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Analysis of Microstrip Transmission Lines by Variational Method in the Fourier Transform Domain

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Abstract:

This paper presents a formal analysis and theoretical framework for evaluating microstrip transmission lines using the variational method in the Fourier transform domain. The approach, originally introduced by Yamashita and Mittra, combines spectral transform techniques with stationary variational formulations to efficiently compute electrostatic parameters such as line capacitance, characteristic impedance (Z), and guide wavelength ratio (λ/λ_0). Formulations for both infinitesimally thin and finite-thickness strip conductors across shielded, unshielded, and double-layer dielectric structures are derived. In addition, a systematic refinement procedure based on Galerkin's method/Rayleigh-Ritz expansion in the transform domain is presented to achieve higher accuracy.

Keywords: *Microstrip lines, Variational method, Fourier transform domain, Line capacitance, Characteristic impedance, Green's function, Galerkin's method.*

1. Introduction:

Microstrip transmission lines are essential fundamental components in modern RF and microwave integrated circuits. Accurate analysis of their propagation characteristics—such as characteristic impedance and phase velocity—requires solving Poisson's equation across inhomogeneous dielectric interfaces. Yamashita and Mittra (1968) introduced a highly efficient numerical method that merges the Fourier transform approach with variational techniques. By translating space-domain convolution integrals into straightforward algebraic products in the spectral domain via Parseval's theorem, this approach substantially reduces computational complexity while maintaining high solution accuracy.

2. Variational Method in the Fourier Transform Domain:

2.1 Problem Formulation and Spectral Formulation:

Consider an insulated double-layer microstrip line structure (as shown in Figure 3.7). The conventional single-layer microstrip line can be derived from this generalized configuration by setting relative dielectric constants $\varepsilon_1^* = \varepsilon_2^*$, $\varepsilon_3^* = 1$, and $d \to \infty$.

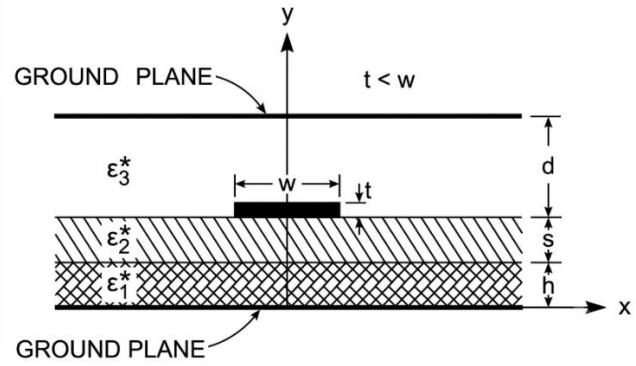


Figure 3.7: Shielded double-layer microstrip line.

To determine the capacitance per unit length along the uniform microstrip transmission line, Poisson's equation for potential distribution $\phi(x, y)$ in the cross-sectional plane is formulated. For an infinitely thin strip positioned at $y = h + s$:

$$\nabla_t^2 \phi(x, y) = -\frac{1}{\varepsilon} \rho(x) \delta(y - h - s - p) \qquad (1)$$

where $\rho(x) = 0$ for $|x| > w/2$, δ is the Dirac delta function, ε is permittivity, and p represents an initial displacement parameter introduced to separate continuity conditions from interface boundaries (taking $p \to 0$ in the final steps).

Applying the continuous Fourier transform defined by:

$$f(\beta) = \int_{-\infty}^{\infty} f(x) e^{j\beta x} \, dx \qquad (2)$$

Equation (1) transforms into the ordinary differential equation:

$$\left(\frac{d^2}{dy^2} - \beta^2\right) \phi(\beta, y) = -\frac{1}{\varepsilon_0} \rho(\beta) \delta(y - h - s - p) \qquad (3)$$

2.2 Boundary Conditions and Green's Function:

The general solution to Equation (3) involves linear combinations of $e^{\beta y}$ and $e^{-\beta y}$. In bounded regions (finite ground plane spacing d), boundary and interface conditions across different media interfaces are expressed as:

$$\phi(\beta,0)=0 \quad (4a)$$

$$\phi(\beta,h+0)=\phi(\beta,h-0) \quad (4b)$$

$$\varepsilon_2^*\frac{d}{dy}\phi(\beta,h+0)=\varepsilon_1^*\frac{d}{dy}\phi(\beta,h-0) \quad (4c)$$

$$\phi(\beta,h+s+0)=\phi(\beta,h+s-0) \quad (4d)$$

$$\varepsilon_3^*\frac{d}{dy}\phi(\beta,h+s+0)=\varepsilon_2^*\frac{d}{dy}\phi(\beta,h+s-0) \quad (4e)$$

$$\phi(\beta,h+s+p+0)=\phi(\beta,h+s+p-0) \quad (4f)$$

$$\varepsilon_3^*\frac{d}{dy}\phi(\beta,h+s+p+0)-\varepsilon_3^*\frac{d}{dy}\phi(\beta,h+s+p-0)=-\frac{1}{\varepsilon_0}\rho(\beta) \quad (4g)$$

$$\phi(\beta,h+s+d)=0 \quad (4h)$$

Applying these conditions and letting $p\to 0$, the spectral domain potential at the strip surface $y=h+s$ is solved as:

$$\phi(\beta,h+s)=\frac{1}{\varepsilon_0}\rho(\beta)g(\beta) \quad (5)$$

where $g(\beta)$ represents the Fourier transform of the domain Green's function evaluated at $y=h+s$:

$$g(\beta)=\frac{\varepsilon_1^*\coth(\beta h)+\varepsilon_2^*\coth(\beta s)}{\beta[\varepsilon_1^*\coth(\beta h)(\varepsilon_3^*\coth(\beta d)+\varepsilon_2^*\coth(\beta s))+\varepsilon_2^*(\varepsilon_2^*+\varepsilon_3^*\coth(\beta d)\coth(\beta s))]} \quad (6)$$

In the spatial domain, the equivalent potential distribution is represented by the convolution integral:

$$\phi(x,h+s)=\frac{1}{\varepsilon_0}\int_{-w/2}^{w/2}\rho\,(x')g(x,x';h+s,h+s)\,dx' \quad (7)$$

2.3 Variational Capacitance Formulation:

In the space domain, line capacitance C is governed by:

$$\frac{1}{C}=\frac{1}{Q^2}\int_{-w/2}^{w/2}\rho\,(x)\phi(x,h+s)\,dx \quad (8)$$

where total charge per unit length is $Q=\int_{-w/2}^{w/2}\rho\,(x)\,dx$. Applying Parseval's relation transforms Equation (8) into the spectral representation:

$$\frac{1}{C}=\frac{1}{2\pi Q^2}\int_{-\infty}^{\infty}\rho^*\,(\beta)\phi(\beta,h+s)\,d\beta \quad (9)$$

This transformation converts spatial convolution into a scalar function product, significantly enhancing computational efficiency.

To account for a finite conductor strip thickness t, an averaging parameter $h(\beta)$ is incorporated:

$$\phi(\beta,h+s+t)=\frac{\sinh[\beta(d-t)]}{\sinh[\beta d]}\phi(\beta,h+s) \quad (10)$$

$$h(\beta) = \frac{1}{2}\left[1 + \frac{\sinh[\beta(d-t)]}{\sinh[\beta d]}\right] \quad (11)$$

The final stationary variational formulation for total line capacitance including strip thickness becomes:

$$\frac{1}{C} = \frac{1}{2\pi Q^2 \varepsilon_0} \int_0^\infty [\rho(\beta)]^2 \, g(\beta) h(\beta) \, d\beta \quad (12)$$

Unloaded capacitance C_0 is evaluated by setting relative permittivities $\varepsilon_1^* = \varepsilon_2^* = 1$ and $\varepsilon_3^* = 1$. Due to the variational property of Equation (12), placing an approximate charge distribution $\rho(\beta)$ yields an upper bound for $1/C$ (underestimating true line capacitance C).

3. Computational Results and Comparative Analysis:

Numerical calculations were performed using specific trial charge distributions:

1. **Uniform/Constant Trial Distribution:**

$$\rho(x) = \rho_0, \quad |x| \le \frac{w}{2} \quad (13a)$$

$$\frac{\rho(\beta)}{Q} = \left[\frac{2\sin(\beta w/2)}{\beta w/2}\right] - \left[\frac{2\sin(\beta w/4)}{\beta w/4}\right]^2 \quad (13b)$$

2. **Higher-Order Edge Approximation:**

$$\rho(x) = 1 + \left|\frac{2x}{w}\right|^3, \quad |x| \le \frac{w}{2} \quad (14a)$$

$$\frac{\rho(\beta)}{Q} = \frac{8}{5}\left[\frac{2\sin(\beta w/2)}{\beta w/2}\right] + \frac{12}{5(\beta w/2)^2}\left[\frac{2\sin(\beta w/2)}{\beta w/2}\cos\left(\frac{\beta w}{2}\right) - \left(\frac{2\sin(\beta w/2)}{\beta w/2}\right)^2 + \frac{\sin^2(\beta w/4)}{(\beta w/4)^2}\right] \quad (14b)$$

Theoretical results computed using trial functions (13b) and (14b) exhibit close agreement with classical modified conformal mapping results established by Wheeler (1965) as well as empirical data from Arditi (1955), Dukes (1956), and Hyltin (1965). Computation times on standard legacy platforms (such as the IBM 7094) averaged under 10 seconds per curve run.

4. Galerkin's Method in the Spectral Domain:

To systematically refine charge distribution approximations beyond simple heuristic trial functions, Galerkin's technique (or the Rayleigh-Ritz method) is applied directly in the transform domain (Ward, Mittra, and Itoh, 1971).

Equation (25) is reformulated as:

$$\phi_i(\beta, h+s) + \phi_0(\beta, h+s) = \frac{1}{\varepsilon_0}\rho(\beta)g(\beta) \quad (15)$$

where $\phi_i(\beta, h+s)$ is the known strip potential transform ($|x| < w/2$) and $\phi_0(\beta, h+s)$ represents the unknown potential transform outside the strip region ($|x| > w/2$).

Expanding $\rho(\beta)$ into a finite set of N known basis functions $\rho_n(\beta)$ with unknown expansion coefficients c_n:

$$\rho(\beta) = \sum_{n=1}^{N} c_n \rho_n(\beta) \quad (16)$$

Basis functions $\rho_n(\beta)$ are selected such that their spatial inverse transforms are bounded exclusively within the strip region ($|x| \leq w/2$):

$$\rho_n(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \rho_n(\beta) e^{-j\beta x}\, d\beta = 0, \quad |x| > \frac{w}{2} \quad (17)$$

Taking the inner product of both sides with basic functions $\rho_m(\beta)$ produces the matrix equation system:

$$b_m = \sum_{n=1}^{N} k_{mn}\, c_n, \quad m = 1,2,\ldots,N \quad (18)$$

where:

$$b_m = \int_{-\infty}^{\infty} \rho_m^*(\beta)\phi_i(\beta, h+s)\, d\beta = \frac{1}{2\pi}\int_{-w/2}^{w/2} \rho_m(x)\phi_i(x, h+s)\, dx \quad (19a)$$

$$k_{mn} = \frac{1}{\varepsilon_0}\int_{-\infty}^{\infty} \rho_m^*(\beta) g(\beta)\rho_n(\beta)\, d\beta \quad (19b)$$

By Parseval's relation, the inner product integration eliminates the unknown term $\phi_0(\beta, h+s)$ because its spatial representation is identically zero over the support interval of $\rho_m(x)$. Solving the inverted matrix system for c_n yields the refined line capacitance:

$$C = \frac{1}{V_t}\sum_{n=1}^{N} c_n \int_{-w/2}^{w/2} \rho_n(x)\, dx \quad (20)$$

where V_t is the prescribed potential difference between the strip conductor and ground plane. Increasing the number of expansion terms N monotonically converges the calculated capacitance toward the exact stationary solution.

Conclusion:

The variational method in the Fourier transform domain provides a robust, computationally fast, and highly accurate analytical framework for microstrip transmission lines. By replacing spatial domain integral equations with spectral algebraic products, calculation times are drastically reduced. Furthermore, incorporating Galerkin's expansion allows for systematic convergence to exact characteristic parameters across diverse geometry and dielectric configurations.

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