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QUEUEING MODEL WITH FUZZY ARRIVAL AND DETERMINISTIC SERVICE PATTERN

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ABSTRACT

In the present communication, a methodology for designing Queue system performance has been proposed in which arrival rates of customers are supposed in fuzzy parameters and service rate is not a random variable but is deterministic value with a single service channel.The α-cut approach and fuzzy arithmetic operationsare used to construct system characteristic membership function. The model finds its application in various real events such as at petrol pump filling station, in general National level polling station, in cinema hall and various other practical situations. The validity of the designed model has been presented through numerical illustrations and graphs.

***Keywords**: Triangular Fuzzy number, Trapezoidal Fuzzy number, steady state, α-cut etc.*

INTRODUCTION

Many researchers have studied a queue model M/D/1/∞/FIFO in which the arrivals follow Poisson pattern and deterministic service rate with single service channel.But in real world situation, for example in a polling station where the persons cast their vote to elect their representatives through ballot box or voting machine, arrival patterns are generally seen uncertain. The arrivals may be slow, fast, very fast depending upon the situation and environment. the arrivals of persons on a booth appear fuzzyin nature i.e. totally uncertain. Mathematically the uncertainty can be resolved using fuzzy set .Our modelling realise of basic assumption of queueing theory that the line formed when the service rate being provided cannot keep pace with arrivals. In every polling station the voters are turn out certain coming with different rate and checking and verification identity ballot all these take a fixed time.

The classical queue model can be applied better in more be fitting situation if it is expanded using fuzzy parameters. Li and Lee[1989],Negi and Lee[1992], Kao[1999], Chen[1985][1995][2005][2006], Singh T.P.[2013] have highlighted fuzzy queue model using Zadeh[1965] extension principle and α-cut. This paper is further an extension of a classical queue model by applying α-cut technique and fuzzy arithmetic operations. The fuzzy parameters are considered in triangular as well as Trapezoidal fuzzy number. The validity of the proposed model has been presented through numerical example and graphs.

PRELIMANIRIES:

Fuzzy Set:

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In the universe of discourse X, a fuzzy subset A on X is defined by the membership function μ_A (X) which maps each element x into X to a real number in the interval [0,1]. μ_A(X) Denotes the grade or degree of membership and it is usually expressed as

$$\mu_A(X): X \rightarrow [0,1]$$

α – Cut:

If a fuzzy set Ãis defined on X, for any $\alpha \in [0,1]$, the α-cuts ${}^{\alpha}A$ is represented by the following crisp set,

Strong α-cuts: ${}^{\alpha+}A = \{x \in X \,/\, \mu_A(x) > \alpha; \alpha \in [0,1]\}$

Weak α-cuts: ${}^{\alpha}A = \{x \in X \,/\, \mu_A(x) \geq \alpha\}; \alpha \in [0,1]\}$

Hence, the fuzzy set Ã can be treated as crisp set ${}^{\alpha}A$ in which all the members have their membership values greater than or at least equal to α. It is one of the most important concepts in fuzzy set theory.

Triangular Fuzzy Number:

A triangular fuzzy number A= (a_1, a_2, a_3,) is defined by the membership function as

$$\mu_{(A)}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \end{cases}$$

Trapezoidal Fuzzy Number:

A trapezoidal fuzzy number A= (a_1, a_2, a_3, a_4) is defined by the membership function as

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & x > a_4 \end{cases} \quad \square$$

Operation of Interval

Operation of fuzzy number can be generalized from that of crisp interval. Let us have a look at theoperations of interval.

∀a1, a3, b1, b3 ∈ R

A = [a1, a3], B = [b1, b3]

Assuming A and B as numbers expressed as interval, main operations of interval are

i) Addition

[a1, a3] (+) [b1, b3] = [a1 + b1, a3 + b3]

ii) Subtraction

[a1, a3] (ñ) [b1, b3] = [a1 ñ b3, a3 ñ b1]

iii) Multiplication

[a1, a3] (•) [b1, b3] = [a1 • b1 ∧ a1 • b3 ∧ a3 • b1 ∧ a3 • b3, a1 • b1 ∨ a1 • b3 ∨ a3 • b1 ∨ a3 • b3]

iv) Division

[a1, a3] (/) [b1, b3] = [a1 / b1 ∧ a1 / b3 ∧ a3 / b1 ∧ a3 / b3, a1 / b1 ∨ a1 / b3 ∨ a3 / b1 ∨ a3 / b3]

excluding the case b1 = 0 or b3 = 0

MODEL FORMULATION

In classical queue systems where service time isconstant, the M/D/1 Queue Model are used. M/D/1 indicates a system where arrivals are a Poisson process with parameter λ, Service time is deterministic or constant and there is one server.

Average number of customers in this system are given by

$L_s=(2\lambda\mu+\lambda^2)/(2\mu(\mu-\lambda))$

Average number of customers in this system are given by

$L_q=\lambda^2/(2\mu(\mu-\lambda))$.

Average waiting time of a customer in this system is given by

$W_s=(2\mu+\lambda)/(2\mu(\mu-\lambda))$.

Average waiting time of a customer in the queue is given by

$W_q=\lambda/(2\mu(\mu-\lambda))$.

For example Let A be a fuzzy set whose membership function is given by

$$\mu_A(x)=\begin{Bmatrix}\dfrac{x-a}{b-a}, a\le x\le b\\ \dfrac{c-x}{c-b}, b\le x\le c\end{Bmatrix}$$

To find α-cut of A, we first set α € [0,1] to both left and right reference functions of A.

That is $\alpha=\dfrac{x-a}{b-a}$ and $\alpha=\dfrac{c-x}{c-b}$.

Expressing x in terms of α, we have x = (b-a) α + a and x = c – (c-b) α

Which gives the α-cut of A is

α_A = [(b-a) α + a, c- (c-b)α].

FUZZIFICATION OF THE MODEL

In real live event λ is approximately known and μ is fixed. λ is represented by fuzzy number λ.

$\tilde{\lambda}(\alpha) = [\tilde{\lambda}_1(\alpha), \tilde{\lambda}_2(\alpha)]$, μ = constant

Expected line length can be written as

$\tilde{L}_s(\alpha) = [\tilde{L}_{S,lower}(\alpha), \tilde{L}_{S,upper}(\alpha)]$,

Similar for

$\tilde{L}_q(\alpha) = [\tilde{L}_{q,lower}(\alpha), \tilde{L}_{q,upper}(\alpha)]$,

$\tilde{W}_s(\alpha) = [\tilde{W}_{S,lower}(\alpha), \tilde{W}_{S,upper}(\alpha)]$,

$\tilde{W}_q(\alpha) = [\tilde{W}_{q,lower}(\alpha), \tilde{W}_{q,upper}(\alpha)]$,

Case I for triangular Fuzzy parameters

$\tilde{\lambda} = (10,12,13)$, μ = 14

$\tilde{\lambda}(\alpha) = [2\alpha + 10, 13 - \alpha]$

$$\tilde{L}_s(\alpha) = \left[\frac{-\alpha^2 + 82\alpha + 111}{112 - 56\alpha}, \frac{-4\alpha^2 - 68\alpha + 264}{28 + 28\alpha}\right]$$

$$\tilde{W}_s(\alpha) = \left[\frac{15 + \alpha}{112 - 56\alpha}, \frac{18 - 2\alpha}{28 + 28\alpha}\right]$$

CONCLUSION

We have discussed queue system with impatient customers and developed the steady state probability equations. The Matrix form of the solution has been derived. We formulate a cost model to determine the optional service rate and total expected cost of the system per unit time. Although this function is too complicated to derived the explicit expression for optimal service rate, even than we have made an attempt to evaluate numerically to performance measures & the optimal service rate for the system

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