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# STUDY OF RETRIAL QUEUEING MODEL WHEN FOR SERVER IS NON-RELIABLE AND WITH SYSTEM BREAKDOWN

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**Abstract: -**Generally it is assumed that in a queueing system service provider is giving seriatim service to the customers. But, many circumstances in real- world when we see the various situation where service gets interrupted due to breakdown in service mechanism. When the customers arrive for the service and the server is busy, instead of would not join queue, the orbit of infinite size. The customers in the orbit will try for service one by one when the server is idle under the classical retrial policy with retrial rate $n\alpha$, where $n$ is the size of the orbit. During a working vacation period, if there are customers in the system at a service completion instant, the vacation will be interrupted. Under the stable condition, the probability generating functions of the number of customers in the orbit are obtained. Various system performance measures are also developed. Finally, some numerical examples and cost optimization analysis are presented.

***Keywords:*** *- Poisson arrival, Service time is exponential, Single server, working queue, idle queue and retrial policy.*

## Introduction:-

Most of the studies of this type of including those of Gaver in 1962 and Thiruvengudan (1963) single server system with exponential distribution. White and Christic (1958) seem to have studied first a queueing model with single channel which provide service with breakdown. The term removable server is just an abbreviation for the system of turning on and turning off the server, depending on number of customer in the system. A non-reliable sever means that a server is typically subject to unpredictable breakdowns. The server is removable under N-Policy; turn the server on whenever N ( N >1 ) or more customer are presents, turn the server off only when no customer are presents. For the non-reliable server, Avi – Itzhak and Naor[1967] studied the ordinary M/M/1 queueing model where the service does not depend upon the number of customer in the queue. Neut and Lucanton [1983] Studied a Markovian queueing system single/multiple server subject to breakdown and repair. In these type of breakdowns the customer are waiting to serve and server goes for vacations and after repairing he come back for service. But here we assume that during the breakdown period the server can do some another work, additional work aside from service of customers arriving at the queue or pending work to utilize that time. The server schedule the service of these tasks during the breakdown period.

In present paper, we will study the optimal operation of a single removable and non-reliable server in a Markovian queueing system under steady-state condition. The system is in idle state before the arrival of customer and after the arrival of customer when system is in working state, it may breakdown and system goes to vacation during that period. After getting some repair facility server again come back for service to customer. Interarrival and service time distributions of customers are assumed to be exponentially distributed. Breakdown and repair time distributions of the server are assumed to be exponentially distributed. The total expected cost function per unit time is developed to obtain the optimal operating policy at minimum cost.

**The queueing system may be described as follows:**

(i) Arrival of customer are according to poisson process with parameter $\lambda$.

(ii) Service time of customer are exponentially distribution with parameter $1/\mu$.

(iii) As long as the server on, it will serve the customer immediately.

(iv) When it is working, it is assumed that breakdown can happen at any time, with poisson rate $\phi_1$.

(v) Whenever breakdown occur it is repaired immediately with repair rate $\phi_2$ where repair rate are assumed to be exponentially distributed.

(vi) The server goes on vacation when there is no customer in the server and come back only when N customer in the queue.

**Solution of the Model**

Let us suppose

i = 0 denotes the server is in idle state when there is no customer in system.

i = 1 denotes the server is in working state.

i = 2 denotes the server is in working state and found to be breakdown.

$p_0(r) \rightarrow$ probability that there are r customers in the system when server is in idle state or turned off, where r = 0, 1, ……………………….………N – 1.

$p_1(r) \rightarrow$ probability that there are r customers in the system when server is turn on and in working state, where r = 1, 2,………

$p_2(r) \rightarrow$ probability that there are customer in the system when system is in working state and found to be broken, where r = 1, 2, ……..

**The steady-state equations:-**

$$\lambda P_0(0) = \mu P_1(1) \tag{1}$$

$$\lambda P_0(r) = \lambda P_0(r-1), \qquad 1 \le r \le N-1 \tag{2}$$

$$(\lambda + \phi_1 + \mu)\, P_1(1) = \mu P_1(2) + \phi_2 P_2(1) \tag{3}$$

$$(\lambda + \phi_1 + \mu)\, P_1(r) = \lambda P_1(r-1) + \mu P_1(r+1) + \phi_2 P_2(r), \qquad 2 \le r \le N-1 \tag{4}$$

$$(\lambda + \phi_1 + \mu) P_1(N) = \lambda P_0(N-1) + \lambda P_1(N-1) + \mu P_1(N+1) + \phi_2 P_2(N) \tag{5}$$

$$(\lambda + \phi_1 + \mu)\, P_1(r) = \lambda P_1(r-1) + \mu P_1(r+1) + \phi_2 P_2(r), \qquad r \ge N+1 \tag{6}$$

$$(\lambda + \phi_2)\, P_2(1) = \phi_1 P_1(1) \tag{7}$$

$$(\lambda + \phi_2)\, P_2(r) = \lambda P_2(r-1) + \phi_1 P_1(r), \qquad r \ge 2 \tag{8}$$

Let generating function techniques may be used to obtain analytical solution $P_0(0)$.Define the respective generating functions of $P_0(r)$, $P_1(r)$, $P_2(r)$ as follows:

$$Q_0(z) = \sum_{r=0}^{N-1} z^r P_0(r) \qquad |z| \le 1$$

$$Q_1(z) = \sum_{r=1}^{\infty} z^r \,.\, P_1(r)$$

$$Q_2(z) = \sum_{r=1}^{\infty} z^r \,.\, P_2(r) \tag{9}$$

In terms of $P_0(0)$, we may express

$$Q_0(z) = \sum_{r=0}^{N-1} z^r \,.\, P_0(r)$$

$$= P_0(0) \left[ \frac{1 - z^N}{1 - z} \right] \tag{10}$$

Using equation (2.1) to (2.8) ,we get

$\lambda.\, zP_0(0) + (\lambda+\phi_1+\mu)z.\, Q_1(z) = \lambda.z\, P_0(0).\, z^N + \lambda\, z^2.\, Q_1(z) + \mu\, Q_1(z) + \phi_2\, z\, Q_2(z)$

$$[\lambda z^2-(\lambda+\phi_1+\mu)z+\mu]Q_1(z)+\phi_2.zQ_2(z)=\lambda z(1-z^N)P_0(0) \quad (11)$$

and

$[\lambda + \phi_2]\, Q_2(z) = \phi_1\, Q_1(z) + \lambda\, z.\, Q_2(z),$

$$\phi_1\, Q_1(z) + (\lambda z - \lambda - \phi_2)\, Q_2(z) = 0 \quad (12)$$

$$Q_2(z) = \frac{\phi_1}{\lambda+\phi_2-\lambda z} Q_1(z) \quad (13)$$

$$Q_1(z) = \frac{\lambda.z(1-z^N).(\lambda z-\lambda-\phi_2).P_0(0)}{[\lambda z^2-(\lambda+\phi_1+\mu)z+\mu][\lambda z-\lambda-\phi_2]-\phi_2 z\phi_1} \quad (14)$$

$$Q_2(z)=\frac{\phi_1\lambda.z(z^N-1)}{[\lambda z^2-(\lambda+\phi_1+\mu)z+\mu](\lambda z-\lambda-\phi_2)-\phi_2 z\phi_1} P_0(0) \quad (15)$$

Let Q(z) denote the generating function of the number of customer in the (queue) system then, adding equation (2.11), (2.14) and (2.15)

$Q(z) = Q_0(z) + Q_1(z) + Q_2(z)$

$$=\left[\frac{1-z^N}{1-z}+\frac{\lambda z(1-z^N)(\lambda z-\lambda-\phi_1-\phi_2)}{[\lambda z^2-(\lambda+\phi_1+\mu)z+\mu][\lambda z-\lambda-\phi_2]-\phi_1 z\phi_2}\right]P_0(0) \quad (16)$$

The value of $P_0(0)$ can be obtained from normalized condition

$$\underset{z\to 1}{\mathrm{lt}}\ Q(z) = 1$$

$$P(z)=\left[\frac{1-z^N}{1-z}+\frac{\lambda z(1-z^N)(\lambda z-\lambda-\phi_1-\phi_2)}{[\lambda z^2-(\lambda+\phi_1+\mu)z+\mu][(\lambda z-\lambda-\phi_2)]-\phi, z\phi_2}\right] P_0(0)$$

$$= \frac{N.\mu\phi_2}{\mu\phi_2-\phi_2\lambda-\lambda\phi_1}$$

Now taking $z\to 1$

$$\text{Hence } P_0(0) = \frac{\mu\phi_2-\lambda(\phi_2+\mu)}{N\mu\phi_2} \quad (17)$$

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**Operational Characteristics: -** is given by

$E_s = E_1 + E_2 + E_3$

Where

$E_1$ : Number of customer in system/queue when system is in idle state

$E_2$ : Number of customer in system when system is in working state

$E_3$ : Number of customers is in the system when the server is in broken state

$E_s$ : Number of customers in system

$$E_1 = \sum_{r=1}^{N-1} r.\, P_0(r)$$

$$E_2 = \sum_{r=1}^{\infty} r\, P_1(r)$$

$$E_3 = \sum_{r=0}^{\infty} r\, P_2(r)$$

$$E_1 = \frac{N-1}{2} - \rho\frac{(N-1)(\phi_1+\phi_2)}{2} \qquad \left[\frac{\lambda}{\mu} = \rho\right] \tag{18}$$

Similarly we will find $E_2$ and $E_3$, by using equation (15) and (16) and

$$E_2 = \frac{\lambda}{\mu}\frac{(N+1)}{2} + \frac{\lambda^2}{\mu^2}\frac{(\lambda+\phi_1+\phi\phi_1+\phi_2^2)}{\phi_2\left[\phi_2 - \frac{\lambda}{\mu}[\phi_1+\phi_2]\right]} \tag{19}$$

$$E_3 = \frac{\lambda}{\mu}.\frac{\phi_2(N+1)}{2P_2} - \left(\frac{\lambda}{\mu}\right)^2 \frac{\phi_2(\lambda-\phi_2-\phi_1-\mu)}{\phi_2\left[\phi_2 - \frac{\lambda}{\mu}[\phi_1+\phi_2]\right]} \tag{20}$$

$$E_S = \frac{(N-1)}{2} + \frac{\rho(\lambda\phi_1+\phi_1\phi_2+\phi_2^2)}{\phi_2(\phi_2-\rho(\phi_1+\phi_2)} \tag{21}$$

Here if we put N = 1 in equation (20) the expected number of customer in the system for the ordinary M/M/1 queueing system with non –

**Proportion of time in the server:**

Let us suppose

$P_{ID}$ : Probability that the server is idle

$P_{BS}$ : Probability that the server is in working state

$P_{BD}$ : Probability that server is breakdown

Then we have the following expression

$$P_{ID} = 1 - \frac{\rho}{\phi_2}[\phi_1 + \phi_2] \tag{22}$$

$$\boxed{P_{BS} = \rho = \frac{\lambda}{\mu}} \tag{23}$$

$$\boxed{P_{BD} = \frac{\phi_1}{\phi_2}\rho} \tag{24}$$

**Conclusion:-** Therefore here we prove that N-policy Markovian queueing system with a non-reliable server, the probability that the server is busy in steady state is equal to the traffic intensity. It is important that Heyman has shown that fraction of time the server is busy for the N policy M/G/1 queueing system with reliable server is traffic intensity too.

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