



Incentre point based method to Solve Multi-Objective Transportation Problems using Generalized Triangular Fuzzy Numbers

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ABSTRACT

The transportation problems (TP) are special kinds of linear programming problems. Real-life situations may have inconsistent supply, demand, and unit transportation costs due to a variety of factors. The decision maker can handle multiple objectives simultaneously in the present time. This paper presents an algorithm for solving the multi-objective transportation problem (MOTP) with generalized triangular fuzzy numbers (GFTN). The fuzzy MOTP is defuzzified into k-Crisp Single Objective Transportation Problem (CSOTP) using the incentre point method. After Defuzzification, the primal solutions of the k-Crisp Single Objective Transportation Problem are obtained by using LINGO 20.0. Using the solution of the k- single objectives Transportation Problem, a set of efficient Solutions to the Fuzzy multi-objective transportation problem is obtained. Among the set of efficient solutions to the multi-objective transportation problem, the decision maker can choose the most suitable optimal solution. This method is easy to use. To illustrate this method, a numerical example is considered.

Keywords: Efficient Solution, Fuzzy transportation problem, Generalised Triangular fuzzy number, Multi-Objective Transportation Problem

1. INTRODUCTION

A key area of operations research is transportation problem (TP), which has applications in lots of fields, including inventory control, communication networks, production planning, scheduling, and personal allocation. In today's highly competitive market, businesses are under increasing pressure to improve how they produce and provide their goods and services. The challenges become more difficult in determining when and how to deliver the items in the quantities the customers desire while remaining cost-effective. Transportation models give an effective framework for dealing with this problem. The TP's purpose is to determine a shipment schedule that minimizes overall shipping costs while meeting supply and demand requirements. The MOTP is a linear optimization problem with several variable objectives and equality-type constraints. The concept of fuzzy transportation problems (FTP) was developed to find the solution to the transportation problem's unpredictable parameters, such as fuel prices, weather conditions, product supply, demand, etc. F.L. Hitchcock [1] practically invented it for the first time in 1941. Lotfi A Zadeh [2] was given the concept of fuzziness in 1965. Zimmermann H.J. [3] was the first to use an appropriate membership function to solve an LP problem with multiple objectives.

Ringuest et al. [4] (1987) proposed two interactive algorithms to Solve the MOTP. Bit et al. [5] solved TP problems with several criteria by using fuzzy programming. Lushu li. Et al. [6] presented a technique that uses standard optimization techniques to solve the fuzzy compromise programming model to produce a non-dominated compromise solution with the highest synthetic membership degree for all objectives. Rouhparvar. H. [7] proposed a new method to defuzzify the general fuzzy number. Maity. G. et al. [8] studied the MOTP under uncertain environments Ahmed J.S. et al. [10] gave a new novel approach to defuzzify the triangular fuzzy numbers (TFN) in the fuzzy TP and modify the Centre of gravity method (COG) for multi-objective linear programming (MOLP). Mitlif RJ [11] solves triangular fuzzy fractional programming problems with a new ranking method for ordinary fuzzy numbers. Bagheri. M. et al. [12] presented the DEA approach to solving FMOTP. Z. Li et. al. [13] presented a Lagrangian Relaxation (LR) heuristic approach that may offer both a lower limit and a nearly optimum solution for each of the single-objective issues is created due to the NP-hardness of the single-objective problems produced from BMTTP. SG Bodke [14] introduced a method to solve fuzzy MOTP after converting it into Crisp MOTP which is based on the Zimmerman technique using the exponential membership function. Sharma, R., & Tyagi, S. L. [15] present an algorithm to solve multi-objective transportation problems with generalized trapezoidal fuzzy numbers based on the ranking function.

An approach for solving multi-objective transportation problems with triangular fuzzy numbers is presented in this paper. In this algorithm, fuzzy MOTP is converted into crisp MOTP by using the incentre point method point. A single objective TP is solved using LINGO 20.0. After finding the primal optimal solution of single objective TP, the set of fuzzy efficient solutions is found using the technique given by Jayalakshmi and Sujatha [9].

2. SOME PRELIMINARIES

2.1 Membership function (MF): Let X be a universal set. The membership function (MF) of fuzzy set $\tilde{A}$ in X is defined as

$$\mu_{\tilde{A}} : X \rightarrow [0, 1]$$

For each $x \in X$, $\mu_{\tilde{A}}(x)$ is the grade of membership of element x in the fuzzy set $\tilde{A}$.

2.2 Fuzzy Number: A fuzzy set $\tilde{A}$ with membership function $\tilde{A} : \mathbb{R} \rightarrow [0,1]$ defined on the set of real numbers is called a fuzzy number if it satisfies the following properties:

- (i) $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}$
- (ii) $\exists a \in \mathbb{R}$ such that $\tilde{A}(a) = 1$.
- (iii) $\tilde{A}$ is piece-wise continuous.

2.3 Generalized triangular fuzzy number (GTFN): A fuzzy set $A = (a_1, a_2, a_3; w)$ such that $(a_1 \leq a_2 \leq a_3)$ is said to be GTFN with centre a_2 , left width $a_2 - a_1 > 0$, right width $a_3 - a_2 > 0$; if its MF is as the following:

$$\mu_A(y) = \begin{cases} w \left(1 - \frac{(a_1 - y)}{(a_2 - a_1)} \right) & a_1 \leq y \leq a_2 \\ w \left(1 - \frac{(y - a_3)}{(a_3 - a_2)} \right) & a_2 \leq y \leq a_3 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

2.4 Incentre Point of a triangle: A point in the triangle where the bisectors of each angle of a triangle meet, called the incentre point of a triangle. Let DEF be a triangle, then the incentre point of DEF is given as

$$I = \frac{D|EF| + E|FD| + F|DE|}{|EF| + |FD| + |DE|} \quad (2)$$

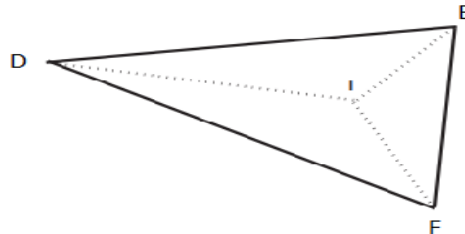


Figure 1 triangle DEF

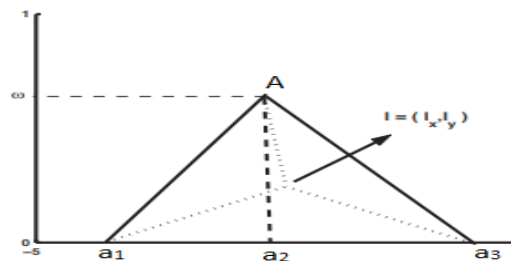


Figure 2. Representation of Incentre point for the triangular fuzzy number

3. RANKING OF GENERALIZED TRIANGULAR FUZZY NUMBER (GTFN) BY INCENTRE POINT METHOD

The ranking of GTFN is defined as:

Let $A = (a_1, a_2, a_3; w)$ is a GTFN. The incentre point of GTFN of A is (I_x, I_y) is given as

$$I_x = \frac{a_1\alpha + a_2\beta + a_3\gamma}{\alpha + \beta + \gamma} \quad (3)$$

$$I_y = \frac{w\beta}{\alpha + \beta + \gamma} \quad (4)$$

$$\text{Where } \alpha = \sqrt{(a_3 - a_2)^2 + w^2}, \beta = a_3 - a_1, \gamma = \sqrt{(a_2 - a_1)^2 + w^2}. \quad (5)$$

as If A and B be the two GTFNs. Then the ranking of GTFN is defined as:

(I) $A > B$ iff $(I_x)_A > (I_x)_B$

(II) $A > B$ iff $(I_y)_A > (I_y)_B$ and $(I_x)_A = (I_x)_B$

(III) $A = B$ iff $(I_y)_A = (I_y)_B$ and $(I_x)_A = (I_x)_B$

Therefore, $I = (I_x, I_y)$ defuzzified A with a defuzzifier $I_x \in (a_1, a_3)$.

4. THE MATHEMATICAL MODEL FOR FUZZY MOTP WITH GTFN

The fuzzy MOTP ($\tilde{T}$) for k objectives can be represented as:

$$(\tilde{T}_u) \quad \text{Min } \tilde{Z}_k(y) = \sum_{i_0=1}^{m_0} \sum_{j_0=1}^{n_0} \tilde{\alpha}_{i_0 j_0}^{(k)} \tilde{y}_{i_0 j_0}$$

for $k = 1, 2, \dots$

Subject to

$$\sum_{i_0=1}^{m_0} \tilde{y}_{i_0 j_0} = \tilde{d}_{j_0} : \text{for fixed } j_0 \text{ such that } j_0 = 1, 2, \dots, n_0$$

$$\sum_{j_0=1}^{n_0} \tilde{y}_{i_0 j_0} = \tilde{s}_{i_0} : \text{for fixed } i_0 \text{ such that } i_0 = 1, 2, \dots, m_0$$

$$\tilde{y}_{i_0 j_0} \geq 0$$

here,

m_0 = total no. of origins

n_0 = total no. of destinations

$\tilde{y}_{i_0 j_0}$ = transportable fuzzy quantity of goods transported from i_0 -th origins to j_0 -th destinations

$\tilde{s}_{i_0}$ = the fuzzy availabilities of goods at i_0 -th origins

$\tilde{d}_{j_0}$ = fuzzy requirements of goods at j_0 -th destinations.

$\tilde{\alpha}_{i_0 j_0}^{(k)}$ = the fuzzy cost for transporting one unit of the given good from i_0 -th origin to j_0 -th destination.

We also assume that $\sum_{i=1}^m \tilde{s}_i = \sum_{j=1}^n \tilde{d}_j$ (i.e., Balanced MOTP) if it is not balanced, we can easily convert it into

balanced MOTP.

Where $\tilde{\alpha}_{i_0 j_0}^{(k)}, \tilde{s}_{i_0}, \tilde{d}_{j_0}$ ($i_0 = 1, 2, \dots, m_0, j_0 = 1, 2, \dots, n_0$) are the generalized triangular fuzzy number.

5. DEFUZZIFICATION OF FUZZY MOTP BY INCENTRE POINT METHOD

Let $A = (a_1, a_2, a_3; w)$ such that $(a_1 \leq a_2 \leq a_3)$ with the membership function $\mu_A(y)$ given by equation (1) is defuzzified by the defuzzifier

$$I_x = \frac{a_1 \alpha + a_2 \beta + a_3 \gamma}{\alpha + \beta + \gamma}$$

Where $\alpha = \sqrt{(a_3 - a_2)^2 + w^2}$, $\beta = a_3 - a_1$, $\gamma = \sqrt{(a_2 - a_1)^2 + w^2}$ and $I_x \in (a_1, a_3)$

By using the defuzzifier I_x , the fuzzy MOTP with GTFN is converted into crisp MOTP.

Using this above procedure, all $\tilde{\alpha}_{i_0 j_0}^{(k)}, \tilde{s}_{i_0}, \tilde{d}_{j_0}$ ($i_0 = 1, 2, \dots, m_0, j_0 = 1, 2, \dots, n_0$) is converted into crisp quantities. The converted Crisp MOTP (T) can be represented mathematically as:

$$(T_u) \text{ Min } \tilde{Z}_k(y) = \sum_{i_0=1}^{m_0} \sum_{j_0=1}^{n_0} \alpha_{i_0 j_0}^{(k)} y_{i_0 j_0}$$

for $k = 1, 2, \dots$

Subject to

$$\sum_{i_0=1}^{m_0} y_{i_0 j_0} = d_{j_0} : \text{for fixed } j_0 \text{ such that } j_0 = 1, 2, \dots, n_0$$

$$\sum_{j_0=1}^{n_0} y_{i_0 j_0} = s_{i_0} : \text{for fixed } i_0 \text{ such that } i_0 = 1, 2, \dots, m_0$$

$$\text{And } y_{i_0 j_0} \geq 0, \quad d_{j_0} \geq 0, \quad s_{i_0} \geq 0.$$

6. PROCEDURE FOR PROPOSED METHOD

The proposed method solves the multi-objective transportation problem (MOTP) in which the transportation cost, availabilities and requirements are represented by generalized triangular fuzzy numbers. The steps of the proposed method are following:

Step I: first, the fuzzy transportation problem ($\widetilde{T}_u$) is formulated mathematically.

Step II: In this step, the fuzzy MOTP ($\widetilde{T}_u$) problem is converted into Crisp MOTP (T_u) using the incentre point method.

Step III: In this step, the Crisp MOTP (T_u) is splitted into k-single objective transportation problems.

Step IV: In this step, the primal optimal solution of single objective TP (T_u), $u = 1, 2, \dots, k$ is obtained using LINGO 20. Consider the optimal transportation solution of TP (T_u) be Y_u^* with lowest cost Z_u^* .

Step V: An optimal solution to the problem (T_u) obtained in step IV is used to find efficient solutions to the problems $T_1, T_2, \dots, T_{v-1}, T_v, \dots, T_k$.

Step VI: Repeat all steps I to step V for all the problems (T_u), $u = 1, 2, \dots, k$, the set of efficient solutions for problem ($\widetilde{T}_u$) is obtained.

To illustrate this method, a numerical example is considered.

Numerical example: Consider a fuzzy MOTP with GFTN with the following characteristics is given as:

Source→ Destination↓	S_1	S_2	S_3	S_4	Availability (s_i)
D_1	(1,1.5,2;1) (3,4,5;1)	(1,2,3;1) (5,7,9;1)	(5,6.5,8;1) (2,3,4;1)	(4,6,8;1) (1,3,5;1)	(3,6,9;1)
D_2	(1,1.5,2;1) (4,5,6;1)	(7,7.5,8;1) (7,8,9;1)	(3,4.5,6;1) (7,8.5,10;1)	(3,4,5;1) (9,10,11;1)	(13.5,15,16.5;1)
D_3	(4,7,10;1) (4,6,8;1)	(7,9,11;1) (1,2,3;1)	(1.5,2,2.5;1) (3,4.5,6;1)	(3,4,5;1) (1,1.5,2;1)	(15,18,21;1)
Requirement (d_j)	(1,5,9;1)	(6,10,14;1)	(10,9,11;1)	(11,15,19;1)	

Step I: first the problem is formulated in mathematical form as:

$$\text{Min } \tilde{Z}_1 = (1,1.5,2;1) y_{11} + (1,2,3;1) y_{12} + (5,6.5,8;1) y_{13} + (4,6,8;1) y_{14} + (1,1.5,2;1) y_{21} + (7,7.5,8;1) y_{22} + (3,4.5,6;1) y_{23} + (3,4,5;1) y_{24} + (4,7,10;1) y_{31} + (7,9,11;1) y_{32} + (1.5,2,2.5;1) y_{33} + (3,4,5;1) y_{34}$$

$$\text{Min } \tilde{Z}_2 = (3,4,5;1) y_{11} + (5,7,9;1) y_{12} + (2,3,4;1) y_{13} + (1,3,5;1) y_{14} + (4,5,6;1) y_{21} + (7,8,9;1) y_{22} + (7,8.5,10;1) y_{23} + (9,10,11;1) y_{24} + (4,6,8;1) y_{31} + (1,2,3;1) y_{32} + (3,4.5,6;1) y_{33} + (1,1.5,2;1) y_{34}$$

Subject to

$$y_{11} + y_{12} + y_{13} + y_{14} = (3,6,9;1)$$

$$y_{21} + y_{22} + y_{23} + y_{24} = (13.5,15,16.5;1)$$

$$y_{31} + y_{32} + y_{33} + y_{34} = (15,18,21;1)$$

$$y_{11} + y_{21} + y_{31} = (1,5,9;1)$$

$$y_{12} + y_{22} + y_{32} = (6,10,14;1)$$

$$y_{13} + y_{23} + y_{33} = (10, 9, 11; 1)$$

$$y_{14} + y_{24} + y_{34} = (11, 15, 19; 1)$$

Step II: the fuzzy MOTP with GFTN is converted into crisp MOTP by the method given in [4]:

$$\text{Here } \tilde{a}_{11}^{(1)} = (a_1, a_2, a_3; w) = (1, 1.5, 2; 1)$$

Then defuzzifier

$$I_x = \frac{\alpha a_1 + \beta a_2 + \gamma a_3}{\alpha + \beta + \gamma}$$

$$\text{Here, } \alpha = \sqrt{(a_3 - a_2)^2 + w^2} = \sqrt{(2 - 1.5)^2 + 1^2} = \sqrt{1.25} = 1.1180$$

$$\beta = (a_3 - a_1) = 2 - 1 = 1$$

$$\gamma = \sqrt{(a_2 - a_1)^2 + w^2} = \sqrt{(1.5 - 1)^2 + 1^2} = \sqrt{1.25} = 1.1180$$

$$\text{Then } I_x = \frac{1 \times 1.11850 + 1.5 \times 1 + 1.1180 + 2}{1.1180 + 1 + 1.1180} = 1.5$$

Then crisp value of $\tilde{a}_{11}^{(1)}$ is $a_{11}^{(1)} = 1.5$

Similarly, all crisp values are obtained.

$$a_{12}^{(1)} = 2, a_{13}^{(1)} = 6.5, a_{14}^{(1)} = 6, a_{21}^{(1)} = 1.5, a_{22}^{(1)} = 7.5, a_{23}^{(1)} = 4.5, a_{24}^{(1)} = 4, a_{31}^{(1)} = 7,$$

$$a_{32}^{(1)} = 9, a_{33}^{(1)} = 2, a_{34}^{(1)} = 4$$

$$a_{11}^{(2)} = 4, a_{12}^{(2)} = 7, a_{13}^{(2)} = 3, a_{14}^{(2)} = 3, a_{21}^{(2)} = 5, a_{22}^{(2)} = 8, a_{23}^{(2)} = 8.5, a_{24}^{(2)} = 10, a_{31}^{(2)} = 6, a_{32}^{(2)} = 2,$$

$$a_{33}^{(2)} = 4.5, a_{34}^{(2)} = 1.5$$

$$\text{Availabilities: } s_1 = 6, s_2 = 15, s_3 = 9$$

$$\text{Requirements: } d_1 = 5, d_2 = 10, d_3 = 9, d_4 = 6$$

Step III: The crisp MOTP is converted into Crisp SOTP

First crisp SOTP (T_1)

$$\text{Min } Z_1 = 1.5 y_{11} + 2 y_{12} + 6.5 y_{13} + 6 y_{14} + 1.5 y_{21} + 7.5 y_{22} + 4.5 y_{23} + 4 y_{24} + 7 y_{31} + 9 y_{32} + 2 y_{33} + 4 y_{34}$$

Subject to

$$y_{11} + y_{12} + y_{13} + y_{14} = 6$$

$$y_{21} + y_{22} + y_{23} + y_{24} = 15$$

$$y_{31} + y_{32} + y_{33} + y_{34} = 18$$

$$y_{11} + y_{21} + y_{31} = 5$$

$$y_{12} + y_{22} + y_{32} = 10$$

$$y_{13} + y_{23} + y_{33} = 9$$

$$y_{14} + y_{24} + y_{34} = 15$$

Second Crisp SOTP (T_2)

$$\text{Min } Z_2 = 4 y_{11} + 7 y_{12} + 3 y_{13} + 3 y_{14} + 5 y_{21} + 8 y_{22} + 8.5 y_{23} + 10 y_{24} + 6 y_{31} + 2 y_{32} + 4.5 y_{33} + 1.5 y_{34}$$

Subject to

$$y_{11} + y_{12} + y_{13} + y_{14} = 6$$

$$y_{21} + y_{22} + y_{23} + y_{24} = 15$$

$$y_{31} + y_{32} + y_{33} + y_{34} = 18$$

$$y_{11} + y_{21} + y_{31} = 5$$

$$y_{12} + y_{22} + y_{32} = 10$$

$$y_{13} + y_{23} + y_{33} = 9$$

$$y_{14} + y_{24} + y_{34} = 15$$

Step IV: In this step, the primal solution of problem is found by using LINGO 20.0.

Solution of T_1 :

$$y_{11}=0, y_{12}=6, y_{13}=0, y_{14}=0, y_{21}=5, y_{22}=4, y_{23}=0, y_{24}=6, y_{31}=0, y_{32}=0, y_{33}=9, y_{34}=9.$$

$$\text{Min } Z_1 = 127.5$$

Solution of T_2 :

$$y_{11}=0, y_{12}=0, y_{13}=6, y_{14}=0, y_{21}=5, y_{22}=7, y_{23}=3, y_{24}=0, y_{31}=0, y_{32}=3, y_{33}=0, y_{34}=15.$$

$$\text{Min } Z_2 = 153.0$$

Step V: Using the optimal transportation of Z_1 in Z_2 the crisp efficient Solution $(Z_1, Z_2) = (127.5, 205)$ and the fuzzy efficient solution of MOTP becomes

$$\tilde{Z}_1 = (97.5, 127.5, 157.5; 1)$$

$$\tilde{Z}_2 = (168, 213, 258; 1)$$

Similarly using the optimal transportation of Z_2 in Z_1 the crisp efficient solution $(Z_1, Z_2) = (150, 153)$ and hence the fuzzy efficient Solution of MOTP becomes

$$\tilde{Z}_1 = (159, 199.5, 240; 1)$$

$$\tilde{Z}_2 = (120, 153, 176; 1)$$

The set of fuzzy efficient solutions of MOTP is

$$\{(97.5, 127.5, 157.5; 1), (168, 213, 258; 1)\}, \{(159, 199.5, 240; 1), (120, 153, 176; 1)\}$$

From the set of efficient solutions, the ideal fuzzy efficient solution of fuzzy MOTP is

$$[(97.5, 127.5, 157.5; 1), (120, 153, 176; 1)]$$

The Crisp efficient solution is (127.5, 153)

7. CONCLUSION

An algorithm for solving a multi-objective transportation problem (MOTP) with a generalized triangular fuzzy number (GTFN) is presented in this paper. The fuzzy MOTP is converted into crisp MOTP by using the incentre point method. the set of efficient solutions is obtained using of primal solutions of a single objective transportation problem. This algorithm is more adaptable than the traditional multi-objective transportation problem-solving techniques and, allows the decision-maker to select the most efficient solution according to his preferences.

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