IMPLEMENTATION OF OFDM AND CHANNEL ESTIMATION USING LS AND MMSE ESTIMATOR

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) provides an effective and low complexity means of eliminating inter symbol interference for transmission over frequency selective fading channels. This technique has received a lot of interest in mobile communication research as the radio channel is usually frequency selective and time variant. In OFDM system, modulation may be coherent or differential. Channel state information (CSI) is required for the OFDM receiver to perform coherent detection or diversity combining, if multiple transmit and receive antennas are deployed. In practice, CSI can be reliably estimated at the receiver by transmitting pilots along with data symbols. This paper discusses the channel estimation in OFDM and its implementation in MATLAB using pilot based block type channel estimation techniques by LS and MMSE algorithms. This paper starts with comparisons of OFDM using BPSK and QPSK on different channels, followed by modeling the LS and MMSE estimators on MATLAB. In the end, results of different simulations are compared to conclude that LS algorithm gives less complexity but MMSE algorithm provides comparatively better results.

Keywords: OFDM, Pilot Based Channel Estimation, LS, MMSE, AWGN Channel.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is most commonly employed in wireless communication systems because of the high rate of data transmission potential with efficiency for high Bandwidth and its ability to combat against multi-path delay. It has been used in wireless standards particularly for broadband multimedia wireless services.

An important factor in the transmission of data is the estimation of channel which is essential before the demodulation of OFDM Signals since the channel suffers from frequency selective fading and time varying factors for a particular mobile communication system [1] The estimation channel is mostly done by inserting pilot symbols into all of the subcarriers of an OFDM symbol or inserting pilot symbols into some of the sub-carriers of each OFDM symbol. The first method is called as the pilot based block type channel estimation and it has been discussed for a slow fading channel. This paper discusses the estimation of the channel for this block type pilot arrangement which is based on Least Square (LS) Estimator and Minimum Mean-Square Error (MMSE) Estimator. [2].

The second method is the comb-type based channel estimation in which pilot symbols are transmitted on some of the sub carriers of each OFDM symbol. This method usually uses different interpolation schemes such as linear, low-pass, spline cubic, and time domain interpolation. In [3] [4], it is shown that second-order interpolation performs better than the linear interpolation.

This paper aims to compare the performance of the pilot based block type channel estimation by using Binary Phase Shift Keying (BPSK) modulation scheme in a slow fading channel. In Section II, the basic system model of OFDM is discussed .In Section III, the estimation of the slow fading channel is performed, based on block-type pilot arrangement. In Section IV, the simulation parameter sand results are discussed. Section V concludes the findings.

II. SYSTEM DESCRIPTION FOR OFDM

The basic OFDM system block diagram under the assumption of frequency domain equalization is shown inFig.1. The binary information is being generated from uniformly distributed random integers with equal probability of either 0 or 1 given as [5]:

$$d_k = [d_0,d_1,d_2,\ldots\ldots d_{N-1}]\ k = 0,\ldots\ldots.N-1 \qquad (1)$$

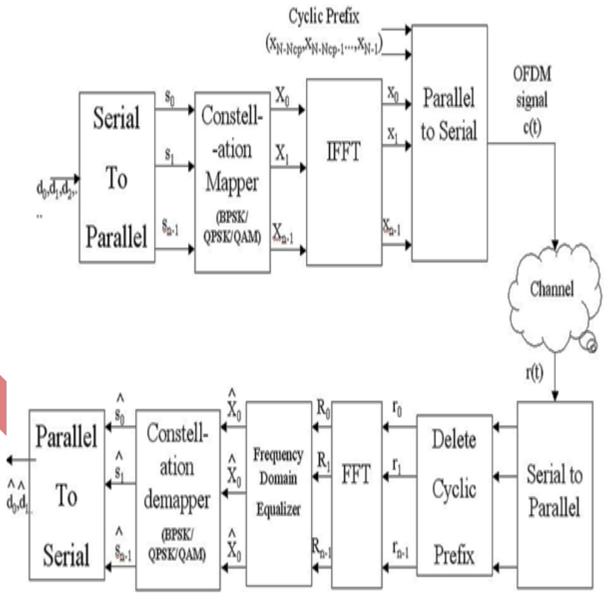


Figure 1: Basic OFDM System

D_k is converted from serial bit stream to parallel and mapped according to the modulation in the block of constellation mapper. The BPSK/QPSK symbols are then superimposed on orthogonal subcarriers using IDFT given as :

$$X(k) = \sum_{n=0}^{N-1} S(n)\sin\left(\frac{2\pi kn}{N}\right) \ - j\sum_{n=0}^{N-1} s(n)\cos\left(\frac{2\pi kn}{N}\right) \qquad (2)$$

Where, S(n) is the BPSK/QPSK symbols and N is the length IDFT. After the IFFT block, cyclic prefix of length D, which is considered to be greater than the impulse response of the channel, it is used to combat inter-symbol interference and inter-carrier interference (ICI). It is given as:

$$\mathbf{x(k) = [x_{cp}(k)\ \ x(K)]} \qquad \mathbf{(3)}$$

The OFDM signal is the constructed by applying the symbol along with CP to parallel to serial converter. It is then transmitted on channel given as:

$$\mathbf{Y(k) = x(k)\otimes h(l) + n(k)} \qquad \mathbf{(4)}$$

where, h(l) is the channel impulse response. The length of channel should be less than the cyclic prefix. For OFDM system, noise is generated in terms of symbols, so it is given as:

$$\mathbf{n(k) =}\ 10^{-Es/20No} * \mathbf{AWGN} \qquad \mathbf{(5)}$$

Where E_S/N_0 is symbol to error ratio (SER) given as :

$$\left(\frac{Es}{No}\right)_{db} = \left(\frac{N}{Ncp+N}\right)_{db} + \left(\frac{Eb}{No}\right)_{db} \qquad \mathbf{(6)}$$

Here, N_{CP} represents the length of cyclic prefix, N_{st} is the no. of used subcarriers and N is the length of FFT or no. of sub-carriers [5]. Since the OFDM signal has overhead in terms of CP, so to compensate for it, we have to scale it so that resultant OFDM signal that is

received is given as :

$$\mathbf{r(k)} = \sqrt{(Ncp+N)/N} \times \mathbf{Y(K)} \qquad \mathbf{(7)}$$

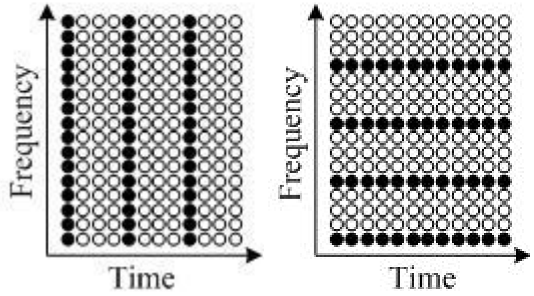
At the receiver the reverse steps are involved and since the OFDM symbols were circularly convolved with channel IR, so after FFT at the receiver [6], the received data is equalized by using the frequency domain equalizer and the equation given as:

$$\mathbf{X(K)} = \frac{Y(K)}{H(K)} \qquad \mathbf{(8)}$$

Where, H(K) is the response of the channel in frequency domain. The frequency domain equalization is useful for equalizing the symbols that were faded as a result of experiencing multipath. The results are discussed in Section V [4].

III. SYSTEM DESCRIPTION FOR CHANNEL ESTIMATION

For an OFDM mobile communication system, the channel transfer function at different subcarriers appears unequal in both frequency and time domains. Therefore, a dynamic estimation of the channel is always required. Pilot-based approaches are widely used to estimate the channel properties and correct the received signal. In this paper, two types of pilot arrangements, as shown in Figure 2 are investigated.



(a) (b)

Figure 2: (a) Block Type Pilot Arrangement & (b) Comb Type Pilot Arrangement

The first kind of pilot arrangement, shown in Figure 2, is denoted as block-type pilot arrangement. This is sent periodically in time-domain and is particularly suitable for slow-fading radio channels. Because the training block contains all pilots, channel interpolation in frequency domain is not required. Therefore, this type of pilot arrangement is relatively insensitive to frequency selectivity. The second kind of pilot arrangement, shown in Figure 2 , is denoted as comb-type pilot arrangement. In this case, the pilot arrangements are uniformly distributed within each OFDM block. Assuming that the payloads of pilot arrangements are the same, the comb-type pilot arrangement has a higher re-transmission rate. Thus, the comb-type pilot arrangement system provides better resistance to fast-fading channels. Since only some sub-carriers contain the pilot signal, the channel response of non-pilot sub-carriers will be estimated by interpolating neighboring pilot sub-channels. Thus, the comb-type pilot arrangement is sensitive to frequency selectivity when comparing to the block-type pilot arrangement system.

In block-type pilot based channel estimation, each subcarrier in an OFDM symbol is used in such a way that all sub-carriers are used as pilots. The estimation of the channel is then done using Least Square Estimator and Minimum Mean Square Error Estimator. [7],[5]. The system shown in Fig. 3 is modeled using the following equation:

$$\mathbf{y = DFT_N\,(IDFT\,(X) \otimes h/\sqrt{N} + \varpi)\,,} \qquad \mathbf{(9)}$$

where

$$\mathbf{x = [x_0\ x_1 \ldots\ldots\ x_{N-1}]^T}$$

$$\mathbf{y = [y_0\ y_1 \ldots\ldots\ y_{N-1}]^T}$$

$$\mathbf{\varpi = [\varpi 0\ \varpi 1 \ldots\ldots\ \varpi N\text{-}1]^T}$$

$$\mathbf{h = [h_0\ h_1 \ldots\ldots\ h_{N-1}]^T}$$

The vector h/√N is the observed channel impulse response when the frequency response of g(t) is sampled and it is given as:

$$\mathbf{h_k} = \frac{1}{\sqrt{N}} \sum\nolimits_m e^{-j\pi/N}\,(k + (N-1)\square m)\,\frac{Sin\,(\pi \square m)}{Sin(\frac{\pi}{N}\,(\square m - k))} \qquad \mathbf{(10)}$$

Where, m is the length of taps, N is the no of sub carriers, and is the value of the tap. If inter symbol interference is eliminated by the cyclic prefix, then the system shown in the Fig. 2 can be modeled using the equation given as [8]:

$$\mathbf{Y_k = H_K X_K + w_k,\ k = 0....N-1} \qquad (11)$$

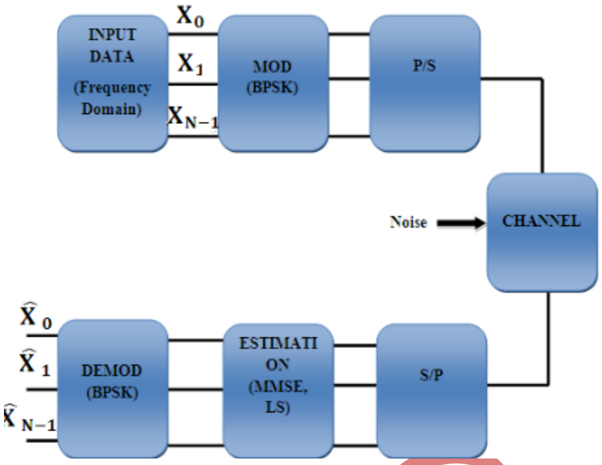


Figure 3: Channel Estimation using Ls/MMSE Algorithm

Where is H_K the Frequency response of given by:

$$\mathbf{H = [H_0\ H_1 \,.....\, H_{N-1}]^T = DFT_N(h)}$$

$$\mathbf{w = [w_0\ w_1 \,.....\, w_{N-1}]^T = DFT_N(\varpi)}$$

Now writing the (11) in Matrix form, it becomes:

$$\mathbf{y = XFh + w} \qquad (12)$$

where ,

$$\mathbf{X = diag\{x_0 x_1 \,....\, X_{N-1}\}}$$

$$\mathbf{y = [y_0\ y_1 \,.....\, y_{N-1}]^T}$$

$$\mathbf{w = [w_0\ w_1 \,.....\, w_{N-1}]^T}$$

$$\mathbf{h = [h_0\ h_1 \,.....\, h_{N-1}]^T}$$

$$\mathbf{F} = \begin{matrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{matrix}$$

F is the matrix of DFT with corresponding weights given as:

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{nk}{N}}$$

If the channel vector h is Gaussian and is it not correlated with the noise of the channel w, then the frequency domain MMSE estimates of h becomes [5].

$$H_{MMSE} = FR_{hy}R_{yy}^{-1}y \qquad (13)$$

Where ,

$$R_{hy} = E\{hy^H\} = R_{hy}F^HX^H$$

$$R_{YY} = E\{yy^H\} = XFR_{hh}F^HX^H + \sigma_n^2 I_N$$

Here R_{hy} is the cross correlation matrix between h and y, R_{yy} is the auto correlation matrix of y with itself and R_{hh} is the auto correlation matrix of the h with itself Since, σ_n^2denotes the noise variance [6]. The factors R_{hh} and σ_n^2 are considered to be known. The LS estimate of the channel is given as:

$$H_{LS} = X^{-1}y \qquad (14)$$

Which minimizes $(Y - XFh)^H(Y - XFh)$. Both estimators suffer from different drawbacks. The MMSE usually suffers from a high complexity, where LS estimator suffers from mean-square error which is high. The MMSE estimator requires to calculate an N X N matrix which results in a high complexity when becomes large [5].It should be noticed that both the estimators are derived under the assumption [6] of known channel correlation and noise variance. In actual scenario these quantities R_{gg} and σ_n^2 , are either considered to be fixed or estimated most commonly in an adaptive way[6].

IV. SIMULATION AND RESULTS

This section discusses the results of the simulation that were performed based on the information and mathematics discussed in the Section II & III respectively. For the simulation of basic OFDM system, we used the following parameters as shown in Table I

.TABLE: SIMULLATION PARAMETER

Parameter	**Specification**
FFT SIZE	2048
No of used Subcarriers	128
No of OFDM Symbol	100
Cyclic Prefix	512
Modulation	BPSK/QPSK
No of Taps / Multipath	8
Channel	AWGN

Similarly for BPSK, again the BER determines how many of the received bits are in error, and then computes it by the number of bits in error divided by the total no of bits in the transmitted signal. As the BER for the Multipath fading is simulated for the (no. of taps) = 8, which is less than the length of the CP, however if we increase the no of taps for the multipath fading then the resultant BER curve would show that the performance is getting worse and more errors would occur.

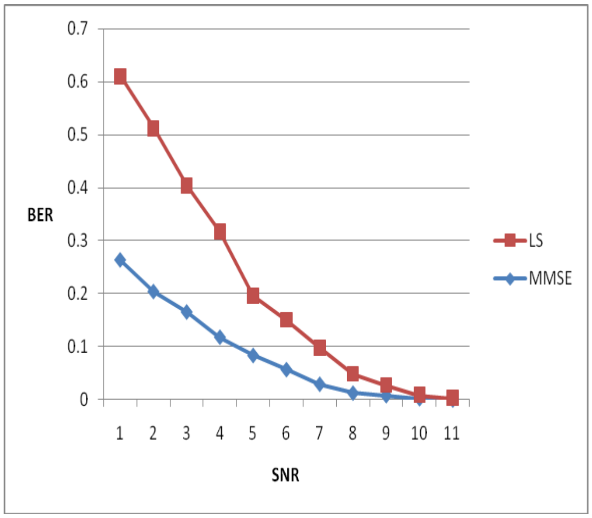


Fig 4

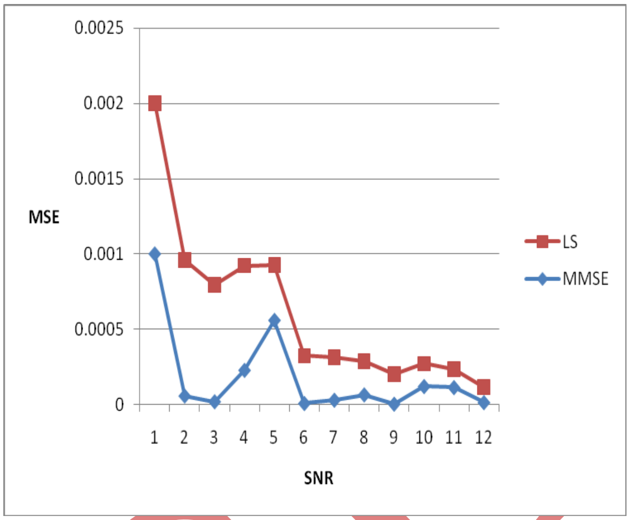


Fig 5

Fig 4: Comparison of BER For OFDM using QPSK With LS/MMSE Estimator

Fig 5: Comparison of Mean Square Error For OFDM using QPSK with LS/MMSE estimator

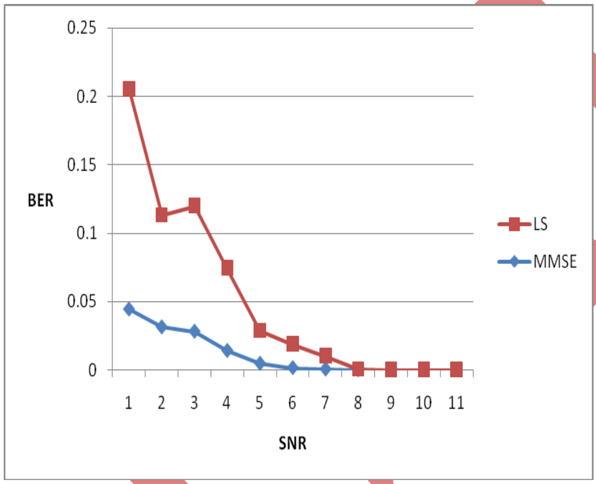


Fig 6

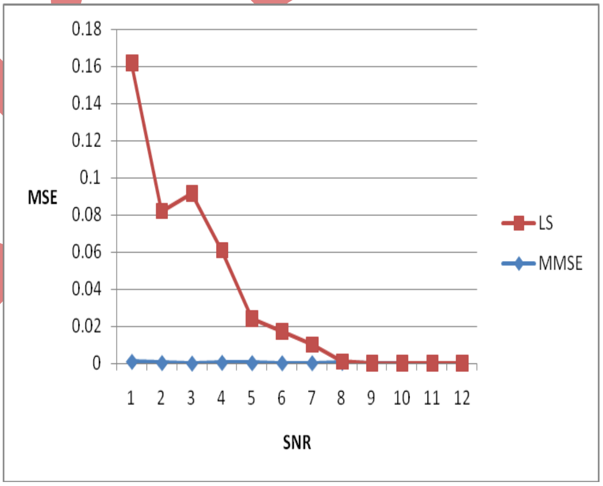


Fig 7

Fig 6: Comparison of BER For OFDM Using BPSK With LS/MMSE Estimator

Fig 7: Comparison of Mean Square Error for OFDM Using BPSK With LS/MMSE Estimator

V. CONCLUSIONS

This paper highlights the channel estimation technique based on pilot aided block type training symbols using LS and MMSE algorithm. The Channel estimation is one of the fundamental issues of OFDM system design. The transmitted signal under goes many effects such reflection, refraction and diffraction. Also due to the mobility, the channel response can change rapidly over time. At the receiver these channel effects must be canceled to recover the original signal.

In section IV, The estimators in this study can be used to efficiently estimate the channel in an OFDM system, given certain knowledge about channel statistics. The MMSE estimators assume a priori knowledge of noise variance and channel covariance. Moreover, its complexity is large compare to the LSE estimator. For high SNRs, the LSE estimator is both simple and adequate. The MMSE estimator has good performance but high complexity. The LSE estimator has low complexity, but its performance is not as good as that MMSE estimator basically at low SNRs.

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