ANALYSIS OF NON-MARKOVIAN PROCESS BY THE INCLUSION OF SUPPLEMENTARY VARIABLES

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Abstract

Certain stochastic processes with discrete states in continuous time can be converted into Markov process by the well-known method of including supplementary variables technique. In this paper we developed a mathematical model of the steel industry which manufactures the stainless steel plates and also made an attempt to improve its availability. The failure and repair rates of different subsystem are arbitrary distributed. Lagrange's method for partial differential equations is used to solve system governing equations. Availability analysis of the system helped in identifying the contribution factors and assessing their impact on the system availability.

Key words: Supplementary variable technique, Chapman- Kolmogorov, variable failure and repair rates, Lagrange's method.

1. Introduction

The study of repairable systems is a basic and important topic in reliability engineering. The system reliability and the system availability play an increasingly important role in power plants, industrial systems, and manufacturing systems .In the earlier studies, a perfect repair model was commonly studied in repairable systems by assuming that the failed system would be repaired as good as new after failures. However in practice, the repairable systems can be brought to one of the possible states following a repair. These states are "as good as new", "as bad as old", "better than old but worse than new", "better than new", and "worse than old" [1].

In the recent past, researcher have recognized to drive more benefits in terms of higher productivity and lower maintenance cost with the application of reliability/availability/maintainability engineering in manufacturing industries. Dayer D. [2] analyzed the unification of reliability/availability/repair-ability models for markov system. Kumar D. et al [3] used markovian approach to model the process of feeding system a component of sugar industry for its production improvement.Islamov [3] proposed a general method for determining the reliability of multiple repairable systems. The Kolmogorov equations with a large number of differential equations are transformed into integral differential equation to obtain solutions.Tsai Y.et al[5] presented a method to study the effect of three types of PM actions-mechanical service, repair and replacement on availability of multiple component system.Sharma and Kumar [6] used RAM analysis on a urea production process plant with an aim to minimize its failure ,plant maintainability requirement and optimize equipment availability.Guo et al [7] applied a more general mathematical model and algorithms for reliability analysis of wind turbines.A three parameter weibull failure rate function is used to model the problem and the parameters are estimated by maximum like hood and least squares.Reza Ahmadi, Mitra Fouladirad [8] discussed Maintenance planning of deteriorating production process. . Gregory Levitin, Liudong Xing[9] developed the reliability and availability of multistate system

In this paper a complex furnishing machine of steel industry is discussed.Probability consideration and supplementary variable technique are used in formulation of the problem. Lagrange's method for partial differential equations is used to solve the governing equations. Numerical results based upon true data collected from industry are represented to illustrate the steady state behavior of the system under different conditions .the result obtained are very informative and can also help in improving the availability of the system.

2.System description, notations and assumptions

2.1.1Sub-system G(Grinding Machine) : There is one machine subjected to major failure. Agrinding machine, often shortened to grinder, is a machine tool used for grinding, which is a type of machining using an abrasive wheel as the cutting tool. Each grain of abrasive on the wheel's surface cuts a small chip from the work piece via shear deformation. Grinding is used to finish work pieces which must show high surface quality (e.g., low surface roughness) and high accuracyof shape and dimension. As the accuracy in dimensions in grinding is on the order of 0.000025mm, in most applications it tends to be a finishing operation and removes comparatively little metal, about 0.25 to 0.50mm depth. However, there are some roughing applications in which grinding removals high volumes of metal quite rapidly. Thus grinding is a diverse field.

2.1.2 Sub-system D(DescalingMachine): These are two identical machine($D_i i = 1,2$) working in parallel.This sub-system can work with one machine in reduced capacity.Steel Strip Descalesis specifically designed to treat steel strips (carbon, alloy or stainless steel) on a continuous passage under the blast streams at a given speed. The Steel Strip Descales have been developed to treat different strip widths (ranging from 50 to 800 mm for the narrow strips and from 800 to 2100 mm for the large strips), horizontally or vertically positioned. The modular blasting cabinets are conveniently arranged and equipped with a number of wheels in order to achieve the required production rate. Automatic blast stream inclination and automatic abrasive flow adjustment devices are available.

2.1.3 Sub-system G (Hot Steckel Mill): These is five non identical machines connected in series .This subsystem can work in reduced capacity. The classical Steckel mill configuration consists of a rougher with an attached edger that jointly roll out slabs to transfer bar thickness of 25-45mm. Next a four high reversing stand rolls the transfer bar to the desired finished strip thickness in 5-9 passes the strip is coiled after each pass and transported into one of the two Steckel furnacesarranged on the entry and exit sides. The heat in the furnaces maintains the strip temperature at a high level.

2.1.1Sub-system C(Cutting Machine): There is one machine can work in reduced capacity. The SM-8 Cutting Machine is designed for in-line cutting of billets, blooms and slabs. This machine is an adaptation of the SM-10 and uses the same tubular and vertical drives. The machine is mounted to a stationary support pad provided by the customer. The product is aligned against fixed stops by the customer, which locates it for the "start of cut" position. Adjustable cut cycle can be provided where a variety of widths are to be cut. Cutting can be done on either hot or cold material (customer must specify). The machine can be operated remotely if desired.

2.2 Notations

o : The Sub-system/unit is running without any failure.

g : Unit is good state but not operative.

m: Unit is under preventive maintenance

r: unit is under repair or repair continued.

G^z: indicate the working state of grinding machine w.r.t z,(z=-o, g, m ,r).

$H_{k\ \ l}^{x\ y}$: indicates the working state of the sub-system H_k and H_l w.r.t $x, y, (x, y = o, g, r)$: : $k = 2,3,4,5,6$ :: $l = 2\ if\ k = 3,4,5,6.\ l = 3\ if\ k = 2,4,5,6; l = 4\ if\ k = 2,3,5,6: ; l = 5\ if\ k = 2,3,4,6: ; ; l = 6\ if\ k = 2,3,4,5,.$

$D_{u\ \ 3-u}^{t\ n}$: indicates the working states of the subsystem D the order pair $\binom{t}{u}$ and $\binom{t}{3-u}$ represents the functioning of the sub-system D w. r. t to "t" and "n"$(u = 1,2; t, = \quad o, r)$.

C^z: *indicate the working state of grinding machine w.r.t z,(z=-o, g, r).*

$\alpha_i(y)$: refers failure rate of the sub-system $D, H, C and\ G$ from normal to failed state $(i = \quad 1, \dots 8))$.

$\delta_1(x)$: referspreventive maintenance rate of the subsystem G and has an elapse repair time repair rate of the sub-system $G$and has an elapsed repair time '

σ_1: refers constant transition state of the subsystem G which transits the system into reduced state.

$\mu_i(x)$: Time dependent repair rates of the subsystem $D, H, C and\ G$ it from failed to normal state and elapsed repair time x,$(i = \ 1, \dots, 8)$

$P_1(t)$: The system is working in full capacity.

$P_9(x, t)$: Probability that the system is in state '9' at time t and has an elapsed repair time 'x'.

$P_k(x, y, t)$: Probability that the system is in state k at time t and has an elapsed failure time 'y' and elapsed repair time 'x'$(k = 2, \dots 8, 10, \dots 25)$

2.3 Assumption

The assumptions, on which the present analysis is based on, are as follows:

(i) Repair and failure rates are independent of each other.

(ii) Failure and Repair rates of the subsystems are taken as variable.

(iii) Performance wise, a repaired unit is as good as new one for a specified duration.

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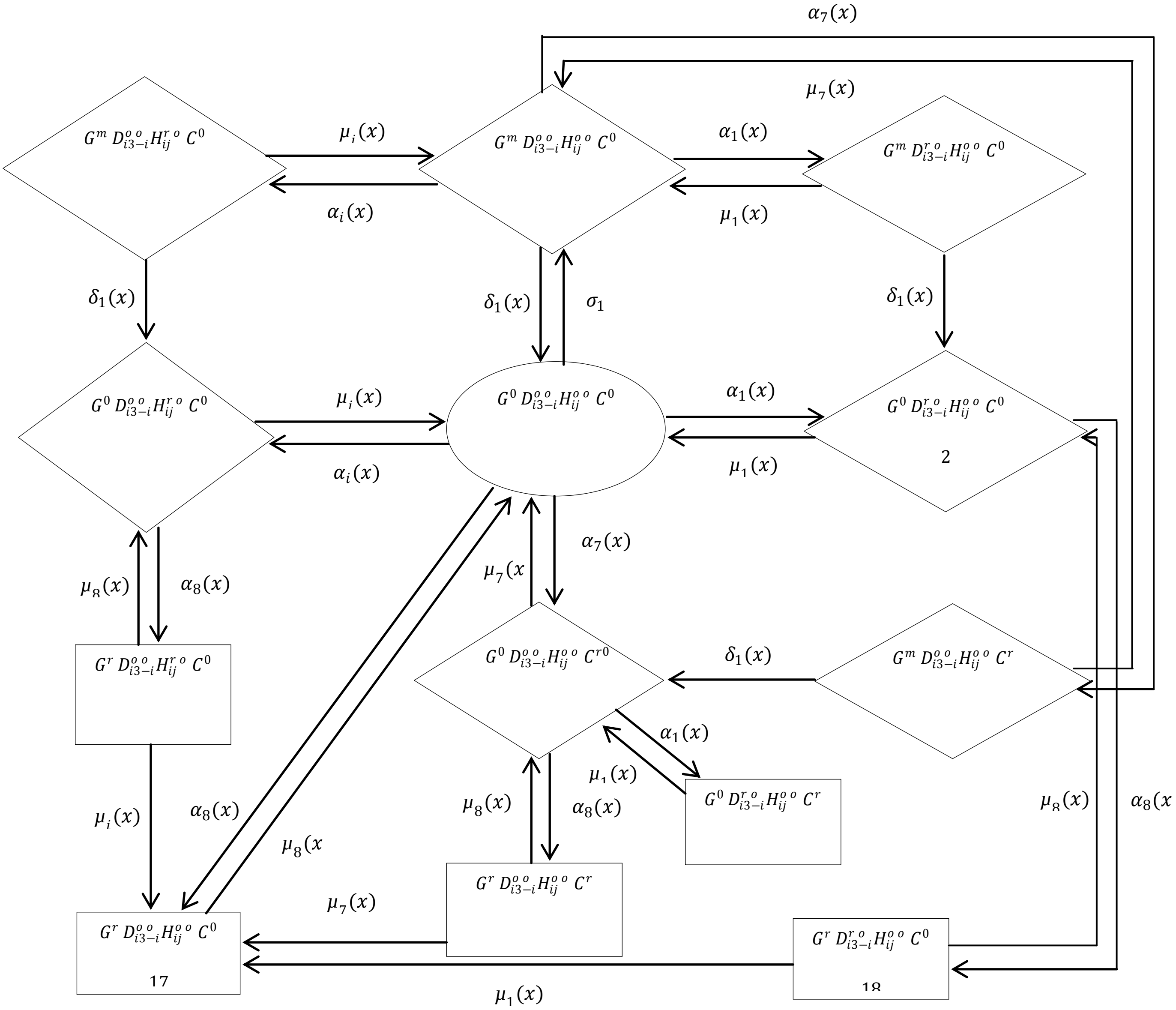
(iv) Sufficient repair facilities are provided.

(v) System can work at reduced capacity also



Mathematical Modeling

$$\left[\frac{d}{dt} + \sum_{i=1}^{8} \alpha_i(y) + \sigma_1\right] P_1(t) = \int \sum_{i=1}^{7} \mu_i(x)\, P_{i+1}(x,y,t)dxdy + \int \delta_1(x) P_9\,(x,t)dx$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_8(y)+\mu_1(x)\right]P_2(x,y,t)$$
$$=\alpha_1(y)P_1(t)+\mu_8(x)P_{18}(x,y,t)+\delta_1(x)P_{10}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_8(y)+\mu_2(x)\right]P_3(x,y,t)$$
$$=\alpha_2(y)P_1(t)+\mu_8(x)P_{19}(x,y,t)+\delta_1(x)P_{11}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_8(y)+\mu_3(x)\right]P_4(x,y,t)$$
$$=\alpha_3(y)P_1(t)+\mu_8(x)P_{20}(x,y,t)+\delta_1(x)P_{12}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_8(y)+\mu_4(x)\right]P_5(x,y,t)$$
$$=\alpha_4(y)P_1(t)+\mu_8(x)P_{21}(x,y,t)+\delta_1(x)P_{13}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_8(y)+\mu_5(x)\right]P_6(x,y,t)$$
$$=\alpha_5(y)P_1(t)+\mu_8(x)P_{22}(x,y,t)+\delta_1(x)P_{14}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_8(y)+\mu_6(x)\right]P_7(x,y,t)$$
$$=\alpha_6(y)P_1(t)+\mu_8(x)P_{23}(x,y,t)+\delta_1(x)P_{15}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\alpha_1(y)+\alpha_8(y)+\mu_7(x)\right]P_8(x,y,t)$$
$$=\alpha_7(y)P_1(t)+\mu_1(x)P_{25}(x,y,t)++\mu_8(x)P_{24}(x,y,t)+\delta_1(x)P_{16}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\sum_{i=1}^{7}\alpha_i(y)+\delta_1(x)\right]P_9(x,y,t)=\sigma_1P_1(t)+\sum_{i=1}^{7}\mu_i(x)\,P_{i+9}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\mu_i(x)+\delta_1(x)\right]P_{i+9}(x,y,t)=\alpha_i(y)P_9(x,t) \qquad i=1\ldots7$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\mu_8(x)\right]P_{17}(x,y,t)=\alpha_8(y)P_1(t)+\sum_{i=1}^{7}\mu_i(x)\,P_{i+17}(x,y,t)$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\mu_i(x)+\mu_8(x)\right]P_{i+17}(x,y,t)=\alpha_8(y)P_{i+1}(x,y,t) \qquad i=1\ldots7$$
$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\mu_1(x)\right]P_{25}(x,y,t)=\alpha_1(y)P_8(x,y,t)$$

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Where

$A_0 = \sum_{i=1}^{8} \alpha_i(y) + \sigma_1; S_0(t) = \int \sum_{i=1}^{7} \mu_i(x) P_{i+1}(x,y,t)dxdy + \int \delta_1(x)P_9(x,t)dx$

$A_1(x,y) = \alpha_8(y)+\mu_1(x)$

$;S_1(x,y,t) = \alpha_1(y)P_1(t)+\mu_8(x)P_{18}(x,y,t) + \delta_1(x)P_{10}(x,y,t)A_2(x,y) = \alpha_8(y)+\mu_2(x)\ ;S_2(x,y,t) = \alpha_2(y)P_1(t)+\mu_8(x)P_{19}(x,y,t) + \delta_1(x)P_{11}(x,y,t)$

$A_3(x,y) = \alpha_8(y)+\mu_3(x)$

$;S_3(x,y,t) = \alpha_3(y)P_1(t)+\mu_8(x)P_{20}(x,y,t) + \delta_1(x)P_{12}(x,y,t)$

$A_4(x,y) = \alpha_8(y)+\mu_4(x)$

$;S_4(x,y,t) = \alpha_4(y)P_1(t)+\mu_8(x)P_{21}(x,y,t) + \delta_1(x)P_{13}(x,y,t)$

$A_5(x,y) = \alpha_8(y)+\mu_5(x)$

$;S_5(x,y,t) = \alpha_5(y)P_1(t)+\mu_8(x)P_{22}(x,y,t) + \delta_1(x)P_{14}(x,y,t)$

$A_6(x,y) = \alpha_8(y)+\mu_6(x)$

$;S_6(x,y,t) = \alpha_6(y)P_1(t)+\mu_8(x)P_{23}(x,y,t) + \delta_1(x)P_{15}(x,y,t)$

$A_7(x,y) = +\alpha_1(y) + \alpha_8(y)+\mu_7(x) \qquad ;A_8(x,y) = \sum_{i=1}^{7} \alpha_i(y) + \delta_1(x)$

$S_7(x,y,t) = \alpha_7(y)P_1(t)+\mu_1(x)P_{25}(x,y,t) + +\mu_8(x)P_{24}(x,y,t) + \delta_1(x)P_{16}(x,y,t)S_8(x,y,t) = \sigma_1 P_1(t) + \sum_{i=1}^{7} \mu_i(x) P_{i+9}(x,y,t);T_0(x) = \mu_1(x) + \delta_1(x)$

$$S_9(x,y,t) = \alpha_8(y)P_1(t) + \sum_{i=1}^{7} \mu_i(x) P_{i+17}(x,y,t)$$

$T_i(x) = \mu_{i+1}(x) + \delta_1(x) \qquad i = 1 \dots 6 \qquad ;T_{i+6}(x) = \mu_i(x)+\mu_8(x) \qquad i = 1 \dots 7$

So That

$$\left[\frac{d}{dt} + A_0\right] P_1(t) = S_0(t)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + A_1(x,y)\right] P_2(x,y,t) = S_1(x,y,t))$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + A_2(x,y)\right] P_3(x,y,t) = S_2(x,y,t)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + A_3(x,y)\right] P_4(x,y,t) = S_3(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+A_4(x,y)\right]P_5(x,y,t)=S_4(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+A_5(x,y)\right]P_6(x,y,t)=S_5(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+A_6(x,y)\right]P_7(x,y,t)=S_6(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+A_7(x,y)\right]P_8(x,y,t)=S_7(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+A_8(x,y)\right]P_9(x,y,t)=S_8(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+T_0(x)\right]P_{10}(x,y,t)=\alpha_1(y)P_9(x,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+T_i(x)\right]P_{i+10}(x,y,t)=\alpha_1(y)P_9(x,t) \qquad i=1\ldots6$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\mu_8(x)\right]P_{17}(x,y,t)=\alpha_8(y)P_1(t)+\sum_{i=1}^{7}\mu_i(x)\,P_{i+17}(x,y,t)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+T_{i+6}(x)\right]P_{i+17}(x,y,t)=\alpha_8(y)P_{i+1}(x,y,t) \qquad i=1\ldots7$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\mu_1(x)\right]P_{25}(x,y,t)=\alpha_1(y)P_8(x,y,t)$$

Initial and boundary Conditions

$P_1(0)=1$

$P_9(x,0)=0 \quad P_i(x,y,0)=0 \qquad i=2\ldots8,10\ldots25$

Boundary Conditions

$P_{i+1}(0,y,t)=\alpha_i(y)P_1(t) \quad i=1..7 \qquad P_9(0,t)=\sigma_1P_1(t)$

$P_{i+9}(0,y,t)=\int\alpha_i(y)P_9(x,t)dx \quad i=1..7 \qquad P_{17}(0,y,t)=\alpha_8(y)P_1(t)$

$P_{i+17}(0,y,t)=\int\alpha_8(y)P_{i+1}(x,y,t)dx \qquad i=1..7P_{25}(0,y,t)=\int\alpha_1(y)P_8(x,y,t)dx$

Solution

$P_1(t)=e^{-A_0t}[1+\int S_0(t)e^{A_0t}dt]$

$P_{i+1}(x,y,t)=e^{-\int A_i(x,y)dx}[\alpha_i(y-x)P_i(t-x)+\int S_i(x,y,t)e^{\int A_i(x,y)dx}dx] \qquad i=1\ldots7$

$$P_9(x,t)=e^{-\int A_8(x,y)dx}[\sigma_1P_1(t-x)+\int S_8(x,y,t)e^{\int A_8(x,y)dx}dx]$$

$P_{10}(0,y,t)=e^{-\int T_0(x)dx}[\int[\alpha_1(y-x)P_9(x,t-x)e^{\int T_0(x)dx}+\alpha_1(y-x)P_9(x,t-x)]dx]$

$$P_{i+10}(x,y,t) = e^{-\int T_i(x)dx}[\int[\alpha_1(y-x)P_9(x,t-x)e^{\int T_i(x)dx} + \alpha_1(y-x)P_9(x,t-x)]dx]$$

$i = 1 \ldots 6$

$$P_{17}(x,y,t) = e^{-\int \mu_8(x)dx}[\alpha_8(y-x)P_1(t-x) + \int S_9(x,y,t)e^{\int \mu_8(x)dx}dx]$$

$$P_{i+17}(x,y,t) = e^{-\int T_{i+6}(x)dx}[\int[\alpha_8(y)P_{i+1}(x,y,t)e^{\int T_{i+6}(x)dx} + \alpha_8(y-x)P_{i+1}(x,y-x,t-x]dx]$$

i=1…7

$$P_{25}(x,y,t) = e^{-\int \mu_1(x)dx}[\int[\alpha_1(y)P_8(x,y,t)e^{\int \mu_1(x)dx} + \alpha_1(y-x)P_{i+1}(x,y-x,t-x)]dx]$$

If the industry provides the failure and repair rates, one can calculate the reliability $R(t)$ in terms of the probability $P_1(t)$ and using Eq. (1). Thus the time dependent Reliability $R(t)$ of the system is given by

$$R(t) = P_1(t) + \int \sum_{i=2}^{16} P_i(x,y,t)dxdy$$

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