International Journal of Advance Research in Science and Engineering
Volume No.07, Issue No.03, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

Busy Period Analysis of a Dissimilar Unit Cold Standby System with Repair or Replacement of Degraded Unit after Inspection

R.K. Bhardwaj[1], Komaldeep Kaur[2]

Department of Statistics, Punjabi University Patiala-147002, India

ABSTRACT:

A semi-Markov model of a cold standby system is developed. The system consists of two dissimilar units and a server. The working unit is taken as standard one whereas standby is taken as some duplicate unit. The standby unit can fail after crossing the maximum redundancy time. After failure the duplicate unit is degraded and went under inspection to check possibility of repair or replacement. The busy period of the server is analyzed using regenerative point technique of renewal theory. A particular is discussed using Weibull distribution.

Keywords-*Busy Period, Cold standby System, Dissimilar Unit, Inspection, Semi-Markov Processes.*

1. INTRODUCTION

The cold standby redundancy technique is widely used by researchers to improve system performance [1-4].In general various past studies considered two main proviso firstly the identical to operating units are used as standby[5-7] and secondly it is assumed that the standby units always found operable when required.Though some papers highlighted the issue of server failure[8-9]. But both ofthese provisions are unrealistic as the first provision increase the system cost and the second need not be essentially true.In fact, housing an original unit as cold standby raise the system cost. Furthermore, a cold standby unit can fail due to adverse environmental effects, if not utilized for a long period of time [10-12].The current paper considered both of these two aspects and investigated a two unit cold standby system model. The system consists of two units, one original as working and a duplicate as standby. The unit in standby can fail after exceeding maximum redundancy time. The duplicate unit becomes degraded after repair and went for inspection, if further fails, to check its suitability for repair or replacement. The operation of original unit is given preference over the duplicate. The theory of renewal processes [13] andsemi-Markov processes [14] are used to study the model at different regenerative points [15].The general expressions are derived for server busy period and expected number of repairs, replacements and inspections. A particular case is discussed using Weibull distribution and results are attained.

2. NOTATIONS

$E / \overline{E}$	: The set of regenerative/ Non-regenerative states
a^d / b^d	: Probability that repair/ replacement of degraded unit is feasible.
$z_i(t) / Z_i(t)$	: pdf / cdf of failure time of i= 1, 2 unit.
$z_{d2}(t) / Z_{d2}(t)$	: pdf / cdf of failure time of degraded unit 2.
$s_2(t) / S_2(t)$	: pdf / cdf of maximum redundancy time of duplicate unit 2.
$s_{d2}(t) / S_{d2}(t)$	: pdf / cdf of maximum redundancy time of degraded unit 2.
$g_i(t) / G_i(t)$	: pdf / cdf of repair time of i=1, 2 unit.
$g_{d2}(t) / G_{d2}(t)$	: pdf / cdf of repair time of degraded unit 2.
$f_{d2}(t) / F_{d2}(t)$	: pdf / cdf of replacement time of degraded unit 2.
$h_{d2}(t) / H_{d2}(t)$	: pdf / cdf of inspection time of degraded unit 2.
$q_{ij}(t) / Q_{ij}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative State S_j or failed state S_j without visiting any other regenerative state in $(0,t]$.
$q_{ij.kr}(t) / Q_{ij.kr}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative state S_j or failed state S_j visiting state S_k, S_r once in $(0,t]$.
$\mu_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	: Probability that server busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states
$[s]/[c]$	: Symbol for Laplace-Stietjes convolution/Laplace convolution
$\sim /*$	: Symbol for Laplace- stietjes Transform (LST)/Laplace transform (LT)
$'(desh)$	: Symbol used to represent alternative result

3. THE MODEL

Considering abovenotations, the following possible transition states diagram of the system model is drawn, as Fig.1, showing various regenerative and non-regenerative states.

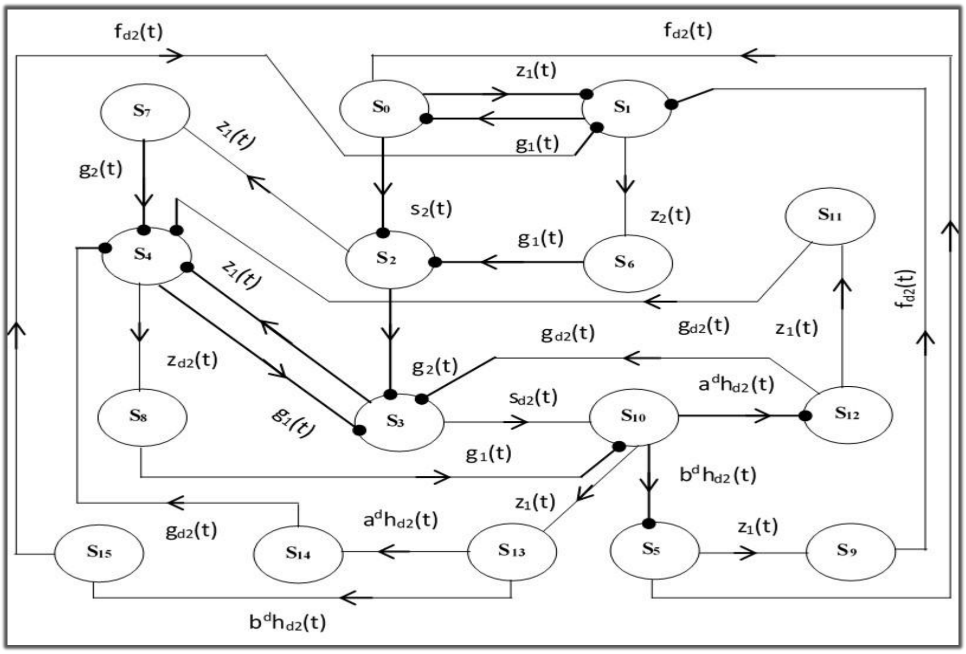

Figure 1: The State Transition Diagram

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements:-

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \qquad (1)$$

$$p_{01} = \int_0^\infty z_1(t)\bar{S}_2(t)dt, \quad p_{02} = \int_0^\infty s_2(t)\bar{Z}_1(t)dt, \quad p_{10} = \int_0^\infty g_1(t)\bar{Z}_2(t)dt, \quad p_{16} = \int_0^\infty z_2(t)\bar{G}_1(t)dt,$$

$$p_{23} = \int_0^\infty g_2(t)\bar{Z}_1(t)dt, \quad p_{27} = \int_0^\infty z_1(t)\bar{G}_2(t)dt, \quad p_{34} = \int_0^\infty z_1(t)\bar{S}_{d2}(t)dt, \quad p_{3,10} = \int_0^\infty s_{d2}(t)\bar{Z}_1(t)dt,$$

$$p_{43} = \int_0^\infty g_1(t)\bar{Z}_{d2}(t)dt, \quad p_{48} = \int_0^\infty z_{d2}(t)\bar{G}_1(t)dt, \quad p_{50} = \int_0^\infty f_{d2}(t)\bar{Z}_1(t)dt, \quad p_{59} = \int_0^\infty z_1(t)\bar{F}_{d2}(t)dt,$$

$$p_{62} = \int_0^\infty g_1(t)dt, \quad p_{74} = \int_0^\infty g_2(t)dt, \quad p_{8,10} = \int_0^\infty g_1(t)dt, \quad p_{91} = \int_0^\infty f_{d2}(t)dt,$$

$$p_{10,5} = \int_0^\infty b^d h_{d2}(t)\bar{Z}_1(t)dt, \quad p_{10,12} = \int_0^\infty a^d h_{d2}(t)\bar{Z}_1(t)dt, \quad p_{10,13} = \int_0^\infty z_1(t)\bar{H}_{d2}(t)dt, \quad p_{11,4} = \int_0^\infty g_{d2}(t)dt,$$

$$p_{12,3} = \int_0^\infty g_{d2}(t)\bar{Z}_1(t)dt, \quad p_{12,11} = \int_0^\infty z_1(t)\bar{G}_{d2}(t)dt, \quad p_{13,14} = \int_0^\infty a^d h_{d2}(t)dt, \quad p_{13,15} = \int_0^\infty b^d h_{d2}(t)dt,$$

$$p_{14,4} = \int_0^\infty g_{d2}(t)dt, \quad p_{15,1} = \int_0^\infty f_{d2}(t)dt, \quad p_{12.6} = p_{16}p_{62}, \quad p_{24.7} = p_{27}p_{74}, \quad p_{4,10.8} = p_{48}p_{8,10}, \quad p_{51.9} = p_{59}p_{91},$$

$$p_{10,1.13,15} = p_{10,13}p_{13,15}p_{15,1}, \quad p_{10,4.13,14} = p_{10,13}p_{13,14}p_{14,4}, \quad p_{12,4.11} = p_{12,11}p_{11,4}$$

For these Transition Probabilities, it can be verified that

International Journal of Advance Research in Science and Engineering
Volume No.07, Issue No.03, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

$$p_{01}+p_{02}=p_{10}+p_{16}=p_{23}+p_{27}=p_{34}+p_{3,10}=p_{43}+p_{48}=p_{50}+p_{59}=p_{62}=p_{74}=p_{8,10}=p_{91}$$
$$=p_{10,5}+p_{10,12}+p_{10,13}=p_{11,4}=p_{12,3}+p_{12,11}=p_{13,14}+p_{13,15}=p_{14,4}=p_{15,1}=p_{10}+p_{12.6}=p_{23}+p_{24.7}$$
$$=p_{43}+p_{4,10.8}=p_{50}+p_{51.9}=p_{10,5}+p_{10,12}+p_{10,1.13,15}+p_{10,4.13,14}=p_{12,3}+p_{12,4.11}=1$$

The Mean sojourn time μ_i in state S_i are given by:

$$\mu_i=E(t)=\int_0^\infty P(T_i>t)dt \tag{2}$$

$$\mu_0=\int_0^\infty \bar{Z}_1(t)\bar{S}_2(t)dt,\ \ \mu_1=\int_0^\infty \bar{G}_1(t)\bar{Z}_2(t),\ \ \mu_2=\int_0^\infty \bar{G}_2(t)\bar{Z}_1dt,\ \ \mu_3=\int_0^\infty \bar{S}_{d2}(t)\bar{Z}_1dt,\qquad \mu_4=\int_0^\infty \bar{G}_1(t)\bar{Z}_{d2}dt,$$
$$\mu_5=\int_0^\infty \bar{Z}_1(t)\bar{F}_{d2}(t)dt\ ,\ \ \mu_{10}=\int_0^\infty \bar{Z}_1(t)\bar{H}_{d2}(t)dt,\ \ \mu_{12}=\int_0^\infty \bar{Z}_1(t)\bar{G}_{d2}(t)dt$$

5. BUSY PERIOD ANALYSIS FOR SERVER

Let $B_i(t)$ be the probability that the server is busy at an instant 't' given that the system entered state S_i at time t=0. The recursive relations for $B_i(t)$ are as follows:

$$B_0(t)=q_{01}(t)[c]B_1(t)+q_{02}(t)[c]B_2(t)$$

$$B_1(t)=W_1^{R_1}(t)+q_{10}(t)[c]B_0(t)+q_{12.6}(t)][c]B_2(t)$$

$$B_2(t)=W_2^{R_2}(t)+q_{23}(t)[c]B_3(t)+q_{24.7}(t)[c]B_4(t)$$

$$B_3(t)=q_{34}(t)[c]B_4(t)+q_{3,10}(t)[c]B_{10}(t)$$

$$B_4(t)=W_4^{R_1}(t)+q_{43}(t)[c]B_3(t)+q_{4,10.8}(t)[c]B_{10}(t)$$

$$B_5(t)=W_5^{Rp_2^d}(t)+q_{50}(t)[c]B_0(t)+q_{51.9}(t)[c]B_1(t)$$

$$B_{10}(t)=W_{10}^{I_2^d}(t)+q_{10,1.13,15}(t)[c]B_1(t)+q_{10,4.13,14}(t)[c]B_4(t)+q_{10,5}(t)[c]B_5(t)+q_{10,12}(t)[c]B_{12}(t)$$

$$B_{12}(t)=W_{12}^{R_2^d}(t)+q_{12,3}(t)[c]B_3(t)+q_{12,4.11}(t)[c]B_4(t) \tag{3}$$

$W_{i\in E}^{R_1/R_2/I_2^d/Rp_2^d/R_2^d}(t)$ is the probability that the server is busy due to repair of unit one/ repair of unit two/ inspection of unit two/ replacement of degraded unit two/ repair of degraded unit two in the regenerative state up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non- regenerative states.

IJARSE
ISSN: 2319-8354

$W_1^{R_1}(t) = \overline{G}_1(t)\overline{Z}_2(t) + (z_2(t)[c]1)\overline{G}_1(t)$, $W_2^{R_2}(t) = \overline{Z}_1(t)\overline{G}_2(t) + (z_1(t)[c]1)\overline{G}_2(t)$,

$W_4^{R_1}(t) = \overline{G}_1(t)\overline{Z}_{d2}(t) + (z_{d2}(t)[c]1)\overline{G}_1(t)$, $W_5^{Rp_2^d}(t) = \overline{Z}_1(t)\overline{F}_{d2} + (z_1(t)[c]1)\overline{F}_{d2}(t)$,

$W_{10}^{I_2^d}(t) = \overline{Z}_1(t)\overline{H}_{d2} + (z_1(t)[c]1)\overline{H}_{d2}(t)$, $W_{12}^{R_2^d}(t) = \overline{Z}_1(t)\overline{G}_{d2}(t) + (z_1(t)[c]1)\overline{G}_{d2}(t)$

Taking LT of relation (3), and solving for $B_0^*(s)$, the time for which server is busy is given by

$$B_0(\infty) = \lim_{s\to 0} sB_0^*(s) = \frac{N_1}{D_1} \qquad (4)$$

$$N_1 = [1 - p_{34}p_{43}][\{p_{10,5} + p_{10,1.13,15}\} - p_{02}p_{50}p_{10,5}]W_1^{R_1^*}(0) + [1 - p_{01}p_{10}][1 - p_{3,10}p_{10,12}p_{12,3} - p_{23}p_{3,10}$$
$$\{p_{10,5} + p_{10,1.13,15}\}]W_4^{R_1^*}(0) + [1 - p_{01}p_{10}][1 - p_{34}p_{43}][\{p_{10,5} + p_{10,1.13,15}\}W_2^{R_2^*}(0) + W_{10}^{I_2^{d*}}(0)$$
$$+ p_{10,12}W_{12}^{R_2^{d*}}(0) + p_{10,5}W_5^{Rp_2^{d*}}(0)\}]$$

$$D_1 = [1 - p_{34}p_{43}][p_{10,5} + p_{10,1.13,15}][\mu_0 + p_{01}\mu_1'] + [1 - p_{34}p_{43}][p_{10,1.13,15} + p_{10,5}p_{51.9}][p_{02}\mu_1' - p_{12.6}\mu_0]$$
$$+ [1 - p_{01}p_{10}][1 - p_{34}p_{43}][\{p_{10,5} + p_{10,1.13,15}\}\mu_2' + p_{10,5}\mu_5' + \mu_{10}' + p_{10,12}\mu_{12}' + [1 - p_{01}p_{10}][\{1 - p_{24.7}$$
$$p_{4,10.8}\}\mu_3 + \{1 - p_{23}p_{3,10}\}\mu_4'] + [1 - p_{01}p_{10}][p_{4,10.8}\mu_3 - p_{3,10}\mu_4'][p_{10,12}p_{12,3} - p_{23}p_{10,4.13,14}$$
$$- p_{23}p_{10,12}]$$

6. EXPECTED NUMBER OF REPAIRS OF THE DUPLICATE UNIT

Let $R_i^2(t)$ be the expected number of repairs of the failed duplicate unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^2(t)$ are as follows:

$$R_0^2(t) = Q_{01}(t)[s]R_1^2(t) + Q_{02}(t)[s]R_2^2(t)$$

$$R_1^2(t) = Q_{10}(t)[s]R_0^2(t) + Q_{12.6}(t)][s]R_2^2(t)$$

$$R_2^2(t) = Q_{23}(t)[s][1 + R_3^2(t)] + Q_{24.7}(t)[s][1 + R_4^2(t)]$$

$$R_3^2(t) = Q_{34}(t)[s]R_4^2(t) + Q_{3,10}(t)[s]R_{10}^2(t)$$

$$R_4^2(t) = Q_{43}(t)[s]R_3^2(t) + Q_{4,10.8}(t)[s]R_{10}^2(t)$$

$$R_5^2(t) = Q_{50}(t)[s]R_0^2(t) + Q_{51.9}(t)[s]R_1^2(t)$$

$$R_{10}^2(t) = Q_{10,1.13,15}(t)[s]R_1^2(t) + Q_{10,4.13,14}(t)[s]R_4^2(t) + Q_{10,5}(t)[s]R_5^2(t) + Q_{10,12}(t)[s]R_{12}^2(t)$$

International Journal of Advance Research in Science and Engineering
Volume No.07, Issue No.03, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

$$R_{12}^{2}(t)=Q_{12,3}(t)[s]R_{3}^{2}(t)+Q_{12,4.11}(t)[s]R_{4}^{2}(t) \quad (5)$$

Taking LST of above relation (5) and solving for $\tilde{R}_0^2(s)$. The expected number of repairs per unit time to failed duplicate unit is given by

$$R_0^2(\infty)=\lim_{s\to 0} s\tilde{R}_0^2(s)=\frac{N_2^{R_2}}{D_1} \quad (6)$$

$$N_2^{R_2}=[1-p_{01}p_{10}][1-p_{34}p_{43}][p_{10,5}+p_{10,1.13,15}]$$

7. EXPECTED NUMBER OF REPLACEMENTS OF THE DEGRADED UNIT

Let $R_i^{C_2^d}(t)$ be the expected number of replacements of the failed degraded unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^{C_2^d}(t)$ are as follows:

$$R_0^{C_2^d}(t)=Q_{01}(t)[s]R_1^{C_2^d}(t)+Q_{02}(t)[s]R_2^{C_2^d}(t)$$

$$R_1^{C_2^d}(t)=Q_{10}(t)[s]R_0^{C_2^d}(t)+Q_{12.6}(t)][s]R_2^{C_2^d}(t)$$

$$R_2^{C_2^d}(t)=Q_{23}(t)[s]R_3^{C_2^d}(t)+Q_{24.7}(t)[s]R_4^{C_2^d}(t)$$

$$R_3^{C_2^d}(t)=Q_{34}(t)[s]R_4^{C_2^d}(t)+Q_{3,10}(t)[s]R_{10}^{C_2^d}(t)$$

$$R_4^{C_2^d}(t)=Q_{43}(t)[s]R_3^{C_2^d}(t)+Q_{4,10.8}(t)[s]R_{10}^{C_2^d}(t)$$

$$R_5^{C_2^d}(t)=Q_{50}(t)[s][1+R_0^{C_2^d}(t)]+Q_{51.9}(t)[s][1+R_1^{C_2^d}(t)]$$

$$R_{10}^{C_2^d}(t)=Q_{10,1.13,15}(t)[s][1+R_1^{C_2^d}(t)]+Q_{10,4.13,14}(t)[s][1+R_4^{C_2^d}(t)]+Q_{10,5}(t)[s]R_5^{C_2^d}(t)+Q_{10,12}(t)[s]R_{12}^{C_2^d}(t)$$

$$R_{12}^{C_2^d}(t)=Q_{12,3}(t)[s]R_3^{C_2^d}(t)+Q_{12,4.11}(t)[s]R_4^{C_2^d}(t) \quad (7)$$

Taking LST of above relation (7) and solving for $\tilde{R}_0^{C_2^d}(t)$. The expected number of replacements per unit time to failed degraded unit is given by

$$R_0^{C_2^d}(\infty)=\lim_{s\to 0} s\tilde{R}_0^{C_2^d}(s)=\frac{N_3^{C_2^d}}{D_1} \quad (8)$$

$$N_3^{C_2^d}=[1-p_{01}p_{10}][1-p_{34}p_{43}][p_{10,5}+p_{10,1.13,15}]$$

8. EXPECTED NUMBER OF INSPECTIONS OF THE DEGRADED UNIT

Let $I_i^{d_2}(t)$ be the expected number of inspections of the failed degraded unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $I_i^{d_2}(t)$ are as follows:

$$I_0^{d_2}(t) = Q_{01}(t)[s]I_1^{d_2}(t) + Q_{02}(t)[s]I_2^{d_2}(t)$$

$$I_1^{d_2}(t) = Q_{10}(t)[s]I_0^{d_2}(t) + Q_{12.6}(t)][s]I_2^{d_2}(t)$$

$$I_2^{d_2}(t) = Q_{23}(t)[s]I_3^{d_2}(t) + Q_{24.7}(t)[s]I_4^{d_2}(t)$$

$$I_3^{d_2}(t) = Q_{34}(t)[s]I_4^{d_2}(t) + Q_{3,10}(t)[s][1 + I_{10}^{d_2}(t)]$$

$$I_4^{d_2}(t) = Q_{43}(t)[s]I_3^{d_2}(t) + Q_{4,10.8}(t)[s][1 + I_{10}^{d_2}(t)]$$

$$I_5^{d_2}(t) = Q_{50}(t)[s]I_0^{d_2}(t) + Q_{51.9}(t)[s]I_1^{d_2}(t)$$

$$I_{10}^{d_2}(t) = Q_{10,1.13,15}(t)[s]I_1^{d_2}(t) + Q_{10,4.13,14}(t)[s]I_4^{d_2}(t) + Q_{10,5}(t)[s]I_5^{d_2}(t) + Q_{10,12}(t)[s]I_{12}^{d_2}(t)$$

$$I_{12}^{d_2}(t) = Q_{12,3}(t)[s]I_3^{d_2}(t) + Q_{12,4.11}(t)[s]I_4^{d_2}(t) \qquad (9)$$

Taking LST of above relation (9) and solving for $\tilde{I}_0^{d_2}(t)$. The expected number of inspections per unit time to failed degraded unit is given by

$$I_0^{d_2}(\infty) = \lim_{s \to 0} s\tilde{I}_0^{d_2}(s) = \frac{N_3^{I_2^d}}{D_1} \qquad (10)$$

Where

$$N_3^{I_2^d} = [1 - p_{01}p_{10}][1 - p_{34}p_{43}]$$

9. EXPECTED NUMBER OF REPAIRS OF THE DEGRADED UNIT

Let $R_i^{d_2}(t)$ be the expected number of repairs of the failed degraded unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^{d_2}(t)$ are as follows:

$$R_0^{d_2}(t) = Q_{01}(t)[s]R_1^{d_2}(t) + Q_{02}(t)[s]R_2^{d_2}(t)$$

$$R_1^{d_2}(t) = Q_{10}(t)[s]R_0^{d_2}(t) + Q_{12.6}(t)][s]R_2^{d_2}(t)$$

IJARSE
ISSN: 2319-8354

$$R_2^{d_2}(t) = Q_{23}(t)[s]R_3^{d_2}(t) + Q_{24.7}(t)[s]R_4^{d_2}(t)$$

$$R_3^{d_2}(t) = Q_{34}(t)[s]R_4^{d_2}(t) + Q_{3,10}(t)[s]R_{10}^{d_2}(t)$$

$$R_4^{d_2}(t) = Q_{43}(t)[s]R_3^{d_2}(t) + Q_{4,10.8}(t)[s]R_{10}^{d_2}(t)$$

$$R_5^{d_2}(t) = Q_{50}(t)[s]R_0^{d_2}(t) + Q_{51.9}(t)[s]R_1^{d_2}(t)$$

$$R_{10}^{d_2}(t) = Q_{10,1.13,15}(t)[s]R_1^{d_2}(t) + Q_{10,4.13,14}(t)[s][1 + R_4^{d_2}(t)] + Q_{10,5}(t)[s]R_5^{d_2}(t) + Q_{10,12}(t)[s]R_{12}^{d_2}(t)$$

$$R_{12}^{d_2}(t) = Q_{12,3}(t)[s][1 + R_3^{d_2}(t)] + Q_{12,4.11}(t)[s][1 + R_4^{d_2}(t)] \tag{11}$$

Taking LST of above relation (11) and solving for $\tilde{I}_0^{d_2}(t)$. The expected number of repairs per unit time to failed degraded unit is given by

$$R_0^{d_2}(\infty) = \lim_{s \to 0} s\tilde{R}_0^{d_2}(s) = \frac{N_3^{R_2^d}}{D_1} \tag{12}$$

Where

$$N_3^{R_2^d} = [1 - p_{01}p_{10}][1 - p_{34}p_{43}][p_{10,12} + p_{10,4.13,14}]$$

10. SPECIAL CASE-WEIBULL DISTRIBUTION

In the following the values of different performance measures are obtained assuming all the random variables as Weibull distributed with common shape parameter (η) and different scale parameters as follows:

$z_1(t) = \lambda_1 \eta t^{\eta-1} \exp(-\lambda_1 t^{\eta})$, $z_2(t) = \lambda_2 \eta t^{\eta-1} \exp(-\lambda_2 t^{\eta})$, $z_{d2}(t) = \lambda_2^d \eta t^{\eta-1} \exp(-\lambda_2^d t^{\eta})$,

$s_2(t) = \mu_2^c \eta t^{\eta-1} \exp(-\mu_2^c t^{\eta})$, $s_{d2}(t) = \mu_2^d \eta t^{\eta-1} \exp(-\mu_2^d t^{\eta})$, $g_1(t) = \beta_1 \eta t^{\eta-1} \exp(-\beta_1 t^{\eta})$,

$g_2(t) = \beta_2 \eta t^{\eta-1} \exp(-\beta_2 t^{\eta})$, $f_{d2}(t) = \gamma_2^d \eta t^{\eta-1} \exp(-\gamma_2^d t^{\eta})$, $h_{d2}(t) = \alpha_2^d \eta t^{\eta-1} \exp(-\alpha_2^d t^{\eta})$,

$g_{d2}(t) = \beta_2^d \eta t^{\eta-1} \exp(-\beta_2^d t^{\eta})$

where $t \geq 0$ and $\eta, \lambda_1, \lambda_2, \lambda_2^d, \mu_2^c, \mu_2^d, \beta_1, \beta_2, \gamma_2^d, \beta_2^d, \alpha_2^d > 0$ respectively.

We can obtain the following result

$$N_1 = \Gamma(1+\frac{1}{\eta}).\frac{\begin{array}{l}[\lambda_1\lambda_2^d + \mu_2^d(\beta_1+\lambda_2^d)]b^d[(\lambda_1+\mu_2^c)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d) - \mu_2^c\alpha_2^d\gamma_2^d]\{(\beta_1)^{\frac{1}{\eta}} + \lambda_2(\beta_1 \\ +\lambda_2)^{\frac{1}{\eta}-1}\}\{\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\lambda_1+\beta_2)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}\{(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)\}^{\frac{1}{\eta}-1} + \\ [\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\{(\lambda_1+\mu_2^d)(\lambda_1+\beta_2) - b^d\mu_2^d\beta_2^d\} - a^d\mu_2^d \\ \beta_2^d\alpha_2^d(\lambda_1+\beta_2)]\{(\beta_1)^{\frac{1}{\eta}} + \lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}\{\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}}\{(\beta_1+\lambda_2)(\lambda_1 \\ +\alpha_2^d)(\lambda_1+\beta_2^d)(\lambda_1+\beta_2)\}^{\frac{1}{\eta}-1} + [\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)](\beta_1)^{\frac{1}{\eta}} \\ \{(\beta_1+\lambda_2)(\beta_1+\lambda_2^d)\}^{\frac{1}{\eta}-1}[\{\beta_2^d\gamma_2^d(\lambda_1+\beta_2^d)(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}}\{b^d(\lambda_1+\alpha_2^d)((\beta_2)^{\frac{1}{\eta}} + \lambda_1(\beta_2 \\ +\lambda_1)^{\frac{1}{\eta}-1})(\alpha_2^d)^{\frac{1}{\eta}}(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1} + \{(\alpha_2^d)^{\frac{1}{\eta}} + \lambda_1(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}\}\{\beta_2(\lambda_1+\beta_2)\}^{\frac{1}{\eta}}\} + \alpha_2^d\{\beta_2 \\ \alpha_2^d(\lambda_1+\beta_2)\}^{\frac{1}{\eta}}(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}\{a^d\{(\beta_2^d)^{\frac{1}{\eta}} + \lambda_1(\lambda_1+\beta_2^d)^{\frac{1}{\eta}-1}\}\{\gamma_2^d(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}} + b^d\{(\gamma_2^d)^{\frac{1}{\eta}} \\ +\lambda_1(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}-1}\}\{\beta_2^d(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}\}]\end{array}}{\begin{array}{l}[\beta_1\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\beta_1+\lambda_2)(\beta_1+\lambda_2^d)(\lambda_1+\beta_2)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)]^{\frac{1}{\eta}} \\ (\lambda_1+\mu_2^c)(\lambda_1+\mu_2^d)\end{array}},$$

$$N_2^{R_2} = \frac{b^d[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]}{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)},$$

$$N_3^{C_2^d} = \frac{b^d[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]}{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)},$$

$$N_3^{I_2^d} = \frac{[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]}{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)},$$

$$N_3^{R_2^d} = \frac{a^d[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]}{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)},$$

$$D_1 = \Gamma(1+\frac{1}{\eta}).\frac{\begin{array}{l}
[\lambda_1\lambda_2^d + \mu_2^d(\beta_1+\lambda_2^d)]b^d\{(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)\}^{\frac{1}{\eta}-1}\{\beta_2\alpha_2^d\beta_2^d\gamma_2^d \\
(\lambda_1+\beta_2)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}[\{(\beta_1(\beta_1+\lambda_2))^{\frac{1}{\eta}}+\lambda_1(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}\{\beta_1^{\frac{1}{\eta}}+\lambda_2(\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\}\}(\lambda_1 \\
+\gamma_2^d)(\lambda_1+\alpha_2^d)+\lambda_1(\lambda_1+\gamma_2^d+\alpha_2^d)\{\mu_2^c(\beta_1^{\frac{1}{\eta}}+\lambda_2(\beta_1+\lambda_2)^{\frac{1}{\eta}-1})(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}-\lambda_2(\beta_1)^{\frac{1}{\eta}} \\
(\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\}]+[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)][\{\alpha_2^d\gamma_2^d(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}} \\
(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}[b^d\{\beta_2^{\frac{1}{\eta}}+\lambda_1(\lambda_1+\beta_2)^{\frac{1}{\eta}-1}\}(\lambda_1+\alpha_2^d)\{\beta_2^d(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}+a^d\alpha_2^d\{(\beta_2^d)^{\frac{1}{\eta}} \\
+\lambda_1(\lambda_1+\beta_2^d)^{\frac{1}{\eta}-1}\}\{\beta_2(\lambda_1+\beta_2)^{\frac{1}{\eta}}\}]+\{\beta_2(\lambda_1+\beta_2)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}[b^d\alpha_2^d\{(\gamma_2^d)^{\frac{1}{\eta}}+\lambda_1 \\
(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}-1}\}\{\alpha_2^d\beta_2^d\}^{\frac{1}{\eta}}(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}+\{(\alpha_2^d\beta_2^d\gamma_2^d)^{\frac{1}{\eta}}+\lambda_1(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}[(\beta_2^d\gamma_2^d)^{\frac{1}{\eta}} \\
+(a^d\gamma_2^d+b^d\beta_2^d)(\alpha_2^d)^{\frac{1}{\eta}}]\}(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}}]](\beta_1)^{\frac{1}{\eta}}\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1 \\
+\lambda_2^d)\}^{\frac{1}{\eta}-1}+[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)]\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}-1} \\
\{\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}}[(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)[\lambda_1\beta_1+(\beta_1+\lambda_2^d)\beta_2(\beta_1)^{\frac{1}{\eta}}(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1} \\
+\{\lambda_1+\beta_2+\mu_2^d\}\lambda_1\{\beta_1^{\frac{1}{\eta}}+\lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}]+\{\lambda_2^d\beta_1^{\frac{1}{\eta}}(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}-\mu_2^d \\
\{\beta_1^{\frac{1}{\eta}}+\lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}\}a^d\{\alpha_2^d\beta_2^d(\lambda_1+\beta_2)-\beta_2(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2)\}]
\end{array}}{\begin{array}{l}
[\beta_1\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\lambda_1+\beta_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d) \\
(\lambda_1+\alpha_2^d)]^{\frac{1}{\eta}}
\end{array}}$$

11. CONCLUSION

A semi-Markov model of a dissimilar unit cold standby system is developed. The original unit is given preference for operation and inspection over the duplicate. The regenerative point technique of renewal theory is employed to derive expressions for the busy period of server and the frequencies of remedial actions. A special case of Weibull distribution is debated to facilitate further numerical demonstration of the theoretical results.

Acknowledgement:The authors are very thankful to the referees for insight review and valuable comments, which indeed helped a lot for refining the firstversion of the paper.

REFERENCES

[1] S. K. Singh and B. Srinivasu, Stochastic analysis of a two unit cold standby system with preparation time for repair, *Microelectronics Reliability, 27(1),* 1987, 55-60.

[2] K. M. El-Said and M. S. El-Sherbeny, Stochastic analysis of a two-unit cold standby system with two-stage repair and waiting time, *Sankhya B, 72(1),* 2010, 1-10.

[3] X. Bao and L. Cui, A study on reliability for a two-item cold standby Markov repairable system with neglected failures, *Communications in Statistics-Theory and Methods, 41(21)*, 2012, 3988-3999.

[4] J. Cao and Y. Wu, Reliability analysis of a two-unit cold standby system with a replaceable repair facility, *Microelectronics Reliability, 29(2),* 1989, 145-150.

[5] R. K. Bhardwaj and S, C. Malik , Asymptotic performance analysis of 2oo3 cold standby system with constrained repair and arbitrary distributed inspection time, *International Journal of Applied Engineering Research, 6(08)*, 2011, 1493-1502.

[6] R. K. Bhardwaj and R. Singh, Semi Markov approach for asymptotic performance analysis of a standby system with server failure, *Int. J Comput. Appl, 98(3),* 2014, 9-14.

[7] R. K. Bhardwaj and R. Singh, Steady state behavior of a cold standby system with server failure and arbitrary repair, replacement & treatment, *International Journal of Applied Engineering Research, 9(24),* 2014, 26563-26578.

[8] R. K. Bhardwaj and R. Singh, An inspection-repair-replacement model of a stochastic standby system with server failure, *Mathematics in Engineering, Science & Aerospace (MESA), 6(2),* 2015, 191-203.

[9] AK Dhankhar , RK Bhardwaj, SC Malik, Reliability Modeling and Profit Analysis of a System with Different Failure Modes and Replaceable Server Subject to Inspection, Int. j. of Stats & Analysis 2 (3), 2012, 245-255.

[10] R. K. Bhardwaj, K. Kaur and S. C. Malik, Reliability indices of a redundant system with standby failure and arbitrary distribution for repair and replacement times, *International Journal of System Assurance Engineering and Management, 8(2),* 2017, 423-431.

[11] R. K. Bhardwaj, K. Kaur and S. C. Malik, Stochastic Modeling of a System with Maintenance and Replacement of Standby Subject to Inspection, *American J. of Theoretical & Applied Statistics, 4(5),* 2015, 339–346.

[12] R. K. Bhardwaj and K. Kaur, Reliability and Profit Analysis of a Redundant System with Possible Renewal of Standby Subject to Inspection, *Int. J. of Stat. & Reliab. Eng, 1(1),* 2014, 36–46.

[13] D. R. Cox, *Renewal theory*, Methuen and co. Ltd., London, 1962.

[14] N. Limnios and G. Opriean, *Semi-Markov processes and reliability*, Springer, 2001.

[15] W. L. Smith, Regenerative stochastic processes, *Proceedings of the Royal Society of London, Series A. Mathematical and Physical Sciences, 232(1188)*, 1955, 6-31.