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A STUDY OF NUMERICAL SOLUTIONS USING WAVELET METHODOLOGY FOR INTEGRAL AND DIFFERENTIAL EQUATIONS IN DIFFERENT DOMAINS

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1. INTRODUCTION

Many branches of science and engineering rely on integral and differential equations, which are fundamental in the expansive field of mathematical modeling. From heat transfer to population dynamics, and from fluid dynamics to quantum mechanics, these equations explain a multitude of phenomena that are governed by physical systems. Despite the usefulness of these equations as a tool for understanding complex systems, analytical solutions may be difficult to obtain, particularly for nonlinear, high-dimensional, or specified on irregular domains systems. Numerical techniques are often required to estimate solutions to these equations because to their complexity. These solutions may then be used for simulation, prediction, and decision-making in real-world applications. A strong and effective strategy for solving integral and differential equations in many different domains has arisen among the many numerical methods that have been developed over the years: wavelet-based approaches. In order to simplify difficult mathematical issues, wavelet approaches make use of the special features of wavelets, which are functions that can represent data at different resolutions. The ability to localize in both time and frequency is what sets wavelets apart from more conventional approaches like Fourier series. In cases when the solution displays abrupt changes, breaks, or singularities, this quality becomes even more important. Although Fourier analysis works well for periodic functions, it has a hard time with localized characteristics like these because it breaks functions down into infinite global sine and cosine functions, which aren't great for capturing the nitty-gritty of irregular or non-smooth functions. A strong and versatile approximation technique, wavelet transformations allow one to examine a function at many scales and locations. Wavelets are ideal for numerical approaches that attempt to solve complicated integral and differential equations because of their flexibility.

Numerical analysis has come a long way since wavelet techniques were applied to integral equations. When an integral is used to relate the unknown function to its own values, an integral equation is often the result. The integral equations in question may be categorized as either Volterra or Fredholm. The former are specified with regard to an upper limit that changes in relation to the independent variable, while the latter have fixed limits. Electromagnetism, fluid dynamics, and signal processing are just a few of the scientific domains that face integral equations. They shine when used to simulate memory-based systems or those involving spatial or temporal cumulative effects. Complex integral kernels and irregular domains make analytical solution of integral equations very difficult. By extending the unknown function and kernel in terms of wavelet series, wavelet techniques may estimate the solution of these equations, which is a huge benefit in this situation. In doing so, we simplify the issue to a set of linear equations amenable to efficient numerical solution.

Wavelet algorithms provide an equally effective strategy for differential equations. Fluid dynamics, wave propagation, and heat diffusion are all examples of dynamic systems whose behavior may be well described by differential equations. Differential equations may be categorized according to whether they contain derivatives with respect to one independent variable (ordinary differential equations, or ODEs) or many variables (partial differential equations, or PDEs). For systems with a large number of dimensions or that are not linear, analytical solutions to these equations may be very challenging, if not impossible.

2. LITERATURE REVIEW

Kumar, B. et al., (2018) A wavelet Galerkin method based on Daubechies compactly supported wavelets is shown here. Numerical issues with linear and nonlinear PDEs of fourth order in two- and three-dimensional spaces may be solved using this technique. The use of two-term connection coefficients has allowed for the accurate and efficient calculation of higher-order derivatives. Because of their characteristics, wavelets cause the global matrix to be sparse. Orthogonality and localization are two of these characteristics. To be more precise, these characteristics greatly reduce the necessary computational effort. The GMRES iterative solver was used to successfully resolve linear systems of equations obtained from discretized equations. The quasi-linearization method has been successfully used to manage nonlinear components in nonlinear biharmonic equations. We have introduced a powerful compression method to reduce the computational resources needed by our method. We have calculated the stability and error estimates. The goal of providing several examples is to prove that the proposed method is accurate.

Mirzaee, Farshid et al., (2018) In order to numerically solve linear stochastic Itô-Volterra integral equations, this work proposes a novel approach based on operational matrices of Bernoulli wavelets. Process, plan, and blueprint. This goal is accomplished by introducing and explaining the characteristics of Bernoulli wavelets and Bernoulli polynomials. The initial calculations of the integral operational matrix and its stochastic equivalent, originated from the Bernoulli wavelet, follow. These matrices might help simplify the original issue to a set of linear algebraic equations that the correct numerical algorithm could solve. Included in this research are some findings about the convergence analysis and error estimate of the suggested system. Being distinct is important Two numerical examples are provided to prove that the procedure is legitimate and beneficial.

Hazim, Nawar et al., (2018) Studying nonlinear systems is crucial to applied mathematics and engineering. The mixed hyperbolic-parabolic non-linear partial differential equation known as the Burger-Fisher equation has applications in a wide variety of scientific and applied disciplines. Modeling gas dynamics and fluid mechanics are two examples of such fields. This study successfully used the Haar wavelet method to find the numerical solution of the Burger-Fisher problem. Compared to other approaches that are presently being used, our methodology shows a faster convergence. The examples shown here demonstrate how to employ wavelet-based approaches to build a robust strategy for solving the Burger-Fisher problem numerically. The suggested strategy is both accurate and practical for solving these types of issues, according to numerical data, exact answers, and solutions produced using specific conventional procedures like the variational iteration method (VIM). This is seen when the numerical findings are compared to the precise answers.

Al-Fayadh, Ali et al., (2017) Studying nonlinear systems is crucial to applied mathematics and engineering. The mixed hyperbolic-parabolic non-linear partial differential equation known as the Burger-Fisher equation has applications in a wide variety of scientific and applied disciplines. Modeling gas dynamics and fluid mechanics are two examples of such fields. This study successfully used the Haar wavelet method to find the numerical solution of the Burger-Fisher problem. Compared to other approaches that are presently being used, our methodology shows a faster convergence. The examples shown here demonstrate how to employ wavelet-based approaches to build a robust strategy for solving the Burger-Fisher problem numerically.

Yan, W. (2017) The field of wavelet analysis has been rapidly growing in popularity among mathematicians, engineers, and applied scientists in the last few years. This is due to the fact that technological advancements are perpetual. Building a large mathematical system and continuously exploring new fields of research are strengthening wavelets' theoretical foundation. Graph and picture processing, quantum physics, artificial intelligence in armament, denoising compression, and wavelet theory and the wavelet transform are some of the areas examined by the study's author. Numerical solutions to partial differential equations in wavelet theory are also explored by the author. We use the Haar wavelet integration method, a new numerical solution to the 1D second-order linear hyperbolical telegraph problem, to verify the technique's functions in the telegraph equation and to evaluate the time domain and approach decomposition of airspace.

Rahimkhani, P. et al., (2016) New functions, generalized fractional-order Bernoulli wavelet functions (GFBWFs), are developed in this paper for the purpose of finding numerical solutions to fractional-order pantograph differential equations over a long interval. The Bernoulli wavelet transform is the basis of these procedures. To further understand fractional derivative, we may use the Riemann-Liouville fractional integral operator in the Caputo sense. Creating generalized fractional-order Bernoulli wavelets is one of the first things to do. Based on these functions and their characteristics, the GFBWF operating matrices for pantograph and fractional integration are subsequently built. Solving the issue using the integral and pantograph operating matrices reduces it to an algebraic equation system. Finally, we have shown the validity and efficacy of our method by presenting several examples.

Yi, Mingxu et al., (2015) The goal of this study is to provide numerical solutions to long-term fractional-order pantograph differential equations using newly-developed functions called generalized fractional-order Bernoulli wavelet functions (GFBWFs). These methods are based on the Bernoulli wavelet transform. The Caputo version of the Riemann-Liouville fractional integral operator could help shed light on fractional derivatives. A good place to start is by making generalized fractional-order Bernoulli wavelets. These functions and their properties are used to construct the GFBWF operating matrices for fractional and pantograph integration. The system of algebraic equations is the result of solving the problem with the integral and pantograph operating matrices. Finally, using the examples we have presented, we can prove that our technique is effective and valid.

Singh, S. et al., (2015) If you have any kind of partial differential equation, the Haar wavelet may help you solve it. The findings are more precise and time-saving when Haar wavelets are used. Using a Haar wavelet approach, which is detailed in this work, we numerically solve the wave equation. Here, we use the current approach to get a numerical answer that is better and more accurate than the one given by Shi.
Aziz, Imran et al., (2014) An innovative numerical approach using the Haar wavelet has been suggested for solving first and second class Volterra-Fredholm, Fredholm, and Volterra-Fredholm integral equations in two dimensions that are nonlinear. The authors Aziz and Siraj-ul-Islam (2013), Siraj-ul-Islam et al. (2013), and Siraj-ul-Islam et al. (2014) proposed an extension of the Haar wavelet approach to two-dimensional nonlinear integral equations (Fredholm, Volterra, and Volterra-Fredholm). This is an extension of the Haar wavelet method. This approach differs from others as it does not involve numerical integration, a typical technique. As a result, the method's precision is significantly enhanced. Applying the approach to several benchmark issues may demonstrate its usefulness. A comparison is made between the numerical findings and other approaches that have been published recently.
Choudhury, A. (2014). The use of semi-discrete approximations has allowed the development of an exceedingly accurate numerical approach for solving one-dimensional parabolic partial differential equations. Using a wavelet-Galerkin approach, the spatial direction is discretized. Using this method, you may get certain kinds of fundamental functions by including Daubechies functions. These fundamental operations are diverseiable and have compact support. The time variable is often discretized using one of many classical finite difference algorithms. Problems like as diffusion, diffusion-reaction, convection-diffusion, and convection-diffusion-reaction with Dirichlet, mixed, and other boundary conditions are used to derive theoretical and numerical conclusions. When compared to the actual ones, the calculated answers are far superior.

3. OBJECTIVE (S) /NEED OF THE STUDY

1. To develop and analyze numerical solutions of fractional-order differential equations using the Haar wavelet method, focusing on accuracy and efficiency.
2. To develop and investigate efficient numerical solutions for singular perturbation boundary value problems using the Haar wavelet collocation method.

3. To construct and evaluate numerical solutions for singular ordinary differential equations using the Legendre wavelet method, emphasizing accuracy and computational efficiency.
4. To develop and analyze numerical solutions for the Klein/Sine-Gordon equations using the Chebyshev wavelet method, emphasizing accuracy and stability.

NEED OF THE STUDY

Many disciplines rely on numerical solutions to model and solve real-world issues. This includes economics, physics, engineering, biology, and many more. Though they have their uses, traditional techniques like spectral, finite difference, and difference approaches aren't without their drawbacks. These include computing inefficiency, problems with singularities, and an inability to resolve localized solution characteristics. Wavelet technique provides a game-changing way to overcome these constraints because of its built-in capability for multi-resolution analysis. A potential tool for solving complicated equations in various domains, it effectively represents functions, captures local behavior, and flexibly handles boundary constraints.

Numerical approaches based on wavelets have shown promise, but there has been little research into their use for solving non-linear, multi-dimensional, time-dependent integral and differential equations. To fully use wavelet approaches, one must have a thorough grasp of how they work in regards to accuracy, stability, and computing efficiency. The purpose of this research is to determine which wavelet families and parameters work best for certain types of issues, verify wavelet-based procedures, and compare them to more conventional methods. The results should help computational mathematics progress and provide a foundation for new approaches to solving difficult scientific problems.

4. PROPOSED METHODOLOGY

The research methodology for solving differential equations of fractional order and singular perturbation boundary value problems (BVPs) using various wavelet methods will follow a structured approach based on approximation techniques and numerical analysis. For the numerical solution of fractional-order differential equations, the Haar wavelet method will be applied. This will involve discretizing the problem domain using Haar wavelets, where the differential operator will be approximated by a set of wavelet basis functions. The resulting system of algebraic equations will then be solved using appropriate numerical techniques, such as Gaussian elimination or iterative solvers, to obtain an approximate solution with high accuracy.

For singular perturbation BVPs, the Haar wavelet collocation method will be utilized. In this case, the solution will be approximated by a linear combination of Haar wavelets, and the collocation method will ensure that the differential equation is satisfied at specific points in the domain. The problem will be carefully handled to account for the small perturbation parameter, ensuring that the correct asymptotic behavior is captured. The solution will then be obtained by solving the resulting system of equations numerically.

When addressing singular ordinary differential equations (ODEs), the Legendre wavelet method will be employed. Here, the ODE will be transformed into an equivalent algebraic system by expanding the solution as a series of Legendre wavelets. These wavelets are orthogonal, providing excellent numerical stability. By applying boundary conditions and solving the resulting system, an accurate numerical solution will be obtained.

For solving Klein/Sine-Gordon equations, the Chebyshev wavelet method will be used. This method will exploit Chebyshev wavelets, which offer high precision and are particularly suited for solving nonlinear and oscillatory partial differential equations. The approach will involve discretizing the equation and transforming it into a system of algebraic equations, which will then be solved using iterative techniques. The Chebyshev wavelets will provide efficient approximation, ensuring accurate solutions even for complex and highly non-linear systems.

Throughout all methods, the accuracy and convergence of the numerical solutions will be analyzed by comparing the results with known analytical solutions or benchmark problems. The methodologies will focus on ensuring efficient computation and high precision in solving complex differential equations.

5. TENTATIVE CHAPTER PLAN

- CHAPTER 1: INTRODUCTION
- CHAPTER 2: NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER USING HAAR WAVELET METHOD.
- CHAPTER 3: NUMERICAL SOLUTION OF SINGULAR PERTURBATION BVP USING HAAR WAVELET COLLOCATION METHOD
- CHAPTER 4: NUMERICAL SOLUTION OF SINGULAR ORDINARY DIFFERENTIAL EQUATIONS USING LEGENDRE WAVELET METHOD

- CHAPTER 5: NUMERICAL SOLUTION OF KLEIN/SINE-GORDON EQUATIONS USING CHEBYSHEV WAVELET METHOD
- CHAPTER 6: CONCLUSION, RECOMMENDATIONS AND FUTURE SCOPE OF THE STUDY

6. EXPECTED OUTCOMES

The expected outcome of the study on numerical solutions using wavelet methodology for integral and differential equations in various domains is to demonstrate the efficacy and versatility of wavelet-based techniques in solving complex mathematical problems. The study aims to show that wavelet methods provide accurate, efficient, and computationally feasible solutions to both integral and differential equations across a range of domains, from simple to complex geometries. By comparing these solutions with traditional methods, such as finite difference and finite element methods, the study intends to highlight the advantages of wavelet techniques, such as their ability to handle irregular domains, provide multiresolution analysis, and improve convergence rates. Additionally, the research expects to address any challenges related to the implementation of wavelet-based approaches, including their numerical stability, computational cost, and scalability in higher-dimensional problems. Ultimately, the study hopes to contribute to the development of more reliable and efficient methods for solving mathematical models in engineering, physics, and applied sciences.

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