IMPROVE IMAGE QUALITY USING HYBRID WAVELET FRACTAL IMAGE CODER

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ABSTRACT

In this paper, the transformation function used in fractal image compression is chosen in such a way that its unique fix point is a close approximation of the input image. Compression occurs because storing the details of the image transform also known as encoding takes up much less space than the original image. Decompression or decoding involves applying the transform repeatedly to an arbitrary starting image, to arrive at a (fractal) image that is either the original, or one very similar to it.image coding can provide a highly reconstructed image quality with a high compression ratio, is independent of resolution, and has fast decoding process [4].Based on the standard fractal transformation in spatial domain, simple relations may be found relating coefficients in detail sub bands in the wavelet domain. In this paper we evaluate a hybrid wavelet-fractal image coder, and we test its ability to compress the different images and decompress the same [2]. A comparative study between the hybrid wavelet fractal coder and standard fractal coding compression technique have been made in order to investigate the compression ratio with high quality of the image using peak signal to noise ratio and also computation (encoding and decoding) time. We find that the superior performance of the hybrid wavelet-fractal coder (WFC).
In The WFC uses the fractal contractive mapping to predict the wavelet coefficients of the higher resolution from those of the lower resolution and then encode the prediction residue with a bit plane wavelet coder. The fractal prediction is adaptively applied only to regions where the rate saving offered by fractal prediction justifies its overhead. A rate-distortion criterion is derived to evaluate the fractal rate saving and used to select the optimal fractal parameter set for WFC.

Keywords: Discrete Wavelet Transforms Fractal Coder, Wavelet-Fractal Coder, Compression Ratio, Peak Signal to Noise Ratio, Mean Square Error.

I INTRODUCTION

Image compression is the reducing bits to represent an image [12]. One of the important factors for image storage or transmission over any communication media. Fractals, it appears, can be used to compress images to such an extent that it is possible to pack photo-realistic images onto a floppy disk and complete photo libraries onto a CD-ROM.
Literature of Fractals is a structure that is made up of similar forms and patterns that occur in many different sizes .The French mathematician Benoit B. Mandelbrot first coined the term fractal in 1975. In the birth of fractal geometry is usually traced to Mandelbrot in 1977. Fractal mathematics is good for generating natural looking images. Fractal coding is the lossy compression techniques. The method consists of the representation of image blocks through the contractive transformation coefficients, using

the self-similarity is the iteration function system (IFS).It produce (e.g. clouds). Iterated Function Systems set the foundation for fractal image compression. The basic idea of an Iterated Function System is to create a finite set of contraction mappings, written as affine transformations, based on what image one desires to create. If these mappings are contractive, applying the IFS to a seed image will eventually produce an attractor of that map. It does not matter what the seed image is for the mappings, the same fixed point will be produced regardless [2].

In this paper, we intend to study fractal image compression and decompression using one transform coding technique, namely, the Haar Wavelet Transform. Transform Coding has gained popularity over the years, and JPEG is one such popular compression algorithm, which uses Transform Coding. Fractal encoding, on the other hand, is a simple and effective way of image compression but is an asymmetrical process, taking much longer to compress an image than to decompress it.

The organisation of the paper is as follow: an overview of a basic fractal coding scheme is given in section II. Section III describes the hybrid wavelet fractal code .Section IV is given the experiment result and comparison result from different images .In section V contains the conclusion.

II FRACTAL BASICS

A fractal is a structure that is made up of similar forms and patterns that occur in many different sizes. The term fractal was first used by Benoit Mandelbrot to describe repeating patterns that he observed occurring in many different structures [6]. These patterns appeared nearly identical in form at any size and occurred naturally in all things. Mandelbrot also discovered that these fractals could be described in mathematical terms and could be created using very small and finite algorithms and data. For example if you look at the surface of an object such as the floor currently beneath your feet, you will notice that there are many repeating patterns in its texture. The floor's surface may be wood, concrete, tile, carpet, or even dirt, but it still contains repeating patterns ranging in size from very small to very large [6][11].

2.1. Fractal image compression

Standard Fractal image compression is a recent technique based on the representation of an image. The self-transformability property of an image is assumed and exploited in fractal coding. It provides high compression ratios and fast decoding. Apart from this it is also simple and is an easily executable technique [4][10]. We here consider a special type of photocopying machine that reduces the input image by a half and reproduces it three times on the copy. What will be happen when we return back the output of the machine to the input? Next figure shows the output images:

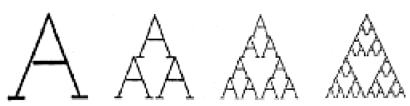


Fig.1 Photocopying Output Image

It is obvious that the output images converge to the Sierpinski triangle. This final image is called attractor for this photocopying machine. Any initial image (letter 'A' in our case) will be transformed to the attractor if we repeatedly run the machine. On the other words, the attractor for this machine is always the same image without regardless of the initial image. This feature is one of the keys to the fractal image compression.

Transformations of the form as follows will help us.

$$w_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix} \tag{1}$$

Such transformations are called affine transformations. Affine transformations are able to skew, stretch, rotate, scale and translate an input image. The machine produces self-similarity images called fractals [1]. We need only a few parameters of affine transformations to produce Sierpinski triangle from any initial image. But how to find parameters for any image.That is inverse problem mentioned earlier Barnsley's dream is to discover algorithm for calculating these parameters. Unfortunately, images from the nature are very inappropriate for generating using affine transformations because natural images are not exactly self-similar. Fractal image compressions based on two methods are as follows

2.2. Iteration function system

The Behaviour of the photocopying machine is described with mathematical model called an Iterated Function System (IFS). An iterated function system consists of a collection of contractive affine transformations [8]. This collection of transformations defines a map

$$W(*) = \bigcup_{i}^{x} w_i(*) \tag{2}$$

For an input set S, we can compute wi for each i, take the union of these sets, and get a new set W(S). Hutchinson proved an important fact in Iterated Function Systems [7]: if the Wi is contractive, then W is contractive. Hutchinson's theorem tells us that the map W will have a unique fixed point in the space of all images. That means, whatever image (or set) we start with, we can repeatedly apply W to it and our initial image will converge to a fixed image. Thus W completely determines a unique image. In other words, given an input image (or set) f0, we can repeatedly apply W (or photocopying machine described with W) and we will get

$$f_1 = W(f_o) \tag{3}$$

As a first copy,

$$f_2 = W(f_1) = W(W(f_o)) = W^{(2)}(f_o) \tag{4}$$

As a second copy and so on.

The attractor, unique image, as the result of the transformations is,

$$|W| = f_\infty = \lim_{x \to \infty} W^{(n)}(f_o) \tag{5}$$

2.3. Partitioned Iteration Function System

For encoding of the image, it is necessary to divide it into non-overlapped pieces (so called domains, D) which are each transformed separately. By partitioning the image into pieces, we must to encode many shapes that are difficult to encode using IFS. Because, we have to use Partitioned Iterated Function Systems (PIFS).PIFS is also based on affine transformation but, now it describes transformation from one piece of the image to another (so called ranges, R). Affine transformation mentioned earlier is able to "geometrically" transform part of the image but is not able to transform grey level of the pixel. That is reason; we have to add a new dimension into affine transformation [3] [6]:

$$w_i\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}a_i & b_i & 0\\c_i & d_i & 0\\0 & 0 & s_i\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}+\begin{bmatrix}e_i\\f_i\\o_i\end{bmatrix} \tag{6}$$

Where Si represents the contrast, Oi the brightness of the transformation z=f(x,y)

III HYBRID WAVELET FRACTAL CODER

In this paper we evaluate fractal code in wavelet domain because the some advantages over fractal coding difficulties to obtain high quality of decoded images and blocking artefacts at low bitrates.DWT is the concept of multi-resolution analysis was put forward by Mallat [12], wavelet transform has always been a precedent in image processing fields. Wavelets are obtained from a single prototype wavelet called mother wavelet. wavelet functions to represent signals in multiple levels of time-frequency resolutions. The discrete wavelet transform (DWT) can be implemented by passing the signal through a combination of low pass and high pass filters and down sampling by a factor of two to obtain a single level of decomposition. Multiple levels of the wavelet transform are performed by repeating the filtering and down sampling operation on low pass branch outputs [13].

We performed the following affine transformation to each block [1][5]:

$$\begin{aligned}&(Di,j)=\alpha Dij+t0\\&where\,\alpha=[0,1],\alpha\in Rand\ to\in[-255,255],\quad t0\in Z\end{aligned} \tag{7}$$

In this case we are trying to find linear transformations of our Domain Block to arrive to the best approximation of a given Range Block. Each Domain Block is transformed and then compared to each Range Block Rk,l. The exact transformation on each domain block:

$$\min\sum_{m,n}(Ri,j)-(\Gamma(Di,j))m,n \tag{8}$$

With respect to α and t_o

$$\alpha=\frac{N_s^{\,2}\sum_{m,n}(Di,j)_{m,n}(Ri,j)_{m,n}-\left(\sum_{m,n}(Di,j)_{m,n}\right)\left(\sum_{m,n}Ri,j\right)_{m,n}}{N_s^{\,2}\sum_{m,n}((Di,j)_{m,n})^2-(\sum_{m,n}(Di,j)_{m,n})^2} \tag{9}$$

$$t_0 = \frac{\left(\sum (D_{i,j})_{m,n}\right)^2 - \left(\sum (R_{i.j})_{m,n}\right)^2}{N_s^{\,2} \sum_{m,n} ((Di,j)_{m,n})^2 - (\sum_{m,n} (Di,j)_{m,n})^2} \tag{10}$$

where m, n, Ns = 2 or 4 (size of blocks) each transformed domain block G(Di,j) is compared to each range block Rk, l in order to find the closest domain block to each range block. This comparison is performed using the following distortion measure.

$$d_{l2}\left(\Gamma(D_{i,j}), R_{k,l}\right) = \sum_{m,n} \left((\Gamma(D_{i,j}) - (R_{k,l})_{m,n}\right)^2 \tag{11}$$

Each distortion is stored and the minimum is chosen. The transformed domain block which is found to be the best approximation for the current range block is assigned to that range block, i.e. the coordinates of the domain block along with its a and to are saved into the file describing the transformation. This is what is called the Fractal Code Book.

$$\Gamma\left(D_{i,j}\right)_{best} \Rightarrow R_{k,l} \tag{12}$$

The 1-D wavelet transform can be extended to a two dimensional wavelet transform using separable filters. The 2-D transform is performed by applying a 1-D transform along the rows then along the columns. The 2-D wavelet decomposition of a signal shown in below fig2.

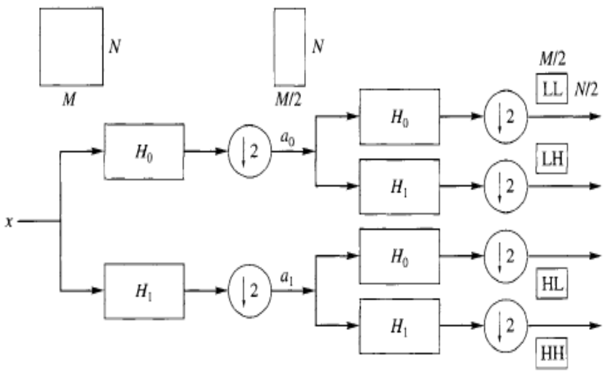


Fig2. 2-D DWT for Image.

In the figure, h0(m)and h1(m)represent the low pass and high pass filter responses, respectively. The filter outputs at level 2 are given by a, dH, dV, dD. The low pass filter output from the previous level serves as the input for the next level of decomposition. The discrete wavelet transform of an image provides a set of wavelet coefficients, which represent the image at multiple scales. The input image is decomposed into four sub images (or sub bands) LH, HL, HH, LL, where the pair letters denotes the row-column filtering operations performed to obtain the sub image [12]. For instance, sub image LH is obtained by low-pass filtering the rows and high-pass filtering the columns, followed by a factor two subsampling in each direction. This procedure can be iterated to obtain a multilevel decomposition of the image. Fig.3 demonstrates the decomposition process. The image is decomposed into different space (horizontal, vertical and diagonal directions) and different frequencies sub-images by multiresolution analysis [12].

The motivation for wavelet-fractal image compression stems from the existence of self similarities in the multi-resolution wavelet representation. In fact, fractal image coding in the wavelet domain has quite different characteristics from the spatial domain coders and can be interpreted as the prediction of a set of wavelet coefficients in the higher frequency sub bands from those in the lower ones[1][3]. Fig3 show that the hybrid wavelet fractal coder in this block diagram we just adding the haar wavelet partitioning block and this block have partitioning the original image into the fixed size scaled by 2 factor which are explained in above section.

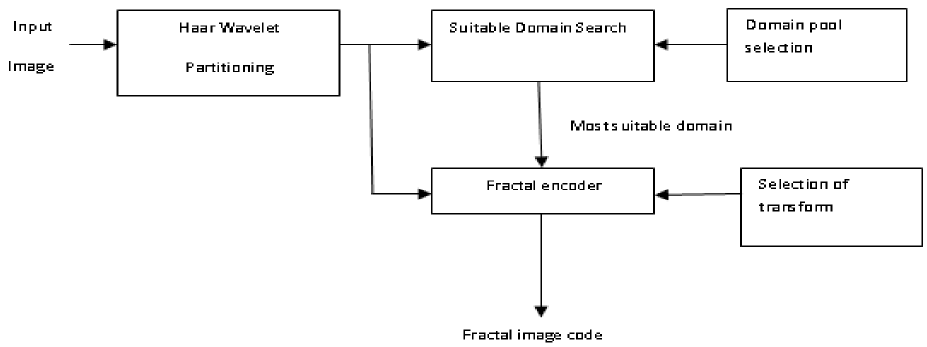


Fig3.Hybrid wavelet fractal coder

The contractive mapping operations carried out in the spatial domain have direct analogy in the wavelet domain. The averaging and subsampling operation S matches the size of the domain tree with that of the range tree. If we use Haar wavelet transform, the subsampling operation is equivalent to moving up the domain block tree by one scale in the wavelet domain, since the Haar transform is exactly the same as combined averaging and subsampling operations [4][12].

The Haar function is:

$$\psi(t) = \begin{cases} 1 & t \in [0, 1/2) \\ -1 & t \in [1/2, 1) \\ 0 & t \notin [0, 1) \end{cases} \quad (13)$$

Mathematically, the wavelet-fractal mapping can be written as [2]:

$$\hat{R} = W(D) = \alpha * \Gamma(S(D)) \quad (14)$$

Where R is a range tree, and D is the domain tree.

The isometric transformation Γ is done within each subband. The contrast scaling factor α is multiplied with each wavelet coefficient of domain tree to reach its correspondence in the range tree. Note that, in wavelet domain, an additive constant is not required as the wavelet tree does not have a constant offset.

IV EXPERIMENT RESULT

The wavelet-fractal image compression algorithm for decomposition of input data is implemented in MATLAB. The performance of the decoded images is evaluated by the PSNR value. The PSNR is calculated using the equation given below [1]:

$$PSNR = \frac{255^2}{MSE} \tag{15}$$

$$MSE = \frac{1}{M * N} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(I(i,j) - I^{\wedge}(i,j) \right)^2 \tag{16}$$

where, I(i,j) is the original image and I^(i,j) is the decompressed one, M and N are the number of columns and rows in the image.

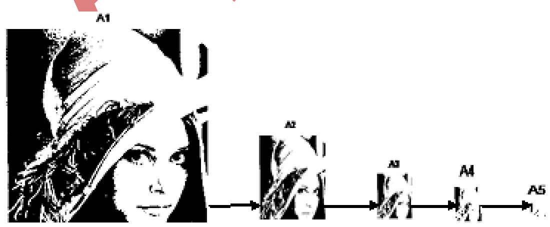
The amount of compression is calculated by compression ratio given by [13]:

$$CR = 100\left(1 - \frac{bpp}{8}\right)\% \tag{17}$$

Where CR is compression ratio and bpp is bits per pixel.

First we decompose the image by 5-level Haar wavelet transform is shown in fig4.and then we reconstruction image is shown in fig5.

Fig4.original image



1- level 2-level 3-level 4-level 5-level

Fig5.5-level decomposition

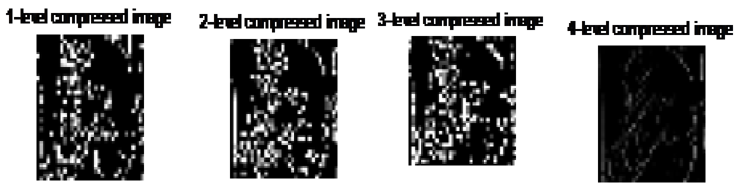


Fig6.1 to 4 level compressed image

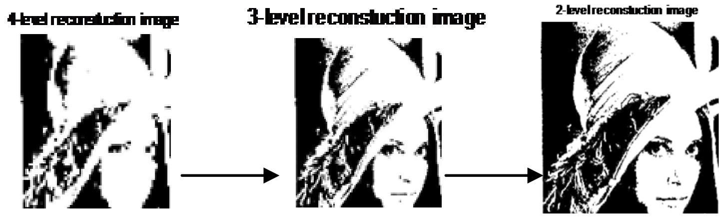


Fig7.4 to 2 level reconstruction images

Fig8.Reconstructed Lena image with 10 iteration

In this paper we have follow the following procedure to implement hybrid WFC : Original lena image with 512*512 size of pixel is applied to the haar wavelet ,this wavelet it produces the four coefficient such as approximation ,vertical,diagonal,horizontal shown in fig5.Now we have select the higher coefficient to making the fractal transform . We have observed the compressed images at each level and shown in fig6.

Now we also observed 4 to 2 level reconstruction images shown in fig7. And the Fig 8. Show that the reconstruction lena image with 10 iteration and original image.

Performance comparison for different images is shown in

TABLE1

FOR LENA IMAGE

Number of iterations	Compression time (sec)	Decompression time(sec)	MSE	PSNR (dB)	Difference
1	393.30	4.56	0.1084	57.78	28414
2	408.20	8.09	0.077	59.26	20186
3	391.22	11.21	0.0495	61.18	12988
4	394.55	15.02	0.0344	62.76	9016
5	383.73	17.97	0.0293	63.45	7690
10	379.96	33.46	0.0293	63.45	7690

Above table containing the only lena with fixed image size (512*512). This table gives the compression time near about 379sec and decompression time is about 33sec, we increasing the number of iteration from 1 to 10 we observed that the MSE is decreased, PSNR is increased, We have observed that when we increases the number of iteration then decoding time will increases and image quality have improved .We got the compression ratio as 227.40:1 and bpp is 0.0044 both the parameter are same for number of iteration. The percentage of compression ratio is 99.96%

V CONCLUSION

In this paper we analyzed the haar wavelet of 5-level decomposition for image compression .We have used various types of standard images for illustration purpose. We used lena with 512*512 size of image. We have got the higher PSNR near about 63.45dB.We have evaluated a hybrid wavelet-fractal coder on images .We have observed that the higher compression in low bit rate.

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Khalid Sayood "*Introduction to Data Compression*" Morgan Kaufmann Publishers, 2005 reconstructed image.