

FIBONACCI SERIES IN MATHEMATICS

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ABSTRACT

In mathematics, the application of the Fibonacci numbers can be seen in the computational analysis of Euclid's algorithm for the evaluation of the greatest common divisor of two integers. A complete sequence is a good example of the Fibonacci numbers which further implies that each and every positive integer can be written as a summation of the Fibonacci numbers where a number can be used only once at most.

Planning poker is another example where these Fibonacci numbers can be used for the evaluation of the projects of the software development in terms of the Scrum method.

Also, the data analysis of the data structure of the Fibonacci heap can be done with the aid of the Fibonacci numbers. There is also an undirected graph which is known as the Fibonacci cube where a series of the Fibonacci number of nodes are used for the parallel computation of the network topology. The current paper highlights the Fibonacci series in mathematics.

KEYWORDS: *Fibonacci, Number, Series, Sequence*

INTRODUCTION

In Mathematics, a Fibonacci series is termed as a sequence of Fibonacci numbers which are generally represented by the F_n . In this series, each number is the additional result of the two preceding numbers which starts from 0 and 1 i.e.

$$F_0=0 \quad F_1=1,$$

and

$$F_n = F_{n-1} + F_{n-2}$$

for $n > 1$.

The beginning of is thus:

(0),1,1,2,3,5,8,13,21,34,55,89,144,.....

n^{th} Fibonacci numbers are represented with the help of Binet's formula in terms of golden ratio and n which indicates that the ratio of two consecutive Fibonacci numbers come in the category of golden ratio with the increment in the term n .

The usage of the Fibonacci numbers can also be observed in the biological settings like branching in trees, leaves arrangement, pine cone arrangement and tree of bees etc. Kepler regarded the Fibonacci sequence as the pentagonal form of some flowers.

There are some rules in terms of the Fibonacci numbers which depict that if an unmated female lays an egg then it may hatch a male or drone bee. Similarly, a female is hatched if a male fertilizes an egg.

According to this theory, all the male bees have a single parent whereas there are two parents of a female bee. On tracing the pedigree of the male bee, it is observed that it has 1 parent, 2 grandparents, 3 great-grandparents, 5 great-great-grandparents, and so on. Hence, a Fibonacci series is termed as this sequence of numbers of parents.

Here, the number of female ancestors is termed as the number of ancestors at each phase, F_n , which is evaluated to be F_{n-1} , with the addition of the number of the male ancestors i.e. F_{n-2} . Here, the hypothesis is considered that the ancestors at each level are unrelated.

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The Fibonacci sequence is defined by the property that each number in the sequence is the sum of the previous two numbers; to get started, the first two numbers must be specified, and these are usually taken to be 1 and 1. In mathematical notation, if the sequence is written $(x_0, x_1, x_2, \dots)$ then the defining relationship is

$$x_n = x_{n-1} + x_{n-2} \quad (n = 2, 3, 4, \dots) \quad (1)$$

with starting conditions $x_0 = 1, x_1 = 1$. On dividing both sides of (1) by x_{n-1} we get

$$1/R_n = 1 + R_{n-1}$$

where $R_n = x_{n-1}/x_n$, the ratio of successive terms.

As $n \rightarrow \infty$, we have $R_n \rightarrow R$ so $1/R = 1 + R$, giving the equation

$$R^2 + R - 1 = 0.$$

This quadratic equation has two roots; the one we need here is obviously between zero and one; it is

$$R = \frac{\sqrt{5} - 1}{2} = 0.61803..., \quad (2)$$

the number known as the *golden Ratio*.

The number R has some remarkable properties; for example, it is expressible as a *continued fraction*:

$$R = \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} \quad (3)$$

In the theory of chaotic dynamical systems, R is recognised as "the most irrational number" between 0 and 1!

The spiral curve shown in the poster is a logarithmic spiral, a curve whose equation in polar coordinates is

$$r = ke^{a\theta}$$

where k and a are constants. The spiral patterns evident in the sunflower are of this form, and the numbers of spirals going in opposite senses are the consecutive Fibonacci numbers 34 and 55.

- The Fibonacci numbers are important in the computational run-time analysis of Euclid's algorithm to determine the greatest common divisor of two integers: the worst case input for this algorithm is a pair of consecutive Fibonacci numbers.^[22]
- Brasch et al. 2012 show how a generalised Fibonacci sequence also can be connected to the field of economics.^[23] In particular, it is shown how a generalised Fibonacci sequence enters the control function of finite-horizon dynamic optimisation problems with one state and one control variable. The procedure is illustrated in an example often referred to as the Brock–Mirman economic growth model.
- Yuri Matiyasevich was able to show that the Fibonacci numbers can be defined by a Diophantine equation, which led to his solving Hilbert's tenth problem.^[24]
- The Fibonacci numbers are also an example of a complete sequence. This means that every positive integer can be written as a sum of Fibonacci numbers, where any one number is used once at most.
- Moreover, every positive integer can be written in a unique way as the sum of *one or more* distinct Fibonacci numbers in such a way that the sum does not include any two

consecutive Fibonacci numbers. This is known as Zeckendorf's theorem, and a sum of Fibonacci numbers that satisfies these conditions is called a Zeckendorf representation. The Zeckendorf representation of a number can be used to derive its Fibonacci coding.

- Fibonacci numbers are used by some pseudorandom number generators.
- They are also used in planning poker, which is a step in estimating in software development projects that use the Scrum methodology.
- Fibonacci numbers are used in a polyphase version of the merge sort algorithm in which an unsorted list is divided into two lists whose lengths correspond to sequential Fibonacci numbers – by dividing the list so that the two parts have lengths in the approximate proportion ϕ . A tape-drive implementation of the polyphase merge sort was described in *The Art of Computer Programming*.
- Fibonacci numbers arise in the analysis of the Fibonacci heap data structure.
- The Fibonacci cube is an undirected graph with a Fibonacci number of nodes that has been proposed as a network topology for parallel computing.
- A one-dimensional optimization method, called the Fibonacci search technique, uses Fibonacci numbers.
- The Fibonacci number series is used for optional lossy compression in the IFF 8SVX audio file format used on Amiga computers. The number series compands the original audio wave similar to logarithmic methods such as μ -law.
- Since the conversion factor 1.609344 for miles to kilometers is close to the golden ratio, the decomposition of distance in miles into a sum of Fibonacci numbers becomes nearly the kilometer sum when the Fibonacci numbers are replaced by their successors. This method amounts to a radix 2 number register in golden ratio base ϕ being shifted. To convert from kilometers to miles, shift the register down the Fibonacci sequence instead.

DISCUSSION

The Fibonacci numbers can be found in different ways among the set of binary strings, or equivalently, among the subsets of a given set.

The number of binary strings of length n without consecutive 1s is the Fibonacci number F_{n+2} . For example, out of the 16 binary strings of length 4, there are $F_6 = 8$ without consecutive 1s – they are 0000, 0001, 0010, 0100, 0101, 1000, 1001 and 1010. By symmetry,

the number of strings of length n without consecutive 0s is also F_{n+2} . Equivalently, F_{n+2} is the number of subsets $S \subset \{1, \dots, n\}$ without consecutive integers: $\{i, i+1\} \not\subset S$ for every i . The symmetric statement is: F_{n+2} is the number of subsets $S \subset \{1, \dots, n\}$ without two consecutive skipped integers: that is, $S = \{a_1 < \dots < a_k\}$ with $a_{i+1} \leq a_i + 2$.

The number of binary strings of length n without an odd number of consecutive 1s is the Fibonacci number F_{n+1} . For example, out of the 16 binary strings of length 4, there are $F_5 = 5$ without an odd number of consecutive 1s – they are 0000, 0011, 0110, 1100, 1111. Equivalently, the number of subsets $S \subset \{1, \dots, n\}$ without an odd number of consecutive integers is F_{n+1} .

- The number of binary strings of length n without an even number of consecutive 0s or 1s is $2F_n$. For example, out of the 16 binary strings of length 4, there are $2F_4 = 6$ without an even number of consecutive 0s or 1s – they are 0001, 0111, 0101, 1000, 1010, 1110. There is an equivalent statement about subsets.

Most identities involving Fibonacci numbers can be proved using combinatorial arguments using the fact that F_n can be interpreted as the number of sequences of 1s and 2s that sum to $n - 1$. This can be taken as the definition of F_n , with the convention that $F_0 = 0$, meaning no sum adds up to -1 , and that $F_1 = 1$, meaning the empty sum "adds up" to 0. Here, the order of the summand matters. For example, $1 + 2$ and $2 + 1$ are considered two different sums.

For example, the recurrence relation

$$F_n = F_{n-1} + F_{n-2}$$

or in words, the n th Fibonacci number is the sum of the previous two Fibonacci numbers, may be shown by dividing the F_n sums of 1s and 2s that add to $n - 1$ into two non-overlapping groups. One group contains those sums whose first term is 1 and the other those sums whose first term is 2. In the first group the remaining terms add to $n - 2$, so it has F_{n-1} sums, and in the second group the remaining terms add to $n - 3$, so there are F_{n-2} sums. So there are a total of $F_{n-1} + F_{n-2}$ sums altogether, showing this is equal to F_n .

Specifically, the first group consists of those sums that start with 2, the second group those that start 1 + 2, the third 1 + 1 + 2, and so on, until the last group, which consists of the single sum where only 1's are used. The number of sums in the first group is $F(n)$, $F(n - 1)$ in the

second group, and so on, with 1 sum in the last group. So the total number of sums is $F(n) + F(n - 1) + \dots + F(1) + 1$ and therefore this quantity is equal to $F(n + 2)$.

CONCLUSION

A *Fibonacci prime* is a Fibonacci number that is prime. The first few are:

2, 3, 5, 13, 89, 233, 1597, 28657, 514229, Fibonacci primes with thousands of digits have been found, but it is not known whether there are infinitely many.

F_{kn} is divisible by F_n , so, apart from $F_4 = 3$, any Fibonacci prime must have a prime index. As there are arbitrarily long runs of composite numbers, there are therefore also arbitrarily long runs of composite Fibonacci numbers. No Fibonacci number greater than $F_6 = 8$ is one greater or one less than a prime number. The only nontrivial square Fibonacci number is 144.

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