



# Analysis of Buckling Behavior in Double-walled Cylindrical Shells under Non-uniform Axial Compression

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## Abstract

*The object of this paper is to formulate and solve the differential equations for non-uniform axial compression in the case of a double-walled cylindrical shell. Numerical results are obtained for deflection and are also shown graphically for different values of  $x/a$ . The buckling loads are also figured.*

**Keywords:** Double walled shell, compressive stress, buckling displacement, axial compression, buckling load, buckling deflection.

## 1. Introduction

Flügge [10] examines the extensively reported phenomenon of buckling in cylindrical shells made of isotropic materials according to traditional shell theory. The advancement of sophisticated structural materials has engendered considerable interest in anisotropic cylindrical shells, particularly those fabricated from boron-epoxy composites or reinforced plastic whiskers, for high-performance engineering applications.

Several researchers, including Tasi [13], Cheng and Kuenzi [1], Hess [11], Thielemann, Schnell, and Pitchler [15], Tasi, Fellmann, and Strange [14], Cheng and Ho [2], and Lei and Cheng [12], have undertaken extensive investigations into the buckling behaviour of these anisotropic cylindrical shells. De [14] achieved significant advancements in resolving the buckling issue associated with grid-work shells. This study expands upon and generalizes prior research by examining double-walled cylindrical shells, utilizing the framework provided by Flügge [10].

This section seeks to examine the buckling behaviour of double-walled cylindrical shells subjected to non-uniform axial compression. We employ Flügge's approach [10] to formulate and resolve the governing differential equations for this setup. Formulas for deflection are found, and a graph shows how they change along the  $x$ -axis. A comparison study is conducted, and the dimensionless buckling stresses are graphically represented to facilitate comprehension and assessment.

## 2. Basic equations

The differential equations for double walled shell are given by <sup>[9]</sup>



$$a_1 u'' + \frac{(1-\nu)}{2} a_2 u'' + \frac{(1+\nu)}{2} a_3 v'' + \nu w'' + k_1 \left[ \frac{(1-\nu)}{2} (u'' + w'') \right] \\ - a_4 w''' - a_5 w''' - q_1 (u'' - w') - q_2' - 2q_3 u' = 0 \quad (1-a)$$

$$a_6 u'' + a_7 v'' + a_8 v'' + a_9 w'' + k_1 (a_{10} v'' - a_{11} w'') \\ - a_{12} w''' - a_{13} w''' - q_1 (v'' + w') - q_2 v'' - 2q_3 (v' + w') = 0 \quad (1-b)$$

$$\nu u' + a_9 v' + a_{13} w' + k_1 \left[ 2w'' - \frac{(1+\nu)}{2} v'' + \frac{(1-\nu)}{2} u'' \right] \\ - a_{12} v''' - a_{14} w''' + a_{15} w''' - a_4 u''' + a_{16} w''' \\ - a_5 (u'' + v'') - a_{17} w'' + q_1 (u' - v' + w'') + q_2 w'' \\ - 2q_3 (v' - w') = 0 \quad (1-c)$$

Here,  $x$  denotes the distance of the point under consideration from a datum plane normal to the generators, and  $\phi$  represents the angular distance of the point from a datum generator. The derivatives with respect to the coordinate's  $x/a$  and  $\phi$  will be denoted by **primes** and **dots**, respectively.

$$a \frac{\partial ( )}{\partial x} = ( )', \quad \frac{\partial ( )}{\partial \phi} = ( ) \cdot$$

and

$$a_1 = \frac{D_x}{D}, \quad a_2 = \frac{aD - S}{aD}, \quad a_3 = \frac{aD + S}{aD}, \quad a_4 = \frac{S_x}{aD},$$

$$a_5 = \frac{S}{aD}, \quad a_6 = \frac{1+\nu}{2} + \frac{\nu S}{aD}, \quad a_7 = \frac{D_\phi}{D} + \frac{S_\phi}{aD},$$

$$a_9 = \frac{D_\phi}{D}, \quad a_8 = \frac{1-\nu}{2} + \frac{3(1-\nu)S}{2aD}, \quad a_{10} = \frac{1-\nu}{2} (1 + a^2),$$

$$a_{11} = \frac{1+\nu}{2} + \frac{1-\nu}{2} a^2, \quad a_{12} = \frac{S_\phi}{aD}, \quad a_{13} = \frac{D_\phi}{D} - \frac{S_\phi}{aD},$$

$$a_{14} = \frac{S_\phi}{aD} - \frac{K_\phi}{a^2 D}, \quad a_{15} = \frac{K_\phi}{a^2 D}, \quad a_{16} = \frac{K_x}{a^2 D}, \quad a_{17} = \frac{2\nu S}{aD}$$

$$q_1 = \frac{pa}{D}, \quad q_2 = \frac{P}{D}, \quad q_3 = \frac{T}{D}, \quad k_1 = \frac{K}{a^2 D}.$$

in which

$$D_\phi = \frac{E}{(1-\nu^2)} \int_s dz + E \int_r \frac{b}{b_1} dz, \quad D_x = \frac{E}{(1-\nu^2)} \int_s \left( 1 + \frac{z}{a} \right) dz + E \int_r \frac{b}{b_1} dz$$

$$D = \frac{E}{(1-\nu^2)} \int_s dz, \quad S_\phi = \frac{E}{(1-\nu^2)} \int_s z dz + E \int_r \frac{b}{b_1} z dz$$

$$S_x = \frac{E}{(1-\nu^2)} \int_s \left( 1 + \frac{z}{a} \right) z dz + E \int_r \frac{b}{b_2} z dz, \quad S = \frac{E}{(1-\nu^2)} \int_s z^2 dz$$

$$K_\phi = \frac{E}{(1-\nu^2)} \int_s z^2 dz + E \int_r \frac{b}{b_1} z^2 dz,$$

$$K_x = \frac{E}{(1-\nu^2)} \int_s z^2 dz + E \int_r \frac{b}{b_2} z^2 dz, \quad K = \frac{E}{(1-\nu^2)} \int_s z^2 dz$$

As given in Flügge [10, p. 301], the basic stresses depend on one of the coordinates, either  $x$  or  $\varphi$ . In this study, we consider the case of non-uniform axial compression.

$$N_{xl} = -P = -P_0 - P_1 \cos \varphi \quad (2)$$

When loads corresponding to this equation are applied to the edges of the Fig-5.6, the force  $N_{xl}$  has the same distribution at any cross-section  $x = \text{constant}$  of the shell, while  $N_{\varphi l} = N_{x\varphi l} = 0$ .

Now, we introduce dimensionless load parameters:

$$q_0 = \frac{P_0}{D}, \quad q_1 = \frac{P_1}{D} \quad (3)$$

and we obtain the differential equations of the problem by replacing  $q_2$  in (1) by  $q_0 + q_1 \cos \varphi$  while dropping the original terms with  $q_1$  and  $q_3$ .

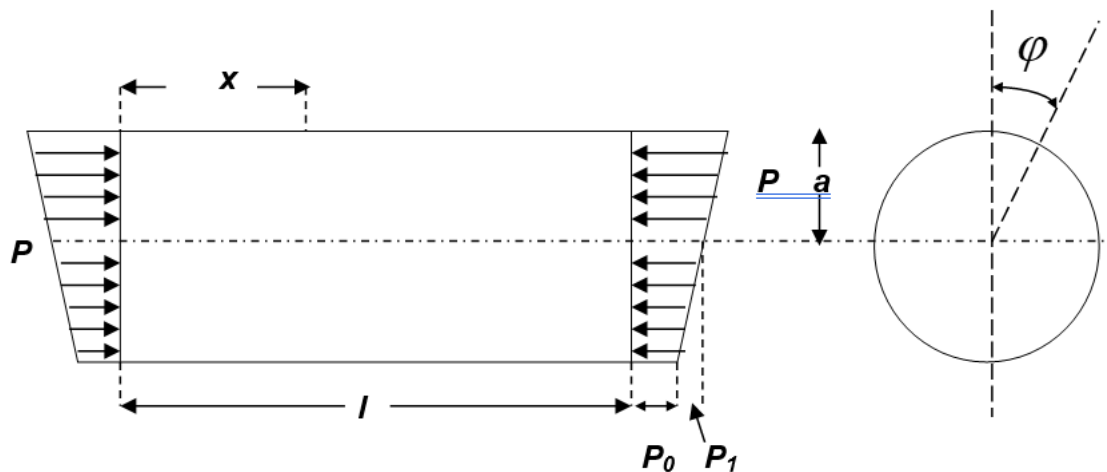


Fig.-1: Cylinder under non-uniform axial compression.

This simple, of course, possible only because we never needed to differentiate any load with respect to  $\varphi$  when deriving (1).

The differential equations thus obtained are

$$\left. \begin{aligned}
 & a_1 u'' + \frac{(1-\nu)}{2} a_2 u'' + \frac{(1+\nu)}{2} a_3 v'' + \nu w' \\
 & + k_1 \left[ \frac{(1-\nu)}{2} (u'' + w'') \right] - a_4 w''' - a_5 w'' - q_0 u'' = q_1 u'' \cos \varphi \\
 & a_6 u' + a_7 v'' + a_8 v'' + a_9 w' + k_1 (a_{10} v'' - a_{11} w'') - a_{12} w''' \\
 & - a_{13} w'' - q_0 v'' = q_1 v'' \cos \varphi \\
 & \nu u' + a_9 v' + a_{13} w + k_1 \left[ 2 w'' - \frac{(1+\nu)}{2} v'' + \frac{(1-\nu)}{2} u'' \right] \\
 & - a_{12} v''' - a_{14} w'' + a_{15} w''' - a_4 u''' + a_{16} w'' - a_5 (u'' + v'') \\
 & - a_{17} w'' + q_0 w'' = -q_1 w'' \cos \varphi
 \end{aligned} \right\} \quad (4)$$

They are still homogeneous in  $u, v, w$  but they contain terms with variable co-efficient.

### 3. Solution of the problem:

Let us put

$$\left. \begin{aligned}
 u &= \cos \frac{\lambda x}{a} \sum_{m=0}^{\infty} A_m \cos (m \varphi) \\
 v &= \sin \frac{\lambda x}{a} \sum_{m=1}^{\infty} B_m \sin (m \varphi) \\
 w &= \sin \frac{\lambda x}{a} \sum_{m=0}^{\infty} C_m \cos (m \varphi)
 \end{aligned} \right\} \quad (5a-c)$$

Upon introducing this into (4) we obtain the following set:

$$\begin{aligned}
 & \sum_{m=0}^{\infty} \left[ A_m \left\{ a_1 \lambda^2 + \frac{(1-\nu)}{2} (a_2 + k_1) m^2 - q_0 \lambda^2 \right\} \right. \\
 & + B_m \left\{ -\frac{(1-\nu)}{2} a_3 \lambda m \right\} + C_m \left\{ -\nu \lambda \right. \\
 & \left. + k_1 \left( \lambda^3 - \frac{(1-\nu)}{2} \lambda m^2 - a_4 \lambda^3 - a_5 \lambda m^2 \right) \right\} \cos (m \varphi) \\
 & = q_1 \lambda^2 \sum_{m=0}^{\infty} A_m \cos (m \varphi) \cos \varphi \\
 & = \frac{1}{2} q_1 \lambda^2 \left[ \sum_{m=0}^{\infty} (A_{m-1} + A_{m+1}) \cos (m \varphi) + A_0 \cos \varphi \right]
 \end{aligned} \quad (6)$$



$$\begin{aligned} & \sum_{m=1}^{\infty} [A_m \{-a_6 \lambda m\} + B_m \{a_7 m^2 + a_8 \lambda^2 + a_{10} k_1 \lambda^2 + q_0 \lambda^2\} \\ & + C_m \{a_9 m + a_{11} \lambda^2 m + a_{12} m^3 + a_5 \lambda^2 m\}] \sin(m\varphi) \\ & = q_1 \lambda^2 \sum_{m=1}^{\infty} B_m \sin(m\varphi) \cos\varphi \\ & = \frac{1}{2} q_1 \lambda^2 \left[ \sum_{m=1}^{\infty} (B_{m-1} + B_{m+1}) \sin(m\varphi) \right] \end{aligned} \quad (7)$$

$$\begin{aligned} & \sum_{m=0}^{\infty} \left\{ A_m \left\{ -\nu \lambda + \frac{(1-\nu)}{2} k_1 \lambda m^2 - a_4 \lambda^3 - a_5 \lambda m^2 \right\} \right. \\ & + B_m \left\{ a_9 m + \frac{(1+\nu)}{2} k_1 \lambda^2 m + a_{12} m^3 + a_5 \lambda^2 m \right\} \\ & + C_m \left\{ a_{13} + 2k_1 \lambda^2 m^2 + 2a_{14} m^2 a_{15} m^4 + a_{16} \lambda^4 + a_{17} \lambda^2 - q_0 \lambda^2 \right\} \left. \right\} \cos(m\varphi) \\ & = q_1 \lambda^2 \sum_{m=0}^{\infty} C_m \cos(m\varphi) \cos\varphi \\ & = \frac{1}{2} q_1 \lambda^2 \left[ \sum_{m=0}^{\infty} (C_{m-1} + C_{m+1}) \cos(m\varphi) + C_0 \cos\varphi \right] \end{aligned} \quad (8)$$

In the second version of the right-hand sides of these equations, it is understood that  $A_{-1} = B_0 = C_{-1} = 0$ . When this second version is used, each of the equations states that two Fourier series in  $\varphi$  must be identically equal. Such an outcome is possible only if their coefficients are the same, and thus each of the three differential equations yields an infinite number of linear algebraic equations for the coefficients  $A_m$ ,  $B_m$ , and  $C_m$ . There are three equations corresponding to each integer  $m$ , and the general triplet is as follows:

$$\left. \begin{aligned} & A_{m-1} \left\{ -\frac{1}{2} q_1 \lambda^2 \right\} + A_m \left\{ a_1 \lambda^2 + \frac{(1-\nu)}{2} (a_2 + k_1) m^2 - q_0 \lambda^2 \right\} \\ & + B_m \left\{ -\frac{(1-\nu)}{2} a_3 \lambda m \right\} + C_m \left\{ -\nu \lambda + k_1 \left( \lambda^3 - \frac{(1-\nu)}{2} \lambda m^2 \right. \right. \\ & \left. \left. - a_4 \lambda^3 - a_5 \lambda m^2 \right) \right\} + A_{m+1} \left\{ -\frac{1}{2} q_1 \lambda^2 \right\} = 0 \\ & B_{m-1} \left\{ -\frac{1}{2} q_1 \lambda^2 \right\} + A_m \{-a_6 \lambda m\} + B_m \{a_7 m^2 + a_8 \lambda^2 \\ & + a_{10} k_1 \lambda^2 + q_0 \lambda^2\} + C_m \{a_9 m + a_{11} \lambda^2 m + a_{12} m^3 + a_5 \lambda^2 m\} \\ & + B_{m+1} \left\{ -\frac{1}{2} q_1 \lambda^2 \right\} = 0 \\ & C_{m-1} \left\{ -\frac{1}{2} q_1 \lambda^2 \right\} + A_m \left\{ -\nu \lambda + \frac{(1-\nu)}{2} k_1 \lambda m^2 - a_4 \lambda^3 - a_5 \lambda m^2 \right\} \\ & + B_m \left\{ a_9 m + \frac{(1+\nu)}{2} k_1 \lambda^2 m + a_{12} m^3 + a_5 \lambda^2 m \right\} \\ & + C_m \left\{ a_{13} + 2k_1 \lambda^2 m^2 + 2a_{14} m^2 + a_{15} m^4 + a_{16} \lambda^4 \right\} \\ & + a_{17} \lambda^2 - q_0 \lambda^2 \left. \right\} + C_{m+1} \left\{ -\frac{1}{2} q_1 \lambda^2 \right\} = 0 \end{aligned} \right] \quad (9)$$



We first introduce an abbreviated notation writing (9) in the following form:

$$\left. \begin{aligned} b_{11,m} A_m + b_{12,m} B_m + b_{13,m} C_m &= \frac{1}{2} q_1 \lambda^2 (A_{m+1} + A_{m-1}) \\ b_{21,m} A_m + b_{22,m} B_m + b_{23,m} C_m &= \frac{1}{2} q_1 \lambda^2 (B_{m+1} + B_{m-1}) \\ b_{31,m} A_m + b_{32,m} B_m + b_{33,m} C_m &= \frac{1}{2} q_1 \lambda^2 (C_{m+1} + C_{m-1}) \end{aligned} \right\} \quad (10)$$

where

$$\begin{aligned} b_{11,m} &= a_1 \lambda^2 + \frac{1-\nu}{2} (a_2 + k_1) m^2 - q_0 \lambda^2 \\ b_{12,m} &= -\frac{1+\nu}{2} a_3 \lambda m \\ b_{13,m} &= -\nu \lambda + k_1 \left( \lambda^3 - \frac{1-\nu}{2} \lambda m^2 - a_4 \lambda^3 - a_5 \lambda m^2 \right) \\ b_{21,m} &= -a_6 \lambda m \\ b_{22,m} &= a_7 m^2 + a_8 \lambda^2 + a_{10} k_1 \lambda^2 + q_0 \lambda^2 \\ b_{23,m} &= a_9 m + a_{11} \lambda^2 m + a_{12} m^3 + a_5 \lambda^2 m \\ b_{31,m} &= -\nu \lambda + \frac{1-\nu}{2} k_1 \lambda m^2 - a_4 \lambda^3 - a_5 \lambda m^2 \\ b_{32,m} &= a_9 m + \frac{1+\nu}{2} k_1 \lambda^2 m + a_{12} m^3 + a_5 \lambda^2 m \\ b_{33,m} &= a_{13} + 2 k_1 \lambda^2 m^2 + 2 a_{14} m^2 + a_{15} m^4 \\ &\quad + a_{16} \lambda^4 + a_{17} \lambda^2 - q_0 \lambda^2 \end{aligned}$$

We then use the first two of these equations to express  $A_m$  and  $B_m$  in terms of  $C_m$  and of  $A_{m-1} \dots B_{m+1}$ . This involves certain determinants of the co-efficient  $b_{11,m} \dots$  etc., and for brevity we shall put the subscript  $m$  only at the end of these determinants and not at all the individual co-efficient.



$$A_m = \alpha_m \left[ s_{1m} C_m + \frac{1}{2} q_1 \lambda^2 \{ b_{22,m} (A_{m-1} + A_{m+1}) - b_{12,m} (B_{m-1} + B_{m+1}) \} \right]$$

$$B_m = \alpha_m \left[ s_{2m} C_m + \frac{1}{2} q_1 \lambda^2 \{ b_{11,m} (B_{m-1} + B_{m+1}) - b_{21,m} (A_{m-1} + A_{m+1}) \} \right]$$

where

$$\alpha_m = \frac{1}{(b_{11}b_{22} - b_{12}b_{21})_m}$$

$$s_{1m} = (b_{12}b_{23} - b_{13}b_{22})_m$$

$$s_{2m} = (b_{21}b_{13} - b_{11}b_{23})_m$$

These equations are used first to eliminate  $A_m$  and  $B_m$  from the third equation of (10). This introduces three terms with  $A_{m-1}$ ,  $A_{m+1}$ ,  $B_{m-1}$ ,  $B_{m+1}$ . The same two equations may now be used again to eliminate these terms. This introduces new terms with  $A_{m-2}$ ,  $A_m$ , ..., but all these terms carry a factor  $q_1^2$  and may be neglected. We thus arrive at the following equation:

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}_m \cdot C_m - \frac{1}{2} q_1 \lambda^2 [\dots C_{m-1} + \dots C_{m+1}] = 0 \quad (11)$$

The co-efficient of  $C_{m-1}$  and  $C_{m+1}$  which have been indicated by a row of dots, are rather lengthy expressions of determinants of  $b_{11}$ , ...,  $b_{33}$ . If they are worked out in detail, it is found that they increase as  $m^4$ , while the co-efficient of  $C_m$  has the form  $m^8 + m^6 + \dots$ . Therefore, if we divide each equation by  $m^4$  and use  $m^4 C_m$  as an unknown, the determinant will fulfill the convergence requirement. It follows that the infinite determinant of (11) must vanish and that this condition is fulfilled with increasing degrees of approximation if finite systems of increasing size of the determinants are set equal to zero. Since the co-efficient of (11) contain the load parameters  $q_0$  and  $q_1$ , the vanishing of the determinant is a relation between these two quantities so that we may assume values of  $q_0$  and calculate from the determinantal equation the corresponding values of  $q_1$  or vice versa. The numerical works may be done in different ways. One may work out the expressions which were indicated in (11) by dots and so obtain the elements of the determinant in general form, or one may write a sufficient segment of (9), introducing numerical values for everything but  $q_1$ , and then perform numerically the elimination of  $A_m$  and  $B_m$  as described; or one may just use (9) as they stand. In each case, one has to solve the eigenvalue problem of a large determinant, and this again can be done in several ways. One may expand the determinant and solve the ensuing algebraic equation, or one may fix a value of  $q_0$  and find by trial an error in the value of  $q_1$  which makes the determinant vanish, or one may use the method of matrix iteration. After a pair  $q_0$ ,  $q_1$  has been found; (11) may be solved for the constants  $C_m$ , which, as in all buckling problems, are determined except for a common factor.

#### 4. Numerical results

We assume <sup>[10]</sup>

$$t_1 = 3 \text{ cm}, \quad t_2 = 2 \text{ cm}, \quad t = t_1 + 2t_2 = 7 \text{ cm},$$

$$k_1 = 10^{-6}, \quad a = 50 \text{ cm}, \quad b = 2 \text{ cm}, \quad b_1 = b_2 = 10 \text{ cm},$$

$$E = 1.28 \times 10^6 \text{ psi}, \quad \nu = 0.378$$

For a shell with  $\frac{l}{a} = \pi, \frac{x}{a} = \frac{\pi}{2}$  it was found that  $q_0 = 0.001, q_1 = 0.049$  represent a buckling load. Solving

(11) and introducing the values  $C_m$  thus obtained into (5-c) yield the following deflection:

$$w = \sin \frac{x}{a} (-0.23559 \cos \varphi + 0.42093 \cos 2\varphi + 0.48630 \cos 3\varphi + 0.66180 \cos 4\varphi - 0.00055 \cos 5\varphi + \dots) \times 10^8$$

This suggests that it would be sufficient to use only a four-by-four determinants made up of the co-efficient of  $C_1, C_2, C_3, C_4$  in four equations (11) for which  $m=1, 2, 3, 4$ .

The buckling displacement as computed from the formula just given is shown in Fig-2 when  $x/a = \pi/2, \pi/3, \pi/4$  and  $\pi/6$ . Although the deflection is not exactly zero in the tensile zone of the shell, it is there exceedingly small, and the largest deflection occurs where  $N_{x1}$  has its largest negative value. For the same shell a number of critical pair's  $q_0, q_1$  have been computed, and the result of this computation is shown in Fig-3. When the cylinder is considered as a tubular bar subjected to an eccentric axial force, then  $q_0$  represents the direct stress and  $q_1$  the bending stress. The ratio of both has been chosen as the abscissa of Fig-3, while the ordinate  $q_0+q_1$  represent the greatest compressive stress. The diagram shows that the latter does not vary much depend on the former and that it increases as the compressive zone of the cylinder decreases in width.

The plane  $\varphi = 0, \varphi = \pi$  is a plane of symmetry for the basic stress pattern (2). From Fig-2 it may be seen that buckling deformation has the same symmetry. This is a necessary consequence of the form (5) which we adopted for the solution of (4). There exists, however, another solution of these equations, which is anti-symmetric with respect to the same plane.

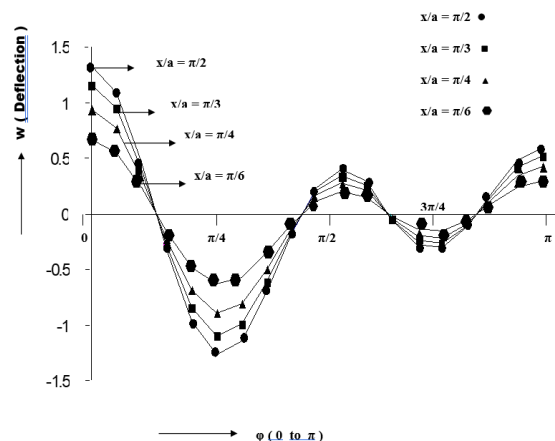


Fig.-2 - Buckling deflection of a cylinder under non-uniform axial compression for different values of  $x/a$ .

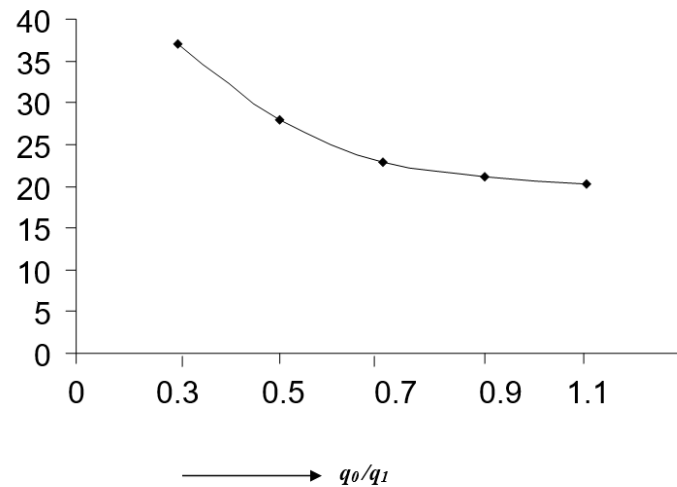


Fig.-3: Dimensionless buckling loads for a cylinder with  $l/a = \pi$ ,  $k_1 = 10^{-6}$ ,  $\nu = 0.378$ .

#### 4. Conclusion

From Fig. 2, we observe that the deflection starts from a maximum positive value, gradually decreases to zero, and then continues to decrease, reaching a minimum negative value. Subsequently, it increases from this minimum, passes through zero again, and rises to a maximum positive value. In other words, the deflection exhibits an oscillatory behavior as it tends toward zero.

As  $x/a$  decreases, the corresponding deflection also gradually decreases. From Fig. 3, we observe that the buckling loads decrease slowly and progressively.

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