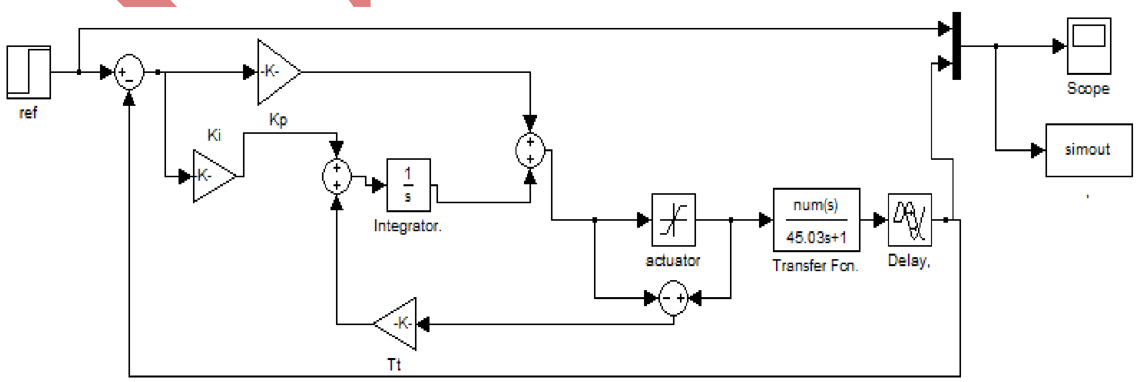


Fig.10 Simulink Model

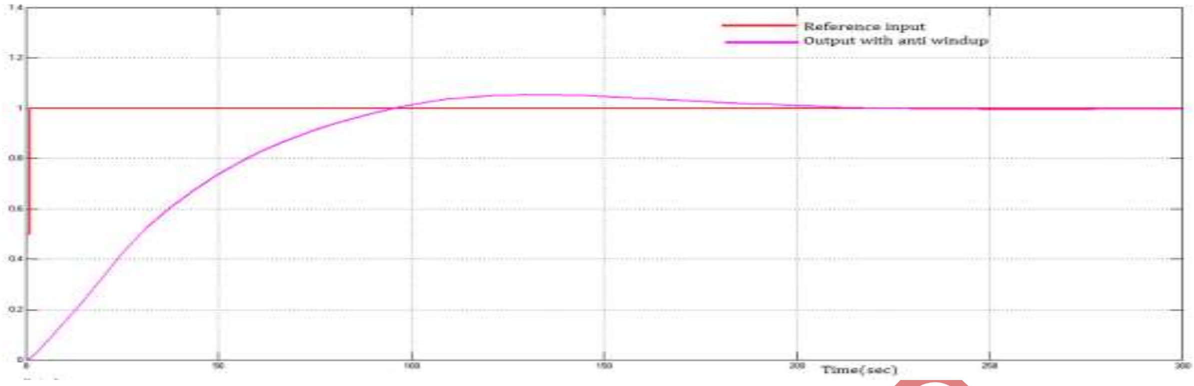


Fig.11 Response

6.4. Automatic Reset Implementation (AS)

When we are using the PI controller is in automatic reset configuration as shown below Fig. the anti- windup technique [9] can be applied easily by inserting the saturation function as shown below in figure. 12 and response of the system is shown in figure. 13

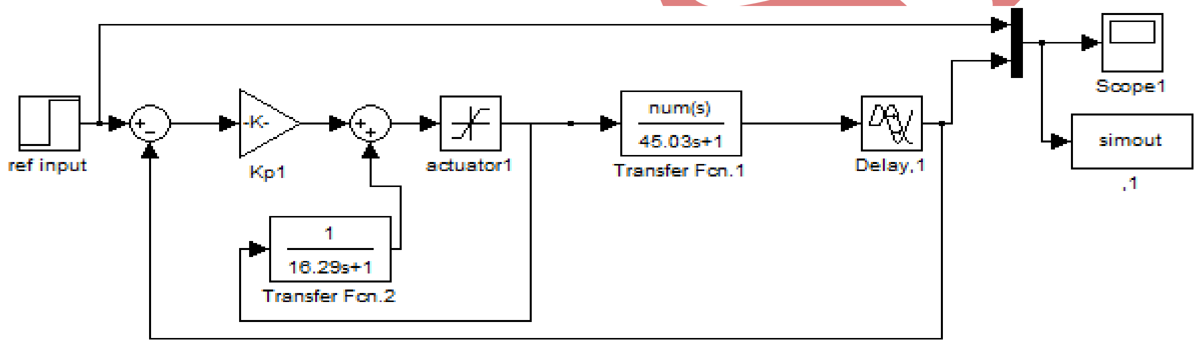


Fig.12 Simulink Model

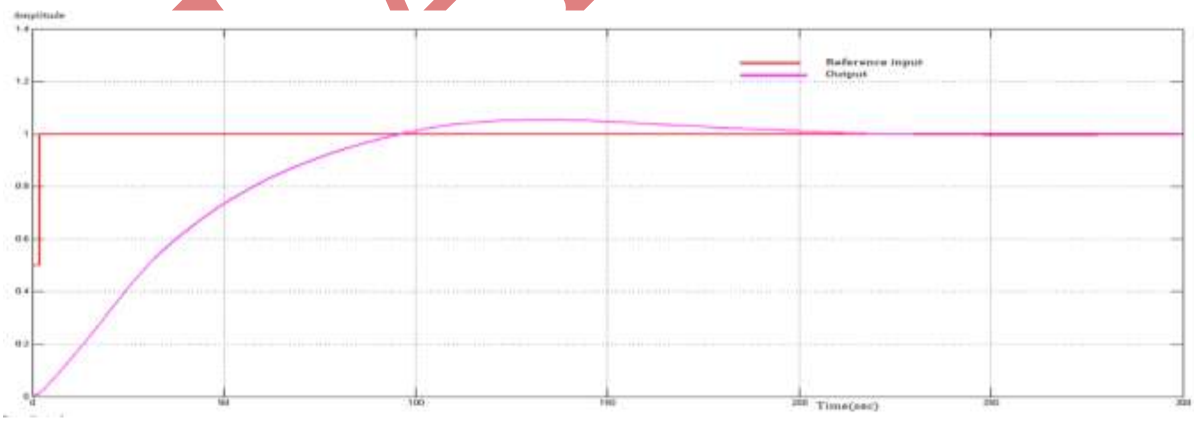


Fig.13 Response

6.5. VSPID Anti-Windup Scheme

Here a proposed method is VSPID anti-windup method that eliminates the weaknesses described in the previous section. When the nominal control u is in saturation, the VSPID control method[10] is used to pushes the integrator so that controller output u lies at the edge of the saturation region. The VSPID controllers make use a type of switching technique between a few different PID controllers the concept of the VSPID controller is used to design the anti-windup scheme. In this scheme we use different Integral gains for different conditions. The VSPID anti-windup scheme is given below. Anti-windup scheme is given below.In this scheme input to the integral controller is given with help of a switch. The switch is operated with the help of logic function. Input to the integral controller

$$I = \begin{cases} \frac{-\alpha(u-u')}{Ki}, \text{if } u \neq u' \text{ and } e(u - ⍰) > 0 \\ \qquad Ki > 0,\ ⍰ = \frac{umax + umin}{2} \\ e \qquad \text{otherwise} \end{cases}$$

Where I is the input to the integral controller α > 0 is a positive constant selected such that u rapidly converges to the nearest extreme value of the interval ($u_{max}u_{min}$).

Simulink model and response of the water tank system using VSPID anti-windup scheme is shown in figure 14 and figure 15.

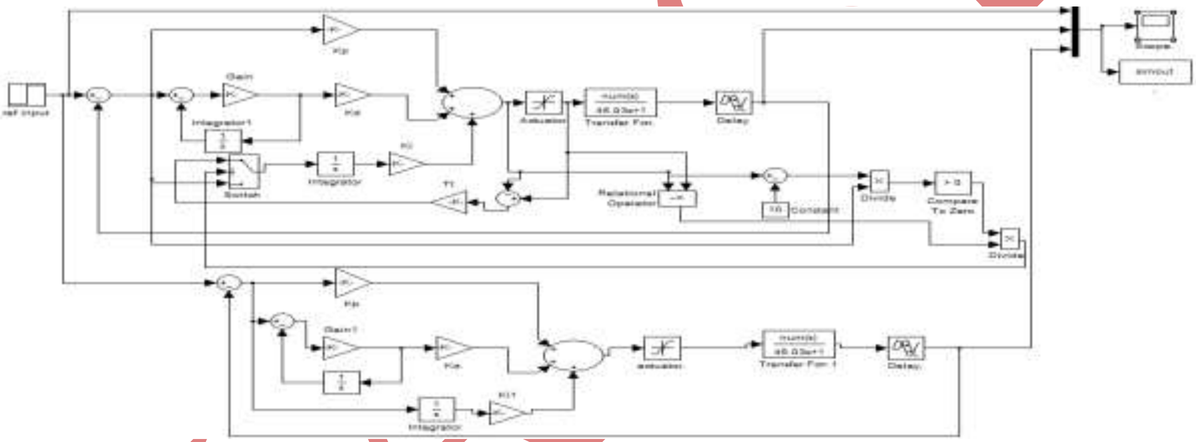


Fig.14 Simulink Model

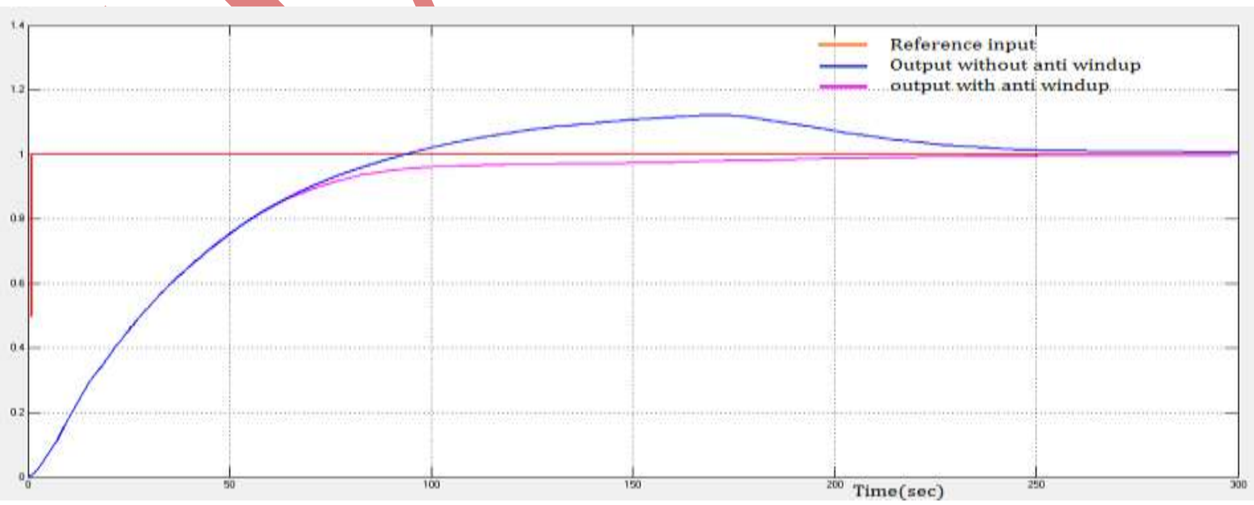


Fig.15 Response

Simulation results for the different anti-windup techniques are plotted. In order to analyze the performance rise time, 2% settling time and the maximum overshoot are represented in table 2

Type	Settling time(min)	Rise time(min)	Overshoot(%)
WAS	3.5	1	6
WAS1	4.33	1.16	11
BC1	3.81	1.30	5
BC2	3.43	1.10	0
VSPID	3.57	1.15	0
CI	3.50	1.06	0
AS	3.82	1.10	5

Table:2

WAS: Without actuator saturation, WAS1:With actuator saturation

VII CONCLUSION

Mathematical modeling of the fluid flow system is performed. Integrator windup effect in the perspective of PID controller has been analyzed and different anti-windup technique have been presented and compared.All the considered methods are effective and each one has particular features. There is not a technique that performs better than the others for all the kind of plant, PID parameters and actuator limit. In some cases overshoot reduced but settling time and the rise time is not improve and in some other cases settling time improve but overshoot is not improve.

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