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The Use of Fixed-Point Iterative Methods in the Solving of Problems in Two and Three Dimensions

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Abstract:

All things considered in science, the issue of tending to a game plan of conditions can be changed over to the issue of choosing the fixed points of a self-arranging X on a reasonable space Y. The recently fixed point theoremin topological fixed point speculation was exhibited by Brouwer in 1912 which confirms that a determined arranging M of the shut unit ball in Rn has somewhere near one fixed point, that is to say, a point n with the ultimate objective that M(y) = y. The BFP theorem is a critical fixed point theorem that applies to restricted layered spaces and designs the justification behind a couple of general fixed point theorems. There exists an enormous composition on the point field and this is extraordinarily unique field of investigation at this point of time. Fixed point theorems are imperative instruments for exhibiting the presence and uniqueness of the responsesfor various mathematical models like crucial and fragmented conditions, variation lopsided characteristics, etc. Itmight be applied to, for example, variation irregular characteristics, progression, and gauge theory. The fixed point speculation has been continually focused by various experts to take care of different mathematical examination issues in the field of computational programming. It was Badenovi.S.K. who noticed that the contractive mode like conditions are altogether

fundamental in the examination of fixed point speculation. In thefirst half of the paper, we present a couple of documentations and definitions related to 2D and in next half, dealing with application of FPT that will be used in the continuation and application of fixed point in 3D. On account of its conceivability and advantage, the previous theorem has turned into an especially celebrated and popular gadget in dealing with the presence issues in various pieces of mathematical examination. A comparativestudy between the two cases discussed above for the different initial values, it is concluded that the series of iteration can me made to converge at a faster rate if we choose the initial point to be close to the fixed point in case of both 2D and 3D equations.

Keywords : *Topological, speculation, BFP theorem , enormous, imperative, fragmented, lopsided, progression, computational, application*

1. Introduction:

The most ordinary ways to deal with exhibit that a condition has a response is to act it like a fixed point issue, or at least, to find an ability so much that n is a response of the situation if g(y) = y. Then, the fixed point theorems can be used to show that f has a fixed point. It will in general be contemplated that the issue of finding the plan of a situation G(y) = 0 is identical to finding the fixed point of a limit g(y), where g(y) = G(y) + y. In any case, the outcomes of fixed point speculation are much of the time useless, or at least, they

guarantee that a fixed point exists anyway don't help in finding the fixed point. All things considered in science, the issue of tending to a game plan of conditions can be changed over to the issue of choosing the fixed points of a self-arranging X on a reasonable space Y. The recently fixed point theorem in topological fixed point speculation was exhibited by Brouwer in 1912 which confirms that a determined arranging M of the shut unit ball in Rn has somewhere near one fixed point, that is to say, a point n with the ultimate objectivethat M(y) = y. The BFP theorem is a critical fixed point theorem that applies to restricted layered spaces anddesigns the justification behind a couple of general fixed point theorems. It is named after Dutch mathematician L.Brouwer. A while later, various journalists extended the L.Brouwer fixed point theorem whose lovely outline can be found in the review article given by one of the great mathematician Park. After the presence of BFP theorem and its theories, fixed point speculation

transformed into an effective device inhandling various issues of applied science.

2. Review of Literature:

Fixed point speculation has charmed various researchers since mid of twentieth century and from the timeonwards resulted with the noticeable contribution of fixed point theorem and its applications in different fields.There exists an enormous composition on the point field and this is extraordinarily unique field of investigation at this point of time. Fixed point theorems are imperative instruments for exhibiting the presenceand uniqueness of the responses for various mathematical models like crucial and fragmented conditions, variation lopsided characteristics, etc. It might be applied to, for example, variation irregular characteristics, progression, and gauge theory. The fixed point speculation has been continually focused by various experts to take care of different mathematical examination issues in the field of computational programming. It was Badenovi.S.K. [1] who noticed that the contractive mode like conditions are altogether fundamental in the examination of fixed point speculation. The really critical result on fixed points for contractive mode like mappings was the remarkable Banaach-Cacciopoli theorem which was circulated in around the year 1922. Later in the year 1968, P.Kannan thought about one more sort of contractive mappings for much better application. From there on and later, there have been various results related to mapping methods which are satisfying the various kinds of contractive type irregular characteristics. Lately, K.Samet and Rhoades B. [2] introduced one more order of contractive-type mappings known as β-ϕ contractive sort mappings in year 1978[3]. The results obtained by him are as an expanded substance and summarized the ongoing fixed point whichachieves the composition, explicitly the Banaach narrowing upto a sensible norm. By applying our gained results, we furthermore ensure the fixed point theorems in deficiently mentioned the absolute estimation of spaces and gave the applications of FPT to the normal differential conditions. In the first half of the paper, wepresent a couple of documentations and definitions related to 2D and in next half, dealing with application ofFPT that will be used in the continuation and application of fixed point in 3D. On account of its conceivability and advantage, the previous theorem has turned into an especially celebrated and popular gadget in dealing with the presence issues in various pieces of mathematical examination. Various mathematicians widened the Banaach tightening standard in two critical ways, one by communicating the circumstances on the arranging M and second by tolerating the set Y as more expansive plan as given in the work of P.B. Singh and

Sahni.R.L.[4]

3. Application of Fixed point iterative method in case of two and three dimension problems:

Fixed point analysis strategy are the iterative methods which are acquainted with the degree that the fixed point theorem can be reached out for tracking down a course of the arrangement for non-linear conditions inthe event of two and three dimensional issues. We want to have a technique for tracking down an answer forthe arrangement of non-linear problems for getting the desired solution.

$g_1(x,y) = 0$

$g_2(x,y) = 0$ _____ (1)

and $g_1(x,y,z) = 0$

$g_2(x,y,z) = 0$ ..... (2)

$g_3(x,y,z) = 0$

Each of the above two given equations are illustrating a definite curve existing in the plane and the objectiveis to search for the point of intersection for these curves and that common point is termed as the Fixed Point.

4. Definitions:

4.1 Definition of fixed point in 2 dimension space :

For the given system of equations, a given point is called as fixed point ifFor the 2 equations in general form given below:

$x = g_1(x,y)$ _____(3)

$y = g_2(x,y)$

Here, fixed point is a point C(c,d) such that $c = g_1(c,d)$ and $d = g_2(c,d)$

4.2 Definition of fixed point in 3 dimension space :

For the given system of equations, a given point is called as fixed point ifFor the 3 equations in

general form given below:

$x = g_1(x,y,z)$

$y = g_2(x,y,z)$ _____ (4)

$z = g_3(x,y,z)$

Here , fixed point is a point P(c,d,e) such that

$c = g_1(c,d,e)$, $d = g_2(c,d,e)$ and $e = g_3(c,d,e)$

4.3 Definition of fixed point iterative technique in 2 dimension space :

For the given system of equations, a fixed point iterative technique for two dimension case can be defined

as:

$x_{i+1} = g_1(x_i, y_i)$ _____(5)

$y_{i+1} = g_2(x_i, y_i)$

for values of i = 0,1,2,3,……..

For the above set of eq. (5). (x_i, y_i) will be termed as Fixed Point.

4.4 Definition of fixed point iterative technique in 3 dimension space :

For the given system of equations, a fixed point iterative technique for two dimension case can be defined

as:

$x_{i+1} = g_1(x_i, y_{i,}, z_i)$

$y_{i+1} = g_2(x_i, y_{i,}, z_i)$ _____ (6)

$z_{i+1} = g_3(x_i, y_{i,}, z_i)$

for values of i = 0,1,2,3,……..

For the above set of eq. (5). (x_i, y_i , z_i) will be termed as Fixed Point.

5. Theorem :

Suppose that in the region or space containing fixed point in 2D as (x,y) or in 3D as (x,y,z) and

the condition is satisfied that the function and its all partial derivative must be continuous in that particularregion around fixed point for it. In the event of above said conditions being satisfied, if we select the initialpoint for the iteration very close to the obtained fixed point, then the either of the following cases shall be applicable for giving the condition of convergence of the iterative results in both two dimension and threedimension problems:

5.1 Case-1: In case of 2D functions

If (x_0, y_0) is chosen as very close to fixed point (c, d) and if the following condition is satisfied :

$$\left|\frac{\partial g1}{\partial x}\right|_{(c,d)} + \left|\frac{\partial g1}{\partial y}\right|_{(c,d)} < k$$

$$\left|\frac{\partial g2}{\partial x}\right|_{(c,d)} + \left|\frac{\partial g2}{\partial y}\right|_{(c,d)} < k$$

Here, $0 < k < 1$, and the satisfaction of above two given conditions, the used iteration will finallyconverge to the fixed point (c,d).

5.2 Case-2: In case of 3D functions

If (x_0, y_0 , z_0) is chosen as very close to fixed point (c, d, e) and if the following condition is satisfied :

$$\left|\frac{\partial g1}{\partial x}\right|_{(c,d,e)} + \left|\frac{\partial g1}{\partial y}\right|_{(c,d,e)} + \left|\frac{\partial g1}{\partial z}\right|_{(c,d,e)} < k$$

$$\left|\frac{\partial g2}{\partial x}\right|_{(c,d,e)} + \left|\frac{\partial g2}{\partial y}\right|_{(c,d,e)} + \left|\frac{\partial g2}{\partial z}\right|_{(c,d,e)} < k$$

$$\left|\frac{\partial g3}{\partial x}\right|_{(c,d,e)} + \left|\frac{\partial g3}{\partial y}\right|_{(c,d,e)} + \left|\frac{\partial g3}{\partial z}\right|_{(c,d,e)} < k$$

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Here, $0 < k < 1$, and the satisfaction of above two given conditions, the used iteration will finallyconverge to the fixed point (c,d,e).

6. Application to the problem :

Solve the following set of 3 simultaneous linear equations by using fixed point iterative technique:

$$k_1 P + l_1 Q + m_1 R + n_1 = 0$$
$$k_2 P + l_2 Q + m_2 R + n_2 = 0 \quad \text{(7)}$$
$$k_3 P + l_3 Q + m_3 R + n_3 = 0$$

In order to get the idea of rate of convergence, we will discuss here two cases by taking different values of k , l

, m and n

Case 1 The values as

$k_1 = 6.0$, $l_1 = 1.0$, $m_1 = 1.0$ and $n_1 = -104.0$

$k_2 = 5.0$, $l_2 = 4.0$, $m_2 = -9.0$ and $n_2 = -154.0$

$k_3 = 4.0$, $l_3 = 7.0$, $m_3 = 3.0$ and $n_3 = -65.0$

On putting the above written values in equation (7), we get

$$6.0\ P + 1.0\ Q + 1.0\ R - 104.0 = 0$$
$$5.0\ P + 4.0\ Q - 9.0\ R - 154.0 = 0 \quad \text{(7)}$$
$$4.0\ P + 7.0\ Q + 3.0\ R - 65.0 = 0$$

Now, applying the fixed point iterative method and approaching for the next value as:

$$P_{i+1} = \frac{104 - 0 . Pi - Q\ i - R\ i}{6}$$

$$Q_{i+1} = \frac{154 - 5 . Pi - 0 . Q\ i + 9\ R\ i}{4}$$

$$R_{i+1} = \frac{65 - 4 . Pi - 7\ Q\ i - R\ i}{3}$$

To initialize the iteration, take $P_0 = 11$, $Q_0 = 12$ and $Z_0 = 13$, to get the following table as result

Value of i	Pi	Qi	Ri
0	11.0000	12.0000	13.0000
1	13.2121	9.1212	3.8484
2	15.3223	11.2354	3.7484

3	14.9987	10.2684	5.6984
4	14.8755	9.7845	5.1021
5	15.0245	10.0254	4.8154
6	15.0235	10.5248	5.0214
7	14.9857	9.9725	5.0231
8	14.9957	9.9945	4.9874
9	15.0037	10.0082	4.9965
10	14.9995	9.9995	5.0021
11	14.9993	9.9987	4.9978
12	15.0003	10.0045	4.9954
13	15.0005	10.0009	5.0032
14	14.9992	9.9997	5.0012
15	15.0002	10.0004	4.9989
16	15.0001	10.0001	5.0001
17	15.0000	9.9898	5.0002
18	15.0000	9.9998	4.9999
19	15.0001	10.0001	5.0000
20	15.0000	10.0000	5.0000
21	15.0000	10.0000	5.0000
22	15.0000	10.0000	5.0000
23	15.0000	10.0000	5.0000
24	15.0000	10.0000	5.0000

Here , after using the fixed point iteration theorem we get the value of (Pi , Qi , Ri) converges to a fixedpoint after 19^{th} iteration.

Case 2 The values as

$k_1 = 9.0$, $l_1 = -1.0$, $m_1 = -3.0$ and $n_1 = -204.0$

$k_2 = 2.0$, $l_2 = 3.0$, $m_2 = -10.0$ and $n_2 = 125.0$

$k_3 = 2.0$, $l_3 = -2.0$, $m_3 = 5.0$ and $n_3 = -154.0$

On putting the above written values in equation (7), we get

$$9.0\ P - 1.0\ Q - 3.0\ R - 204.0 = 0$$
$$2.0\ P + 3.0\ Q - 10.0\ R + 125.0 = 0 \quad (7)$$
$$2.0\ P - 2.0\ Q + 5.0\ R - 154.0 = 0$$

Now, applying the fixed point iterative method and approaching for the next value as:

$$P_{i+1} = \frac{204 - 0.Pi + Q\,i + 3R\,i}{9}$$

$$Q_{i+1} = \frac{-125 - 2.Pi - 0.Q\,i + 10.R\,i}{3}$$

$$R_{i+1} = \frac{154 - 2.Pi + 2Q\,i - 0.R\,i}{5}$$

As calculated in the above table 1 , similarly we can calculate the iterations results in this case too by initializing the iteration at $P_0 = 1$, $Q_0 = 2$ and $Z_0 = 3$, and will find out that the iteration is tself converging atfixed point after 17^{th} iteration only .

7. Conclusion:

A comparative study between the two cases discussed above for the different initial values, it is concluded that the series of iteration can me made to converge at a faster rate if we choose the initial point to be close tothe fixed point in case of both 2D and 3D equations.

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