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Quadruple Fixed Point Theorem in Intuitionistic Fuzzy Metric Space using E. A. along with mixed monotone Property

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Abstract: In this research paper we establish some quadruple coincidence fixed point results using E.Aproperty along with mixed monotone property satisfying $(\varphi, \emptyset)$ contraction in intuitionistic fuzzy metric space. In future we can use theses application in real life.

1. INTRODUCTION

Banach contraction principle is one of the core subjects. It has so many different generalizations with different approaches. Very recently, inspired form the notion of φ-contraction and $\emptyset$-contraction. Contraction mapping is very useful in fixed point theorems. Fixed point theorems are very useful in the existence theory of integral calculus, differential Calculus, functional equations, partial differential equation, random differential equation and other related areas. Existence of coincidence points in intuitionistic fuzzy metric space. The concept of intuitionistic fuzzy metric space was introduced and studied by J. H. Park in (2004, [17])that generalizes the concept of fuzzy metric space due to George and Veeramani. Rajput.A. et.al (2010,[28]) Common fixed points of Compatible Self Maps in Complete intuitionistic Fuzzy Metric Space.In this paper is to givethe new result which is used concept and proved a common fixed point theorem. Rajput.A. et.al (2011,[24]) proved Common Fixed Point Theorems for Multivalued Compatible Maps in IFMS.In this paper to obtain the notion of multivalued weakly compatible maps and prove common fixed point theorems for single and multivalued maps by using a contractive condition of integral type in intuitionistic fuzzy metric space. Rajput.A.et.al. (2011,[25]) provedCommon Fixed point theorems for multivalued maps use in cone intuitionistic fuzzy metric spaces. In this paper to introduce the concept of complete cone metric space for a multivalued transformation. Also proved common fixed point theorems for two multivalued maps in complete cone intuitionistic fuzzy metric space with normal constant M=1. Rajput.A.et.al(2011,[26])proved Fixed Points and Best Proximity Points for Multivalued Mapping Use in IFMS and Satisfying Cyclical Condition. In this paper the concept of cyclical single-valued mapping are extended to multivalued mapping and then we study the existence of fixed points and best proximity points. Rajput.A. et.al (2011,[27])proved Common fixed points of compatible maps in intuitionistic fuzzy metric space of integral type In this paper to obtain a new common fixed point theorems in an intuitionistic fuzzy metric space for point wise R-weakly commuting mappings using contractive condition of integral type.

Alaca et al. (2006,[2]) have established intuitionistic fuzzy versions of Banach contraction principle and Edelstein fixed point theorem. Rajput.A. et.al(2012,[18]), proved Common Fixed Points End Point Theorems use in Intuitionistic Fuzzy Metric Spaces.In this paper shows a fixed point theorems for set valued fuzzy contraction type maps in complete intuitionistic fuzzy metric space which extends and improves some well known results in literature. Then presented an endpoint result we initiate end point theory for fuzzy contraction maps in intuitionistic fuzzy metric spaces.Aamri and El Moutawakil (1988,[1]) defined a property (E.A) which generalizes the concept of non-compatible mappings and gave some common fixed point theorems under strict contractive conditions. Tripathi.N. et.al(2012,[16])proved Intuitionistic Fuzzy Metric Space Using Concept of α-Fixed Point. In this paper we introduce the notion of common property EA in intuitionistic fuzzy metric spaces with the help of α-fixed point. Further we prove some common α-fixed point theorems for hybrid pair of single and multivalued maps under hybrid contractive conditions.Specifically, Lakshmikanthan (2009,[12]) established coupled fixed point for mixed monotone operator in partially ordered metric spaces. Afterward, Lakshmikanthan and Ciric (2009,[5]) extended the results of coupled coincidence and coupled fixed point theorem for two commuting mappings having mixed g-monotone property. Rajput.A. et.al (2012,[20]) proved Fixed Point Theorem for Multivalued Mapping in intuitionistic Fuzzy Metric Space. The main objective of this paper is to obtain some common fixed point theorems for Multivalued mappings in complete intuitionistic fuzzy metric space.

Rajput.A. et.al(2012,[22]) proved Common alpha- Fixed Points Theorems for Multivalued Mappings in intuitionistic Fuzzy Metric Spaces. In this paper to investigate a common α-fixed point theorems in intuitionistic Fuzzy Metric Spaces.Recently, Rajput.A.et.al.(2012,[19]) proved Non Compatible Mappings in intuitionistic fuzzy metric space. In this paper, the new concepts normal product of two multivalued mappings, s-weakly compatible, s-common point and the (EAs) property for two pairs of multivalued mappings are introduced and the common fixed point existence theorems for two pairs of multivalued noncompatible mappings under strict contractive condition are proved without appeal to continuity of any map involved there in and completeness of underlying space. The results presented in this paper generalize, improve and unify some recent results in this field. The purpose of present study is to investigate quadruple common fixed point for mappings satisfying E.A. along with mixed property and possessing monotonicity type properties, in the context of intuitionistic fuzzy metric space which combine method of contraction principle with method of monotone iterations.

In what follows, we collect some relevant definitions, results, examples for our further use.

Definition 1.1 A fuzzy set A in X is a function with domain X and Values in [0, 1].

Definition 1.2 A continuous t-normis a binary operation $*$:$[0, 1]\times [0,1] \to [0,1]$ satisfying the following conditions:

(i) $*$ is a commutative and associative;

(ii) $a * 1 = a$ for all $a \in [0,1]$;

(iii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$;

(iv) $*$ is continuous

Definition 1.3 A continuous t-conorm is a binary operation $\diamond$: $[0, 1]\times [0,1] \to [0,1]$ satisfying the following conditions:

(i) $\diamond$ is a commutative and associative;

(ii) $a \diamond 0 = a$ for all $a \in [0,1]$;

(iii) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$;

(iv) ⋄ is continuous.

Definition 1.4A intuitionistic fuzzy metric space is a 5-tuple (X, M, N, $*$,⋄), where X is a nonempty set, $*$ is a continuous t-norm, ⋄ is a continuous t-conorm and M, N is a fuzzy set on $X^2 \times (0, \infty)$ such that the following axioms holds:

(i) $M(x, y, t) > 0$ $(x, y \in X)$

(ii) $M(x, y, t) = 1$ for all $t > 0$ iff $x = y$

(iii) $M(x, y, t) = M(y, x, t)$ $(x, y \in X, t > 0)$

(iv) $M(x, y, \cdot): [0, \infty) \to [0, 1]$ is continuous for all $x, y \in X$

(v) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$ for all $x, y, z \in X$ and $s, t > 0$.

(vi) $N(x, y, t) \geq 0$ $(x, y \in X)$

(vii) $N(x, y, t) = 0$ for all $t > 0$ iff $x = y$

(viii) $N(x, y, t) = N(y, x, t)$ $(x, y \in X, t > 0)$

(ix) $N(x, y, \cdot): (0, \infty) \to (0, 1]$ is continuous for all $x, y \in X$

(x) $N(x, z, t + s) \leq N(x, y, t) * N(y, z, s)$ for all $x, y, z \in X$ and $s, t > 0$.

Notice that (M, N) is called an intuitionistic fuzzy metric on X. The value M(x, y, t) can be thought of as degree of nearness between x and y and N(x, y, t) as degree of non-nearness between x and y with respect to t respectively.

Definition 1.5 Let $F : X^3 \to X$. An element (x, y, z) is called a tripled fixed point of F if

$$F(x, y, z) = x \,, F(y, x, y) = y \text{ and } F(z, y, x) = z$$

Definition 1.6 Let $F: X^3 \to X$ and $g: X \to X$ An element (x, y, z) is called a tripled coincidence point of F and g if

$$F(x, y, z) = gx \,, F(y, x, y) = gy \text{ and } F(z, y, x) = gz$$

(gx, gy, gz) is said a tripled point of coincidence of F and g.

Definition 1.7 Let $F: X^3 \to X$ and $g: X \to X$. An element (x, y, z) is called a tripled common fixed point of F and g if

$$F(x, y, z) = gx = x \,, F(y, x, y) = gy = y \text{ and } F(z, y, x) = gz = z$$

Definition 1.8 Let X be a nonempty set and $F: X^4 \to X$ be a given mapping. An element (x, y, z, w) ∈ X × X × X × X is called a quadruple fixed point of F if

$$F(x, y, z, w) = x,\ F(y, z, w, x) = y,\ F(z, w, x, y) = z \text{ and } F(w, x, y, z) = w.$$

Definition 1.9 Let $F: X^4 \to X$ and $g : X \to X$. An element (x, y, z, w) is called a quadruple coincidence point of F and g if

$$F(x, y, z, w) = gx,\ F(y, z, w, x) = gy,\ F(z, w, x, y) = gz \text{ and } F(w, x, y, z) = gw.$$

(gx, gy, gz, gw) is said a quadruple point of coincidence of F and g

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Definition 1.10 Let $F : X^4 \to X$ and $g : X \to X$. An element (x, y, z, w) is called a quadruple common fixed point of F and g if

$$F(x, y, z, w) = gx = x, \qquad F(y, z, w, x) = gy = y$$
$$F(z, w, x, y) = gz = z \text{ and } \quad F(w, x, y, z) = gw = w.$$

Definition 1.11 Let X be a non-empty set. Then we say that the mappings $F : X^4 \to X$ and $g : X \to X$ are commutative if for all x, y, z, w ∈ X

$$g(F(x, y, z, w)) = F(gx, gy, gz, gw).$$

Definition 1.12 The mappings F and g where F: X × X × X × X → X and g: X → X, of a intuitionistic fuzzy metric space (X, M, N, ∗, ⋄) has g-mixed monotone property if F is monotone g-nondecreasing in fist argument and is monotone g-nonincreasing in second argument.

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Now, we introduce the notion of compatibility, weakly compatible and E.A property. for g-mixed monotone mapping in intuitionistic fuzzy metric space.

Definition 1.13 The mappings F and g where F: X × X× X× X → X and g: X → X, over intuitionistic fuzzy metric space (X, M, N, ∗, ⋄) are said to be compatible if

$$\lim_{n\to\infty} M(g(F(x_{n,}y_n)),F(g(x_n),\mathrm{g}(y_n)),\mathrm{t})=1$$

$$\lim_{n\to\infty} M(g(F(y_{n,}x_n)),F(g(y_n),\mathrm{g}(x_n)),\mathrm{t})=1$$

and

$$\lim_{n\to\infty} N(g(F(x_{n,}y_n)),F(g(x_n),\mathrm{g}(y_n)),\mathrm{t})=0$$

$$\lim_{n\to\infty} N(g(F(y_{n,}x_n)),F(g(y_n),\mathrm{g}(x_n)),\mathrm{t})=0$$

whenever $\{x_{n,}\}$ and $\{y_{n,}\}$ are sequences in X, such that $\lim_{n\to\infty} \mathrm{F}(\mathrm{x_n},\mathrm{y_n},\mathrm{z_n},\mathrm{w_n}) = \lim_{n\to\infty} \mathrm{g}(\mathrm{x_n}) = \mathrm{x}$ and $\lim_{n\to\infty} \mathrm{F}(\mathrm{y_n},\mathrm{x_n},\mathrm{z_n},\mathrm{w_n}) = \lim_{n\to\infty} \mathrm{g}(\mathrm{y_n}) = \mathrm{y}$, for all x, y ∈ X are satisfied.

Definition 1.14 The bivariate self mapping, i.e., F: X × X → X and self mapping g: X → X of a intuitionistic fuzzy metric space (X, M, N, ∗, ⋄) are said to be weakly compatible if they commute at there coincidence points, that is, if for all x, y ∈ X and t > 0

F(x, y) = g(x) for some x∈X, then F(g(x), g(y)) = g(F(x, y))

and

F(y, x) = g(y) for some y∈X, then F(g(y), g(x)) = g(F(y,x)).

Definition 1.15 The mappings F and g where F: X × X× X× X → X and g: X → X, of an intuitionistic fuzzy metric space (X, M, N, ∗, ⋄) satisfy E.A. property, if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X, such that

$$\lim_{n\to\infty} \mathrm{F}(\mathrm{x_n},\mathrm{y_n},\mathrm{z_n},\mathrm{w_n}) = \lim_{n\to\infty} \mathrm{g}(\mathrm{x_n}) = g(u) \text{ and}$$

$$\lim_{n\to\infty} \mathrm{F}(\mathrm{y_n},\mathrm{x_n},\mathrm{z_n},\mathrm{w_n}) = \lim_{n\to\infty} \mathrm{g}(\mathrm{y_n}) = g(v) \text{for u, v} \in \text{X and t} > 0.$$

Example 1 Let (X, M, N, ∗, ⋄) be intuitionistic fuzzy metric space and g : $R^+ \to R^+$ is an increasing continuous function. For m > 0, we define the function M, N by

(1.1) $M(x,y,t)=\frac{g(t)}{g(t)+m\cdot d(x,y)}$ and $N(x,y,t)=\frac{m\cdot d(x,y)}{g(t)+m\cdot d(x,y)}$

Then for a ∗ b = a·b and a⋄ b = min {1, a + b}, (X, M, N, ∗, ⋄) is an intuitionisticfuzzy metric on X.

As a particular case if we take g(t) = t^n where n ∈ I^+ and m = 1. Then (1.1) becomes

(1.2) $M(x,y,t)=\frac{t^n}{t^n+d(x,y)}$ and $N(x,y,t)=\frac{d(x,y)}{t^n+d(x,y)}$

Then for a ∗ b = min {a, b} and a⋄ b = max {a, b}, (X, M, N, ∗, ⋄) is an intuitionisticfuzzy metric on X.

If we take n = 1 in (1.2), the well-known intuitionistic fuzzy metric obtained.

Example 1.15 Let X = [0, ∞). Consider (X, M, N, ∗, ⋄) be an intuitionistic fuzzy metric spaces as in example 1. We define F: X × X → X and g: X → X as

$$F(x,y)=\begin{cases}\frac{x-y}{5} & if\ x \geq y\\ 0 & if\ x > y\end{cases}\ and\ g(x)=\frac{2x}{5}, for\ x,y \in X$$

F obeys mixed g-monotone property

Let $\{x_{n,}\}$ and $\{y_{n,}\}$ be two sequences in X defined as

$x_n = \frac{1}{n}$ and $y_n = \frac{1}{2n}$

then $\lim_{n\to\infty} F(x_n,y_n) = \lim_{n\to\infty} g(x_n)$= g(0) and $\lim_{n\to\infty} F(y_n,x_n)=\lim_{n\to\infty} g(y_n)$ = g(0) for 0 ∈ X and t > 0, i.e., F and g satisfy E. A. property.

Let the class ∅ of all mappings φ: [0, 1] → [0, 1] satisfying the following properties:

(i) φ is continuous and nondecreasing on [0, 1];

(ii) φ (x) > x for all x ∈ (0, 1).

We note that φ ∈∅, then ∅(1) = 1 and φ(x) ≥ x for all x ∈ [0, 1].

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Let φ be class of function φ: $[0, +\infty) \to [0, +\infty)$ satisfying the following properties:

(iii) φ is continuous and nondecreasing on $[0,+\infty)$;

(iv) $\varphi(t) < t$ for each $t > 0$ and φ (t) = 0 if t =

2. MAIN RESULTS.

In this section, we prove quadruple fixed point for E.A. along with mixed monotone property satisfying for $\emptyset$-contraction, φ -contraction. E.A. property developed the minimize the commutative conditions of the maps to commutativity at their points of coincidence. Moreover, E.A. along with mixed monotoneproperty allows replacing the completeness requirement of theintuitionistic fuzzy metric space with more natural condition of closeness of ranges.

Theorem 2.1 Let (X, M, N, $*$, $\diamond$) be intuitionistic fuzzy metric space and F: $X^4 \times X^4 \to X$,g: X $\to$ X such that F is continuous and has the g mixed monotone property. Assume also that there exist $\varphi \in \Phi$, $\psi \in \Psi$ and x, y, u, v $\in$ X, t > 0

$$(1). M(F(x,y,z,w),F(u,v,h,l),t)$$
$$\geq \emptyset(\min\{M(F(u,v,h,l),g(u),t)\,, M(gy,gv,t), M(gz,gh,t), M(gw,gl,t)\}$$
$$*\{M(F(x,y,z,w),g(u),t), M(F(y,x,z,w),g(v),t), M(F(z,x,y,w),g(h),t), M(F(w,x,y,z),g(l),t)\}$$

$$(2). N(F(x,y,z,w),F(u,v,h,l),t)$$
$$\leq \varphi(\max\{N(F(u,v,h,l),g(u),t)\,, N(gy,gv,t), N(gz,gh,t), N(gw,gl,t)\}$$
$$\lozenge\{N(F(x,y,z,w),g(u),t), N(F(y,x,z,w),g(v),t), N(F(z,x,y,w),g(h),t), N(F(w,x,y,z),g(l),t)\}$$

If F and g satisfy E. A. along with mixed monotone property and g is a closed subspace of X, then F and g have a quadruple common fixed point.

Proof: Since F and g satisfy E. A. property along with mixed monotone, therefore, we can find sequences $\{x_{n,}\}$,$\{y_{n,}\}$,$\{z_{n,}\}$, $\{w_n\}$ in X and the point u, v,h,l in X such that

$$\lim_{n\to\infty} F(x_n, y_n, z_n, w_n) = \lim_{n\to\infty} g(x_n) = g(u)$$

$$\lim_{n\to\infty} F(y_n, x_{n,} z_n, w_n) = \lim_{n\to\infty} g(y_n) = g(v)$$

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$$\lim_{n\to\infty} \mathrm{F}(\mathrm{z_n}, \mathrm{x_n}, \mathrm{y_n}, \mathrm{w_n}) = \lim_{n\to\infty} g\,(z_n) = g(h)$$
$$\lim_{n\to\infty} \mathrm{F}(\mathrm{w_n}, \mathrm{x_n}, \mathrm{y_n}, \mathrm{z_n})\ \lim_{n\to\infty} g\,(w_n) = g(l)$$

Then, using (1) and (2), we obtain

$M(F(x_n, y_n, z_n, w_n), F(u, v, h, l), t)$

$\geq \emptyset(\min\{M(F(u, v, h, l), g(u), t)\,, M(g(y_n), g(v), t), M(g(z_n), gh, t), M(g(w_n), g(l), t)\}$

$* \{M(F(x_n, y_n, z_n, w_n), g(u), t), M(F(y_n, x_n, z_n, w_n), g(v), t), M(F(z_n, x_n, y_n, w_n), g(h), t), M(F(w_n, x_n, y_n, z_n), g(l), t)\}$

$N(F(x_n, y_n, z_n, w_n), F(u, v, h, l), t)$

$\leq \varphi(\max\{N(F(u, v, h, l), g(u), t)\,, N(g(y_n), g(v), t), N(g(z_n), gh, t), N(g(w_n), g(l), t)\}$

$\Diamond \{N(F(x_n, y_n, z_n, w_n), g(u), t), N(F(y_n, x_n, z_n, w_n), g(v), t), N(F(z_n, x_n, y_n, w_n), g(h), t), N(F(w_n, x_n, y_n, z_n), g(l), t)\}$

Taking the limit as n tends to infinity in the above inequality,

$$M(F(g(u), F(u, v, h, l), t)$$
$$\geq \emptyset(\min\{M(F(u, v, h, l), g(u), t)\,, M(g(v), g(v), t), M(g(h), gh, t), M(g(l), g(l), t)\}$$
$$* \{M(F(g(u), g(u), t), M(F(g(v), g(v), t), M(F(g(h), g(h), t), M(F(g(l), g(l), t)\}$$
$$\geq \emptyset(\min\{M(F(u, v, h, l), g(u), t)\,, 1,1,1\} * \{1,1,1,1\}\}$$

$$\geq \emptyset(\min\{M(F(u, v, h, l), g(u), t)\,, 1,1,1\} * 1\}$$
$$\geq \emptyset(\min\{M(F(u, v, h, l), g(u), t)\,, 1,1,1\}$$
$$\geq \emptyset M(F(u, v, h, l), g(u), t)$$

Similarly

$$N(F(g(u), F(u, v, h, l), t)$$
$$\leq \varphi(\max\{N(F(u, v, h, l), g(u), t)\,, N(g(v), g(v), t), N(g(h), gh, t), N(g(l), g(l), t)\}$$
$$\Diamond \{N(F(g(u), g(u), t), N(F(g(v), g(v), t), N(F(g(h), g(h), t), N(F(g(l), g(l), t)\}$$
$$\leq \varphi(\max\{N(F(u, v, h, l), g(u), t)\,, 0,0,0\} \Diamond \{0,0,0,0\}\}$$
$$\leq \varphi(\max\{N(F(u, v, h, l), g(u), t)\,, 0,0,0\} \Diamond 0\}$$

$$\leq \varphi(\max\{N(F(u, v, h, l), g(u), t), 0,0,0)\}$$
$$\leq \varphi N(F(u, v, h, l), g(u), t)$$

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Now, if F(u, v,h,l) ≠ g(u), then 0 < M(F(u, v,h,l), g(u), t) < 1 and N(F(u, v,h,l), g(u), t) > 0, that is,

$\emptyset$(M(F(u, v,h,l), g(u), t)) > M(F(u, v,h,l), g(u), t),

and φ(N(F(u, v,h,l), g(u), t)) < N(F(u, v,h,l), g(u), t),

contradicting the above inequality. This proves that $M(F(u,v,h,l),g(u),t)=1$ and $N(F(u,v,h,l),g(u),t)=0$, which implies due to (ii) and (vii) of definition 1. $F(u,v,h,l)=g(u)$. Similarly it can proved that $F(v,u,h,l)=g(v)$

Our next step to show that $\{g(x_n),g(y_n),g(z_n)\ g(w_n)\}$ are Cauchy sequence in (X, M, N, ∗, ⋄). Since(X, M, N, ∗, ⋄) is complete there exist x,y,z,w∈ X such that

$$\lim_{n\to\infty} g(x_n) = x, \lim_{n\to\infty} g(y_n) = y, \lim_{n\to\infty} g(z_n) = z, \lim_{n\to\infty} g(w_n) = w \text{ ....eq.(i)}$$

From eq.(i) and the continuity of g we have

$$\lim_{n\to\infty} g(gx_n) = gx, \lim_{n\to\infty} g(gy_n) = gy, \lim_{n\to\infty} g(gz_n) = gz, \lim_{n\to\infty} g(gw_n) = gw$$

Let us take $x_0, y_0, z_0, w_0 \in X$ such that

$$gx_0 \le F(x_0, y_0, z_0, w_0), \qquad gy_0 \ge F(y_0, z_0, w_0, x_0),$$
$$gz_0 \le F(z_0, w_0, x_0, y_0), \quad \text{and} \quad gw_0 \ge F(w_0, x_0, y_0, z_0),$$

Since $F(X^4) \subset g(X)$,then we can choose $x_1, y_1, z_1, w_1 \in X$ such that

$$gx_1 = F(x_0, y_0, z_0, w_0), \qquad gy_1 = F(y_0, z_0, w_0, x_0),$$
$$gz_1 = F(z_0, w_0, x_0, y_0), \quad \text{and} \quad gw_1 = F(w_0, x_0, y_0, z_0),$$

Taking into account $F(X^4) \subset g(X)$,by continuing this process, we can construct sequences $\{x_n\}$,$\{y_n\}$, $\{z_n\}$ and $\{w_n\}$ in X such that

$$gx_{n+1}=F(x_n, y_n, z_n, w_n), \qquad gy_{n+1}=F(y_n, z_n, w_n, x_n),$$
$$gz_{n+1}=F(z_n, w_n, x_n, y_n), \qquad gw_{n+1}=F(w_n, x_n, y_n, z_n). \qquad \text{Eq.(ii)}$$

From eq.(ii) and the commutativity of f and g we have

$$g(gx_{n+1})=g(F(x_n,y_n,z_n,w_n)=F(gx_n,gy_n,gz_n,gw_n)$$

$g(gy_{n+1})=g(F(y_n,z_n,w_n,x_n)=F(gy_n,gz_n,gw_n,gx_n)$

$g(gz_{n+1})=g(F(z_n,w_n,x_n,y_n)=F(gz_n,gw_n,gx_n,gy_n)$

and

$g(gw_{n+1})=g(F(w_n,x_n,y_n,z_n)=F(gw_n,gx_n,gy_n,gz_n)$

now we shall show that

$$gx = F(x,y,z,w), gy = F(y,z,w,x), gz = F(z,w,x,y) and\ gw = F(w,x,y,z)$$

By letting $n\to \infty$

$$gx=\lim_{n\to\infty} g(gx_{n+1})=\lim_{n\to\infty} F(gx_n,gy_n,gz_n,gw_n)$$

$$=F(\lim_{n\to\infty} gx_n,\lim_{n\to\infty} gy_n,\lim_{n\to\infty} gz_n,\lim_{n\to\infty} gw_n)$$

$$=F(x,y,z,w)$$

$$gy=\lim_{n\to\infty} g(gy_{n+1})=\lim_{n\to\infty} F(gy_n,gz_n,gw_n,gx_n)$$

$$=F(\lim_{n\to\infty} gy_n,\lim_{n\to\infty} gz_n,\lim_{n\to\infty} gw_n,\lim_{n\to\infty} gx_n)$$

$$=F(y,z,w,x)$$

$$gz=\lim_{n\to\infty} g(gz_{n+1})=\lim_{n\to\infty} F(gz_n,gw_n,gx_n,gy_n)$$

$$=F(\lim_{n\to\infty} gz_n,\lim_{n\to\infty} gw_n,\lim_{n\to\infty} gx_n,\lim_{n\to\infty} gy_n)$$

$$=F(z,w,x,y)$$

$$gw=\lim_{n\to\infty} g(gw_{n+1})=\lim_{n\to\infty} F(gw_n,gx_n,gy_n,gz_n)$$

$$=F(\lim_{n\to\infty} gw_n,\lim_{n\to\infty} gx_n,\lim_{n\to\infty} gy_n,\lim_{n\to\infty} gz_n)$$

$$=F(w,x,y,z)$$

By denoting $F(u,v,h,l) = g(u) = x$, $F(v,u,h,l) = g(v) = y$,$F(h,u,v,l) = g(h) = z$and $F(l,u,v,h) = g(l) = w$. Since $F and\ g$ are weakly compatible then we obtain that $F(x,y,z,w) = gx$, $F(y,x,z,w) = g(y)$,$F(z,x,y,w) = g(z)$and $F(w,x,y,z) = g(w)$.Let us prove that x=$F(x,y,z,w)$

Indeed we obtain by (1) and (2)

$$M(F(x,y.z,w),x,t) = M(F(x,y,z,w),F(u,v,h,l),t)$$

$$\geq \emptyset(\min\{M(F(u,v,h,l),g(u),t)\,, M(y,y,t), M(z,z,t), M(w,w,t)\}$$

$$*\{M(x,x,t), M(y,y,t), M(Fz,z,t), M(w,w,t)\}$$

$$\geq \emptyset M(F(x,y.z,w),x,t)$$

Similarly

$$N(F(x,y.z,w),x,t) = N(F(x,y,z,w),F(u,v,h,l),t)$$

$$\leq \varphi(\max\{N(F(u,v,h,l),g(u),t)\,, N(y,y,t), N(z,z,t), N(w,w,t)\}$$

$$\Diamond \{N(x,x,t), N(y,y,t), N(Fz,z,t), N(w,w,t)\}$$

$$\leq \varphi N(F(x,y.z,w),x,t)$$

If $F(x,y,z,w) \neq x$ then from (ii) and (vii) of definition 1.40 $< M(F(x,y,z,w),x,t) < 1$ and $N(F(x,y,z,w),x,t) \geq 0$ for all t> 0 and therefore

$$\emptyset\big(M(F(x,y,z,w),x,t)\big) > M(F(x,y,z,w),x,t)$$

And

$$\varphi\big(N(F(x,y,z,w),x,t)\big) < N(F(x,y,z,w),x,t)$$

Which contradicts the above inequality.Thus we obtain that $F(x, y. z, w) = x$.Hence x is quadruple fixed point of F and g. Similarly it can be proved that y, z and w is common fixed point of F and g.

Corollary 2.2 Let (X, M, N, ∗, ⋄) be intuitionistic fuzzy metric space and F: X × X× X× X → X, g: X →X be weakly compatible maps of X such that, for some φ ∈ Φ and x, y, z, w, u, v, h, l ∈ X, t > 0,(2.2) M(F(x, y), F(u, v), t) ≥ φ(min{M(g(x), g(u), t)}),
and

N(F(x, y), F(u, v), t) ≤ ψ (max{N(g(x), g(u), t)})
If F and g satisfy E. A. property along with mixed monotone property and g is a closed subspace of X, then F and g have a unique quadruple common fixed point.

3. **Results&Conclusion**

Results: The main finding of this paper is the identification of unique quadruple common fixed point and use with Intuitionistic fuzzy metric spacethe relevant application with appropriate supporting examples.

Conclusion

In this research paper, we have provedQuadruple FixedPoint Theorem in Intuitionistic Fuzzy Metric Space using E. A. along with mixed monotone Property. In future we can use theses application in real life.

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