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ADVANCED MATHEMATICAL MATRICES

REVIEWING STRUM SEQUENCE FOR CONSTRUCTION OF PYRAMID

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ABSTRACT

Matrices have vast use in Engineering field for solving difficult problems in simpler way. In which Eigen values and Eigen Vector problems have wide variety of applications like Power Method whichis used to compute the Largest Eigen value and corresponding Eigen vector of a given system, that tool is used to solve various Damped and Undammed system of equations for Dynamic Structure. House Holder Method and Given Method, both are used to compute atridiagonal matrix which is further applied to get diagonal matrix by Ruti-Shausher method. These all methods have very large applications in Engineering fields for various calculations in various fields like Robotics (DH Table), Finite Element Method, Vibrations, Structural Dynamics, Complicated Elasticity and Plasticity Problems and many more. This paper explains the method of Strum Sequence which will be used for calculation of Eigen Values which can be further used to construct pyramid and can be very helpful in Civil Engineering field for estimation of building height by this method.

Keywords: *Civil Engineering, Eigen Values, House Holder's Method, Matrices, Strum Sequence.*

I. INTRODUCTION:

This paper describes the application of Eigen values and Eigen Vectors with respect to Civil Engineering field. The Advanced mathematical matrix methods help us to solve many engineering problems and the method of Strum Sequence is described in this paper which can be used to find the height or elevation of the building/bridge. The method of Strum sequence not only can be used to find the building height but can also be used to find many solutions related to various other engineering branches. [*]We can construct the matrix for total effective work done by the employees of the company in a particular period of time and eventually finding the total work done by them in specified time. The effect of earthquake on a structure can also be determined and hence the building variations can be calculated from Eigen Values and Eigen Vector. Strum sequence,can be solved for formulated tridiagonal matrixand then we can get the eigen valuesfor all the equations and the pyramid structure can be generated from those values.

II. STRUM SEQUENCE METHOD:

1. This method is applicable for tridiagonal matrix. But a symmetric matrix has to be converted into tridiagonal matrix.
2. To convert symmetric matrix in Tridiagonal matrix, Householder Method andGiven methodshould be used.
3. Considering Tridiagonalmatrix, we will be using the steps given below for eigen values using Strum sequence.

Consider a tridiagonal matrix

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$$A=\begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & a_{23} \\ 0 & a_{23} & a_{33} \end{bmatrix}$$

To find eigen value for A,

The characteristic equation of A is $|A - \lambda I| = 0$

Consider the characteristic function $F_3(\lambda)$ as

$$F_3(\lambda)=\begin{vmatrix} a_{11}-\lambda & a_{12} & 0 \\ a_{12} & a_{22-\lambda} & a_{23} \\ 0 & a_{23} & a_{33-\lambda} \end{vmatrix}$$

Where,

$F_0(\lambda) = 1$

$F_1(\lambda) = (a_{11-\lambda})$

$= (a_{11-\lambda})F_0(\lambda)$

$$F_2(\lambda) = \begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{12} & a_{22-\lambda} \end{vmatrix}$$

$=(a_{22-\lambda})(a_{11}-\lambda) - a_{12}^2$

$=(a_{22-\lambda})F_1(\lambda) - a_{12}^2 F_0(\lambda)$

$$F_3(\lambda)=\begin{vmatrix} a_{11}-\lambda & a_{12} & 0 \\ a_{12} & a_{22-\lambda} & a_{23} \\ 0 & a_{23} & a_{33-\lambda} \end{vmatrix}$$

$$=(a_{33-\lambda})\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{12} & a_{22-\lambda} \end{vmatrix} - (a_{23})\begin{vmatrix} a_{11}-\lambda & 0 \\ a_{12} & a_{23} \end{vmatrix}$$

$=(a_{33-\lambda})F_2(\lambda) - a_{23}^2(a_{11}-\lambda)$

$=(a_{33-\lambda})F_2(\lambda) - a_{23}^2 F_1(\lambda)$

In General;

$F_k(\lambda)= (a_{kk} - \lambda)\, F_{k-1}(\lambda) - (a_{k-1,\,k} - \lambda)^2\, F_{k-2}(\lambda),\ 2\le k \le n$

The equation $F_k(\lambda) = 0$ is the characteristic equation of A of (n*n) order. Then the roots of $F_k(\lambda)=0$ will be called Eigen Values of the Tridiagonal Matrix A.

Now, the sequence

$\{F_k(\lambda): 0\le k \le n\}$ is called "Strum Sequence"

To construct Pyramid: -

$F_1(\lambda) = (a_{11-\lambda}) = 0$

$\Rightarrow \lambda = a_{11} = \lambda_1^{(3)}$

$F_2(\lambda) = 0$

$\Rightarrow \lambda = \lambda_1^{(2)}$ and $\lambda_2^{(2)}$

$F_3(\lambda) = 0$

$\Rightarrow \lambda = \lambda_1^{(1)}, \lambda_2^{(1)}$ and $\lambda_3^{(1)}$

Conditions-

$\lambda_1^{(1)} < \lambda_1^{(2)} < \lambda_1^{(3)}$

$\lambda_2^{(1)} < \lambda_2^{(2)}$

$\lambda_3^{(1)}$

Pyramid Structure-

$\lambda_1^{(3)}$

$\lambda_1^{(2)} < \lambda_2^{(2)}$

$\lambda_1^{(1)} < \lambda_2^{(1)} < \lambda_3^{(1)}$

Example 1-

Construct Pyramid using strum sequence

$$A=\begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

Solution-

To find eigen value for A,

Consider the characteristic function $F_3(\lambda)$ as

$$F_3(\lambda)=\begin{bmatrix} 4-\lambda & 2 & 0 \\ 2 & 6-\lambda & 2 \\ 0 & 2 & 4-\lambda \end{bmatrix}$$

Characteristic equation –

$|A - \lambda I| = 0$

Where,

$F_0(\lambda) = 1$

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$F_1(\lambda) = (4-\lambda)$

$= (4-\lambda)F_0(\lambda)$

$\lambda = 4$

$F_2(\lambda) = \begin{vmatrix} 4-\lambda & 2 \\ 2 & 6-\lambda \end{vmatrix}$

$=(4-\lambda)(6-\lambda)-2^2$

$=(6-\lambda)F_1(\lambda)-4F_0(\lambda)$

λ = 7.24 and 2.76

$F_3(\lambda) = \begin{vmatrix} 4-\lambda & 2 & 0 \\ 2 & 6-\lambda & 2 \\ 0 & 2 & 4-\lambda \end{vmatrix}$

$=(4-\lambda)\begin{vmatrix} 6-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} - (2)\begin{vmatrix} 4-\lambda & 0 \\ 2 & 2 \end{vmatrix}$

$=(4-\lambda)F_2(\lambda)-2(4-\lambda)(2)$

λ = 8, 4 and 2

Roots-

{$(4-\lambda)$, $(\lambda^2 - 10\lambda + 20)$, $(-\lambda^3 + 14\lambda^2 - 56\lambda + 64)$} is Strum Sequence for A

Pyramid-

$\lambda_1^{(1)} < \lambda_1^{(2)} < \lambda_1^{(3)}$

$\lambda_2^{(1)} < \lambda_2^{(2)}$

$\lambda_3^{(1)}$

$2 < 2.76 < 4$

$4 < 7.24$

8

Pyramid Structure-

4

2.76<7.24

2<4<8

Example 2-

Solve by using strum sequence

$$A=\begin{bmatrix} 6 & 3 & 0 \\ 10 & 7 & 4 \\ 14 & 11 & 8 \end{bmatrix}$$

Solution-

<u>Step I</u>- Convert the given A matrix to Symmetric matrix and we get

$$A=\begin{bmatrix} 6 & 6.5 & 7 \\ 6.5 & 7 & 7.5 \\ 7 & 7.5 & 8 \end{bmatrix}$$

<u>Step II</u>- By using House Holder's method, converting the symmetric matrix to Tridiagonal Matrix

$$A=\begin{bmatrix} 6 & -9.53 & 0.015 \\ -9.53 & 14.95 & 0.0323 \\ 0.015 & 0.0323 & -0.016 \end{bmatrix}$$

<u>Step III</u>- Solving this Tridiagonal matrix A by Strum Sequence

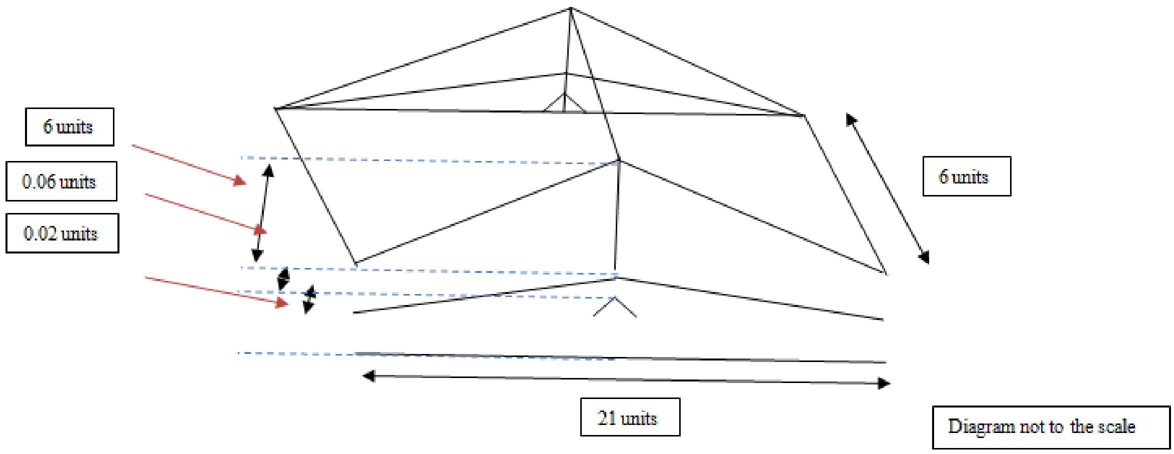
On condition we get the result below,

6

-0.06<21.018

-20.88<-0.025<-0.020

Step IV-Output Diagram from Eigen Values



III.APPLICATIONS:

1. Construction of Pyramid
2. Estimating Height of Building / Bridge
3. In Civil Engineering for studying various mode shape due to Dynamic loading conditions in structural Dynamics
4. Studying the Effect of Earthquake on building by estimating the shape of building which we obtain from Eigen values and Eigen Vectors.

IV.CONCLUSION:

The strum sequence problem shows us the application which can be used in the construction field for estimating the pyramid shape by Eigen values and Eigen vectors and can be useful for deriving the bridge's dimension if matrix has been chosen from the concept of [*] in introduction. This strum sequence can further be used into Finite element Analysis based softwares in future which can estimate the results by adding the matrix into it and can give results based on strum sequence. That future workcan also be used in cross checking the manual calculations done in Civil Engineering field with the software analysis done by strum sequence. Even also the stability of the diagram [step IV] is my future work.

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