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FIXEDPOINT THEOREMS FOR MULTIVALUED MAPPINGS ON MENGER SPACES

Reenu Ahuja[1], Anil Rajput[2], Abha Tenguria[3] and Savita Jain[4]

[1]*Dept. of Mathematics, VNS Engineering College, Bhopal, MP India*

[2]*Dept. of Mathematics, CSA Govt. PG College, Sehore, MPIndia*

[3]*Dept. of Mathematics, Govt.Girls MLB College, Bhopal, MPIndia*

[4]*Jain Sadan, Gandhi Nagar Itarsi, District Hoshangabad MP India*

Abstract :

In the present work, we prove a fixed-point theorem in Menger spaces through weak compatibility.In this paper, using the idea of weak compatibility due to Singh and Jain and the idea of compatibility due to Mishra, we prove a common fixed-point theorem of six maps under the condition of weak compatibility and compatibility in Menger space and give an example to illustrate the theorem.

1.1 Introduction: Fixed-point theory concern itself with a very basic mathematical setting. It is also well known that one of the fundamental and most useful results in fixed-point theory is Banach fixed-point theorem. This result has been extended in many directions for single and multivalued cases and various contraction conditions [3-6,9-12] on a metric space.There have been a number of generalizations of metric space. One such generalization in Menger space introduced in 1942 by Menger who was use distribution function instead of non-negative real number as values of the metrics.

The concept of metric space has been extended in various ways. **Gahler [3**], in which a positive real number is assigned to every three elements of the space, has made one such extension. He initiated the study of 2-metric spaces and some fixed-point theorems in 2-metric spaces were proved in [6,10].

Probabilistic 2-metric space is the probabilistic generalization of 2-metric space**. Zeng** [**12]** first introduced the concept of probabilistic 2-metric space and so it isof interestto develop the fixed point theory in such spaces. Many fixed-point theorems for single valued and multivalued mappings in probabilistic 2-metric spaces have been proved [4-6].

Menger space is a particular type of probabilistic metric space in which the triangular inequality is postulated with the help of a t-norm. The theory of Menger spaces is an important part of stochastic analysis

In an interesting paper [5], Hicks observed that fixed point theorems for certain contraction mappings on a Menger space endowed with a triangular t-norm may be obtained from corresponding results in metric spaces.

Jungck introduced the concept of compatible maps. Moreover, this condition has further been weakened by introducing the notion of weakly compatible mappings by Jungck and Rhoades [7].

Mishra [8] extended this notion of compatibility to probabilistic metric space. Several common fixed-point theorems have been proved for compatible maps in probabilistic metric space.Recently in this line, Singh and Jain [11] introduced the notion of weakly compatible maps in Menger space to establish a common fixed-point theorem.

Cho et al. [1] studied the notion of compatible mappings of type (A) (introduced by Jungck et al. [25] in metric spaces) in Menger spaces which is equivalent to the concept of compatible mappings under some conditions.

Recently **O. Hadzic**[4] proved the existence of a common fixed point for the sequence of self-mappings $\{Aj\}(j = 1,2,\dots), SandT$where Aj commutes with S and T. His result is as follows:

Let (M, d) be a complete metric space,$S, T: M \to M$ be continuous, $Aj: M \to SM \cap TM(j =$ 1,2 so that A_j commutes with$S$ and T and for every i,j(i =j,i,j =1,2….) and every $z, y \in M$:

$d(A_i x, A_j y) < qd(Sx, Ty), 0 < q < 1$ (1.1)

We recall some definitions and known results in Menger probabilistic metric space [2,6]

1.1 Definition: A triangular norm * (shortly t-norm) is a binary operation on the unit interval [0,1] such that for all a, b, c, d ∈ [0,1] the following conditions are satisfied:

(a) $a * 1 = a$;

(b) $a * b = b * a$;

(c) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$;

(d) $a * (b * c) = (a * b) * c$.

1.2 Examples of t-norms are $a * b = \max\{a + b - 1, 0\}$ and $a * b = \min\{a,b\}$.

1.3 Definition A distribution function is a function F: $[-\infty, \infty] \rightarrow [0,1]$ which is left continous on R, non decreasing and $F(-\infty) = 0, F(\infty) = 1$.

We will denote by Δ the family of all distribution functions on $[-\infty, \infty]$. H is a special element of Δ defined by

$$H(t) = \begin{cases} o \; if \; t \leq o \\ 1 \; if \; t > o \end{cases}$$

If X is a nonempty set, $F : X \times X \rightarrow \Delta$ is called a probabilistic distance on X and F (x,y) is usually denoted by F_{xy}.

1.5 Definition The ordered pair (X,F) is called a probabilistic metric space (shortly PM-space) if X is a nonempty set and F is a probabilistic distance satisfying the following conditions:

for all x, y, z $\in X$ and t, s > 0,

(FM-0) $F_{xy}(t) = 1 \Leftrightarrow x = y$;

(FM-1) $F_{xy}(0) = 0$;

(FM-2) $F_{xy} = F_{yx}$;

(FM-3) $F_{xz}(t) = 1, F_{zy}(s) = 1 = F_{xy}(t + s)$=1

The ordered triple $(X, F,*)$ is called Menger spaces if (X, F) is a PM- space, $*$ is a t-norm and the following condition is satisfies: for all $x, y, z \in X$ and $t > 0$. If t-norm $*$ is

$$a * b = min\{a, b\} for\ all\ a, b \in [0,1]$$

then $(X, F,*)$ is a Menger space. Further, $(X, F,*)$ is a complete Menger space if(X, d) is complete.

1.6 Definition: Let $(X, F,*)$ be a Menger space and $*$ be a continuous t-norm.

a) A sequence $\{x_n\}$ in X said to be converge to a point x in X (written $x_n \to x$) iff for every $\in > 0\ and \lambda \in (0,1)$there exists an integer$n_0 = n_0 (\in \lambda)$ such that $F_{xn,x}(\in) > 1 - \lambda foralln \geq n_0$.

b) A sequence $\{x_n\}$ in X said to be Cauchy if for every $\in > 0\ and\lambda \in (0,1)$ there exists an integer $n_0 = n_0 (\in \lambda)$ such that

$$F_{xn,xn+p}(\in) > 1 - \lambda foralln \geq n_0\ andp > 0.$$

c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

1.7 Definition[8]: Self-maps A and B of a Menger space $(X, F,*)$ are said to be compatible $F_{ABxn,BAxn}(t) \to 1$ for all t > 0 , whenever $\{x_n\}$is a sequence in X such that $F_{Axn,Bxn}(t) \to$ x for some x in X as n → ∞.

1.8 Definition: A Probabilistic 2-metric space is an ordered pair(X, F), where $X$is an arbitrary set and F is a mapping from X^3 into the set of distribution functions. The distribution function $F_{x,y,z}(t)$ will denote the value of Fx,,at the positive real number t. The function, is assumed to satisfy the following conditions:

(i) $F_{x,y,z}(0) = 0$ for all $x, y, z \in X$;

(ii) $F_{x,y,z}(t) = 1$for all $t > 0$ if and only if atleast two of the three points x, y, z are equal;

(iii) for distinct points $x, y \in X$, there exists a point $z \in X$ such that$Fx,y,z(t) \neq 1$ for $t > 0$

(iv) $F_{x,y,z}(t) = F_{y,z,x}(t) = F_{z,x,y}(t) = \cdots$ for all $x, y, z \in X$ and $t > 0$;

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(v) if $F_{x,y,w}(t_1) = 1$, $F_{x,w,z}(t_2) = 1$, and $F_{w,y,z}(t_3)$=1, then $F_{x,y,z}(t_1 + t_2 + t_3) = 1$ for all $x, y, z, w \in X$ and $t_1, t_2, t_3 > 0$.

1.9 Definition:ψ- function-A function ψ: [0, 1] x [0, 1] → [0, 1] is said to be a ψ - function if

(i) ψ -is monotone increasing and continuous,

(ii) $\psi(1,1,x,x,x) = x$

(iii) $\psi(1,1,1,1,1) = 1, \psi(0,0,0,0,0) = 0$

(iv) $\psi(0,x,0,x,x) > x$

2. Main result

2.1 Theorem:-- Let (X,F,Δ) be a compatible Menger space, where,Δ is the 3rd order minimum t-norm and the mapping T: X→ X be a self mapping which satisfies the following inequality for all x, y, P ∈ X,

$$F_{Tx,Ty,p}\left(\emptyset(t)\right) \geq \psi\left(F_{x,Tx,p}\left(\emptyset\left(\frac{t_1}{a}\right)\right), F_{y,Ty,p}\left(\emptyset\left(\frac{t}{b}\right)\right), F_{x,y,p}\left(\emptyset\left(\frac{t_3}{c}\right)\right),\right.$$
$$\left. F_{x,ty,p}\left(\emptyset\left(\frac{t_4}{d}\right)\right), F_{Tx,y,p}\left(\emptyset\left(\frac{t_5}{e}\right)\right)\right)$$

Where $t_1, t_2, t_3, t_4, t_{5,}$ t> 0 with t= $t_1 + t_2 + t_3 + t_4 + t_{5,}$ $a, b, c, d, e > 0$ with 0 < a+b+c+d+e < 1, ψ is a ψ- function and $\emptyset isa \emptyset$ function.Then$T$has a unique fixed point of$X$.

Proof:-- Let $x_0 \in X, wenowasequence \{x_n\}\ asfollows: x_n = Tx_{n-1}, n$ ∈ X.

When N is the set of all positive integers.

Now we have for $t_, t_1, t_2, t_3, t_4, t_5 > 0\ witht = t_1 + t_2 + t_3 + t_4 + t_{5,}$

$$F_{x_{n+1},x_n,p}\left(\emptyset(t)\right) = F_{Tx_n,Tx_{n-1},p}\left(\emptyset(t)\right)$$

$$\geq \Psi\left(F_{x_n,Tx_n p},\left(\emptyset\left(\frac{t_1}{a}\right)\right), F_{x_n,Tx_{n-1},p}\left(\emptyset\left(\frac{t_2}{b}\right)\right), F_{x_n,Tx_{n-1},p}\left(\emptyset\left(\frac{t_3}{c}\right)\right),\right.$$

$$\left. F_{x_n,Tx_{n-1},p}\left(\emptyset\left(\frac{t_4}{d}\right)\right), F_{x_n,x_{n+1},p}\left(\emptyset\left(\frac{t_5}{d}\right)\right)\right)$$

$$= \Psi\left(F_{x_n,x_{n+1}p},\left(\emptyset\left(\frac{t_1}{a}\right)\right), F_{x_n,x_n,p}\left(\emptyset\left(\frac{t_2}{b}\right)\right), F_{x_n,x_n,p}\left(\emptyset\left(\frac{t_3}{c}\right)\right),\right.$$

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$$F_{x_n,x_n,p}(\emptyset(\frac{t_4}{d}) \quad ,F_{x_n,x_{n+1},p}(\emptyset\left(\frac{t_5}{d}\right))$$

$$=\Psi\ (F_{x_{n+1},x_n,p}\left(\emptyset\left(\frac{t_1}{a}\right)\right) \quad ,\ F_{x_n,x_n,p}\left(\emptyset\left(\frac{t_2}{b}\right)\right), F_{x_n,x_n,p}\left(\emptyset\left(\frac{t_3}{c}\right)\right),$$

$F_{x_n,x_n,p}(\emptyset(\frac{t_4}{d}) \quad ,F_{x_{n+1},x_n,p}(\emptyset\left(\frac{t_5}{d}\right))$ (2.2)

Let $t_1 = \frac{at}{a+b+c+d+e}$, $t_2 = \frac{bt}{a+b+c+d+e}$

$t_3 = \frac{ct}{a+b+c+d+e}$, $t_4 = \frac{dt}{a+b+c+d+e}$

$t_5 = \frac{et}{a+b+c+d+e}$, and f= a+b+c+d+e, then obviously we have $0 < f < 1$.

Thenwe have

$$F_{xn+1,xn,}P^{(\emptyset(t))} \geq \Psi F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t1}{f}\right)\right),}F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t2}{b}\right)\right)}, F_{xn+1,xn-1,}P^{\left(\emptyset\left(\frac{t5}{e}\right)\right)}\ldots \quad (2.3)$$

We now claim that foe all t> 0,

$F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}$(2.4)

If possible, let for some a t> 0,

$$F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)<}F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}$$

$$F_{xn+1,xn,}P^{(\emptyset(t))} \geq \Psi \quad (F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}, F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)},$$

$$F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} > F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq F_{xn+1,xn,}P^{(\emptyset(t))}$$

Where is a contradiction, since $0 < c < 1$, $\emptyset isstrictlyincreasingandFisnondecreasing.$

There for all $t > 0$

$$F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}$$

$$F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}$$

Hence, using (2.4) we have from (2.3),

$$F_{xn+1,xn,}P^{(\emptyset(t))} \geq \Psi(F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)},$$

$$F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}, F_{xn+1,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq \Psi(F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)},$$

$$F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}, F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq F_{xn,xn-1,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)}(2.5)$$

By repeated application of (2.5) we have after n steps.

$$F_{xn+1,xn,}P^{\left(\emptyset\left(\frac{t}{f}\right)\right)} \geq Fx_1, x_0, p^{\left(\emptyset\left(\frac{t}{f^n}\right)\right)}$$

Therefore, $\lim_{n\to\infty} F_{xn+1,xn,}P^{(\emptyset(t))=1}$ for all t $> 0(2.6)$

By virtue of property of $\emptyset and F$, we can choose$s > 0$ such that $s > Then for all P \in X and t > 0$ we have,

$$\lim_{n\to\infty} F_{xn,xn+1,}P^{(s)} = 1 \ \ (2.7)$$

We next prove that $\{ x_n\}$be not a Cauchy sequence. If possible, let $\{x_n\}$ be not a Cauchy sequence. Then there exist $\in > 0$ and $0 < \lambda < 1$ for m(k) $> n(k) >$ k such that

$F_{xm(k)}, x_{n(k)}, p(\in) \leq 1 - \lambda$ (2.8)

We take m(k) corresponding to n(k) to be the smaller integer satisfying (2.8), so that

$F_{xm(k)-1}, x_{n(k)}, p(\in) < 1 - \lambda$(2.9)

If $\in_1 < \in$ $then we have$

$$F_{xm(k)}, x_{n(k)}, p^{(\in_1)} \leq F_{xm(p^{(\in)}k)}, x_{n(k)},$$

We conduct that it is possible to construct $\{ x_{m\,(k)}\}$ and $\{ x_{n\,(k)}\}$ of $\{x_n\}$ with m(k) $>$ $n(k) >$ k such that and satisfying (2.8), (2.9) whenever $\in$ isreplaced by a smaller positive value.

As $\emptyset$ is continous at o and strictly monotone increasing with$\emptyset(0)$ 0, it is possible to obtain$\in_2 > 0$ $such that \emptyset(\in_2) < \in$.

Then, by the above argument, it is possible to obtain an increasing sequence of integers {m(k)} and {n(k)} with m(k) $> n(k) >$ k such that

$$F_{xm(k)}, x_{n(k)}, p^{(\,\emptyset\,(\in_2))} \leq \ 1 - \lambda$$

$$\text{And } F_{xm(k)-1}, x_{n(k)}, p^{(\,\emptyset\,(\in_2))} > \ 1 - \lambda$$

Now, from we get $1 - \lambda \geq F_{xm(k)}, x_{n(k)}, p^{(\,\emptyset\,(\in_2))}$

$$= F_{xm(k)-1}, x_{n(k)}, p^{(\,\emptyset\,(\in_2))}$$

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$$\geq \Psi F_{xm(k)-1}, Tx_{m(k)-1}, p^{\left(\emptyset\left(\frac{\epsilon^1}{a}\right)\right)},$$

$$F_{xn(k)-1}, Tx_{n(k)-1}, p^{\left(\emptyset\left(\frac{\epsilon^2}{b}\right)\right)}, F_{xm(k)-1}, x_{n(k)-1}, p^{\left(\emptyset\left(\frac{\epsilon^3}{c}\right)\right)},$$

$$F_{xm(k)-1}, Tx_{n(k)-1} p\emptyset\left(\frac{\epsilon^4}{d}\right), F_{xm(k)-1}, Tx_{n(k)-1} p\ \left(\emptyset\left(\frac{\epsilon^5}{d}\right)\right)$$

(Where $\epsilon_2 = \epsilon^1 + \epsilon^2 + \epsilon^3 + \epsilon^4 + \epsilon^5$)

$$= \Psi F_{xm(k)-1}, x_{m(k)}, p^{\left(\emptyset\left(\frac{\epsilon^1}{a}\right)\right)}, F_{xn(k)-1}, x_{n(k)}, p^{\left(\emptyset\left(\frac{\epsilon^2}{b}\right)\right)}, F_{xm(k)-1}, x_{n(k)-1} p\emptyset\left(\frac{\epsilon^3}{c}\right),$$

$$F_{xm(k)}, x_{n(k),} p\ \left(\emptyset\left(\frac{\epsilon^4}{d}\right)\right) F_{xm(k)}, x_{n(k)-1} p\ \left(\emptyset\left(\frac{\epsilon^5}{e}\right)\right.$$

By the property of $\emptyset$, we can choose $\eta_1, \eta_2 > 0\ such that \eta_3, \eta_4, \eta_5$

$\emptyset\ (\frac{\epsilon^1}{a}) = \eta_{1,} \emptyset\ (\frac{\epsilon^2}{b}) = \eta_{2,} \emptyset\ (\frac{\epsilon^3}{c}) = \eta_{3,} \emptyset\ (\frac{\epsilon^4}{d}) = \eta_{4,} \emptyset\ (\frac{\epsilon^5}{e}) = \eta_{5,}$

Then from (2.9) we have,

$$1-\ \lambda \geq \Psi\ (F_{xm(k)-1}, x_{m(k)}, p^{(\eta_{1,})},$$

$$F_{xn(k)-1}, x_{n(k)}, p^{(\eta_{2,})}, F_{xm(k)-1}, x_{n(k)-1}, p^{(\eta_3)}, F_{xm(k)}, Tx_{n(k),} \eta^4\ , F_{xm(k)}, Tx_{n(k),}), \eta_5$$

By (2.7) we can choose $\lambda_1\ with\ 0 < \lambda_1 < \lambda_2 < \lambda_3 < \lambda_4 < \lambda_5 < \lambda_6 < \lambda_7 < 1$, such that

$$(F_{xm(k)-1}, x_{m(k)}, p^{(\eta_{1,})} \geq 1 - \lambda_1 \text{ and}$$

$$F_{xn(k)-1}, x_{n(k)}, p^{(\eta_{2,})} \geq 1 - \lambda_1$$

$$F_{xm(k)-1}, x_{n(k)-1}, p^{(\eta_3)} \geq 1 - \lambda_1$$

$$F_{xm(k)}, x_{n(k),} \eta^4 \geq 1 - \lambda_1$$

$$F_{xm(k)}, x_{n(k),} \eta_5 \geq (1 - \lambda_1)$$

Therefore,

1-$\lambda \geq \mathbf{\Psi}\ (1 - \lambda_1, 1 - \lambda_1, 1 - \lambda_1, 1 - \lambda_1\ \ 1 - \lambda_1\ > 1 - \lambda)$

(by the property of $\mathbf{\Psi}$), which is a contradiction. Hence $\{x_n\}$ is a Cauchy sequence. As (X,F, Δ) is a complete 2- menger space we have $\{x_n\}$ is convergent in X.

Let $\lim_{n\to\infty} x_n$=z (2.10)

We show that $T_z = z$, if possible,

Let $0 < F_{z,Tz}, P^{(\emptyset(t))} < 1$ for some t > 0.

By virtue of the property of $\emptyset wecanchoose$

$\xi_1\xi_2, \xi_{3,}\xi_4, \xi_{5,} t_1, t_2, t_3, t_{4,} t_5 > 0$

$such that \emptyset(t) = \xi_1\xi_2, \xi_{3,}\xi_4, \xi_5 + \emptyset\,(t_1 + t_2 + t_3 + t_{4+} t_5)$

Again since $0 < b < 1, \emptyset\left(\frac{t_2}{b}\right) > \emptyset\,(t)$

$$\emptyset\left(\frac{t_3}{c}\right) > \emptyset\,(t)$$

$$\emptyset\left(\frac{t_4}{d}\right) > \emptyset\,(t)$$

$$\emptyset\left(\frac{t_5}{e}\right) > \emptyset\,(t)$$

Then, we have $F_{z,Tz,p}(\emptyset(t)) > F_{z,Tz,p}(\emptyset(t))$,

which is contradiction.

Hence $F_{z, Tz, p}(\emptyset(t)) = 1$ for all $t > 0$ which implies that z = Tz.

For uniqueness, let z and u be two fixed points. Therefore, for all $t > 0$,

$F_{z, u, p}(\emptyset(t)) = F_{Tz, Tu, p}(\emptyset(t))$

$\geq \psi(F_{z, Tz, p}(\emptyset(\frac{t_1}{a}), F_{u, Tu, p}(\emptyset(\frac{t_2}{b}), F_{z, u, p}(\emptyset(\frac{t_3}{c}), F_{z, Tu, p}(\emptyset(\frac{t_4}{d}), F_{Tz, u, p}(\emptyset(\frac{t_5}{e}))$

$$=\psi(F_{z, z, p}(\emptyset(\frac{t_1}{a}), F_{u, u,p}(\emptyset(\frac{t_2}{b}), F_{z, u, p}(\emptyset(\frac{t_3}{c}), F_{z, u, p}(\emptyset(\frac{t_4}{d}), F_{z, u, p}(\emptyset(\frac{t_5}{e}))$$

For $t_1+t_2+t_3+t_4+t_5 = t$. $\forall\, t_i > 0$, where i = 1, 2, 3, 4, 5

$$= \psi\,(1, 1, F_{z, u, p}(\emptyset(\frac{t}{c}), F_{z, u, p}(\emptyset(\frac{t}{d}), F_{z, u, p}(\emptyset(\frac{t}{e}))$$

Now by using definition of ψ function we have

$$F_{z, u, p}(\emptyset(t)) > F_{z, u, p}(\emptyset(t))$$

Which is contradiction hence z = u. So that T has a unique fixed point.

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