Reliability and Availability Analysis of a Dissimilar Units Cold Standby System with Repair or Replacement of Degraded Unit after Inspection

R.K. Bhardwaj[1], Komaldeep Kaur[2]

Department of Statistics, Punjabi University Patiala-147002, India

ABSTRACT:

In this paper a stochastic model of a cold standby system is investigated. The system has two dissimilar units and a service facility. An original unit is in working mode and another duplicate unit is in cold standby mode. The working unit fails after a pre specified time limit. The duplicate unit becomes degraded after post failure repair. The degraded unit went under inspection to check feasibility of repair or replacement, after failure. The semi-Markov processes are used to investigate the model at different regenerative points. A special case of Weibull distribution is discussed to derive the results.

Keywords-*Cold Standby, Degradation, Dissimilar Unit, Inspectionand Stochastic Processes.*

1. INTRODUCTION

The performance of a working system can be outlined in terms of its reliability and availability [1]. The reliability of a system is the measure of probability that the system performs its intended function without any failure for a stated period of time [2]. The reliability can be estimated with the idea of average time to system failure. Similarly the availability is also the measure of probability that the system is available for use [3]. There are various ways to amplify these two characteristics of the system. The use of cold standby redundancy is one such mean to improve system performance. The cold standby system models are debated by various researchers in the literature [4-7]. Some researchers raised the issue of server failure [8-12]. In these studies identical units are taken as standby irrespective of its impact on system costs. Some studies considered the concept of failure of unit in cold standby mode [13-15]. Therefore keeping the behavioral aspect in view in this paper a cold standby system is considered with a duplicate unit, having low price, as cold standby. Furthermore, the worth of original unit is utilized by giving it priority for operation as well repairas advocated in some studies [16]. The system starts its operation with the original unit in working and duplicate unit in standby mode. The standbyduplicate unit fails after crossing maximum redundancy time and goes under repair and become degraded after repair. If it further fails then goes under inspection for checking the feasibility of repair orreplacement.The original unit works as new after each repair. All the random variables follow general distribution. The switch is perfect and prompt.The expressions are derived using semi-Markov processes [17] and regenerative point technique [18] of stochastic processes. A special case is discussed by considering Weibull distribution for all the random variables and results are obtained to facilitate further numerical analysis of results if desired.

2. NOTATIONS

$E / \overline{E}$	The set of regenerative/ Non-regenerative states
$No_1/No_2/Do_2$	Original unit/ Duplicate unit/ Degraded unit in operation.
Cs_2/DCs_2	Duplicate unit/ Degraded unit in cold-standby mode.
F_{ur_i}/F_{UR_i}	Failed unit i= 1, 2 under repair /under repair continuously from previous state.
F_{wr_i}/F_{WR_i}	Failed unit i= 1, 2 waiting for repair / waiting for repair continuously from previous state.
DF_{ui_2}/DF_{UI_2}	Degraded failed unit under inspection / under inspection continuously from previous state.
DF_{wi_2}/DF_{WI_2}	Degraded failed waiting for inspection / waiting for inspection continuously from previous state.
DF_{ur_2}/DF_{UR_2}	Degraded failed unit under repair / under repair continuously from previous state.
DF_{urp_2}/DF_{URp_2}	Degraded failed unit under replacement / under replacement continuously from previous state.
a^d / b^d	Probability that repair/ replacement of degraded unit is feasible.
$z_i(t)/Z_i(t)$	pdf / cdf of failure time of i= 1, 2 unit.
$z_{d2}(t)/Z_{d2}(t)$	pdf / cdf of failure time of degraded unit 2.
$s_2(t)/S_2(t)$	pdf / cdf of maximum redundancy time of duplicate unit 2.
$s_{d2}(t)/S_{d2}(t)$	pdf / cdf of maximum redundancy time of degraded unit 2.
$g_i(t)/G_i(t)$	pdf / cdf of repair time of i=1, 2 unit.
$g_{d2}(t)/G_{d2}(t)$	pdf / cdf of repair time of degraded unit 2.
$f_{d2}(t)/F_{d2}(t)$	pdf / cdf of replacement time of degraded unit 2.
$h_{d2}(t)/H_{d2}(t)$	pdf / cdf of inspection time of degraded unit 2.
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of first passage time from regenerative state S_i to regenerative State S_j or failed state S_j without visiting any other regenerative state in $(0,t]$.
$q_{ij.kr}(t)/Q_{ij.kr}(t)$	pdf/cdf of first passage time from regenerative state S_i to regenerative state S_j or failed state S_j visiting state S_k, S_r once in $(0,t]$.
$\mu_i(t)$	Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to

	any regenerative state
$[s]/[c]$	Symbol for Laplace-Stietjes convolution/Laplace convolution
pdf/cdf	Probability density/distribution function

Considering these symbols, the following are possible transition states of the system model

The regenerative states:

$$S_0=(N_{O_1},C_{S_2}),\ S_1=(F_{ur_1},N_{O_2}),\ S_2=(N_{O_1},F_{ur_2}),\ S_3=(N_{O_1},DC_{S_2}),\ S_4=(F_{ur_1},D_{O_2}),\ S_5=(N_{O_1},DF_{urp_2}),$$
$$S_{10}=(N_{O_1},DF_{ui_2}),\ S_{12}=(N_{O_1},DF_{ur_2})$$

The non-regenerative states:

$$S_6=(F_{UR_1},F_{wr_2}),\ S_7=(F_{wr_1},F_{UR_2}),\ S_8=(F_{UR_1},DF_{wi_2}),\ S_9=(F_{wr_1},DF_{URp_2}),\ S_{11}=(F_{wr_1},DF_{UR_2}),$$
$$S_{13}=(F_{wr_1},DF_{UI_2}),\ S_{14}=(F_{WR_1},DF_{ur_2}),\ S_{15}=(F_{WR_1},DF_{urp_2})$$

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij}=Q_{ij}(\infty)=\int_0^\infty q_{ij}(t)dt \qquad (1)$$

$$p_{01}=\int_0^\infty z_1(t)\bar{S}_2(t)dt,\quad p_{02}=\int_0^\infty s_2(t)\bar{Z}_1(t)dt,\quad p_{10}=\int_0^\infty g_1(t)\bar{Z}_2(t)dt,\quad p_{16}=\int_0^\infty z_2(t)\bar{G}_1(t)dt,$$

$$p_{23}=\int_0^\infty g_2(t)\bar{Z}_1(t)dt,\quad p_{27}=\int_0^\infty z_1(t)\bar{G}_2(t)dt,\quad p_{34}=\int_0^\infty z_1(t)\bar{S}_{d2}(t)dt,\quad p_{3,10}=\int_0^\infty s_{d2}(t)\bar{Z}_1(t)dt,$$

$$p_{43}=\int_0^\infty g_1(t)\bar{Z}_{d2}(t)dt,\quad p_{48}=\int_0^\infty z_{d2}(t)\bar{G}_1(t)dt,\quad p_{50}=\int_0^\infty f_{d2}(t)\bar{Z}_1(t)dt,\quad p_{59}=\int_0^\infty z_1(t)\bar{F}_{d2}(t)dt,$$

$$p_{62}=\int_0^\infty g_1(t)dt,\quad p_{74}=\int_0^\infty g_2(t)dt,\quad p_{8,10}=\int_0^\infty g_1(t)dt,\quad p_{91}=\int_0^\infty f_{d2}(t)dt,$$

$$p_{10,5}=\int_0^\infty b^d h_{d2}(t)\bar{Z}_1(t)dt,\quad p_{10,12}=\int_0^\infty a^d h_{d2}(t)\bar{Z}_1(t)dt,\quad p_{10,13}=\int_0^\infty z_1(t)\bar{H}_{d2}(t)dt,\quad p_{11,4}=\int_0^\infty g_{d2}(t)dt,$$
$$p_{12,3}=\int_0^\infty g_{d2}(t)\bar{Z}_1(t)dt,\quad p_{12,11}=\int_0^\infty z_1(t)\bar{G}_{d2}(t)dt,\quad p_{13,14}=\int_0^\infty a^d h_{d2}(t)dt,\quad p_{13,15}=\int_0^\infty b^d h_{d2}(t)dt,$$
$$p_{14,4}=\int_0^\infty g_{d2}(t)dt,\quad p_{15,1}=\int_0^\infty f_{d2}(t)dt,\ p_{12.6}=p_{16}p_{62},\quad p_{24.7}=p_{27}p_{74},\quad p_{4,10.8}=p_{48}p_{8,10},$$
$$p_{51.9}=p_{59}p_{91},\quad p_{10,1.13,15}=p_{10,13}p_{13,15}p_{15,1},\quad p_{10,4.13,14}=p_{10,13}p_{13,14}p_{14,4},\quad p_{12,4.11}=p_{12,11}p_{11,4}$$

For these Transition Probabilities, it can be verified that

$$p_{01}+p_{02}=p_{10}+p_{16}=p_{23}+p_{27}=p_{34}+p_{3,10}=p_{43}+p_{48}=p_{50}+p_{59}=p_{62}=p_{74}=p_{8,10}=p_{91}$$
$$=p_{10,5}+p_{10,12}+p_{10,13}=p_{11,4}=p_{12,3}+p_{12,11}=p_{13,14}+p_{13,15}=p_{14,4}=p_{15,1}=p_{10}+p_{12.6}=p_{23}+p_{24.7}$$
$$=p_{43}+p_{4,10.8}=p_{50}+p_{51.9}=p_{10,5}+p_{10,12}+p_{10,1.13,15}+p_{10,4.13,14}=p_{12,3}+p_{12,4.11}=1$$

The Mean sojourn time μ_i in state S_i are given by:

$$\mu_i=E(t)=\int_0^\infty P(T_i>t)dt \qquad (2)$$

$$\mu_0 = \int_0^\infty \bar{Z}_1(t)\bar{S}_2(t)dt,\ \ \mu_1 = \int_0^\infty \bar{G}_1(t)\bar{Z}_2(t),\ \mu_2 = \int_0^\infty \bar{G}_2(t)\bar{Z}_1 dt,\ \mu_3 = \int_0^\infty \bar{S}_{d2}(t)\bar{Z}_1 dt,$$

$$\mu_4 = \int_0^\infty \bar{G}_1(t)\bar{Z}_{d2} dt,\ \ \mu_5 = \int_0^\infty \bar{Z}_1(t)\bar{F}_{d2}(t)dt\ , \qquad \mu_{10} = \int_0^\infty \bar{Z}_1(t)\bar{H}_{d2}(t)dt,\ \ \mu_{12} = \int_0^\infty \bar{Z}_1(t)\bar{G}_{d2}(t)dt$$

4. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t)[s]\phi_1(t) + Q_{02}(t)[s]\phi_2(t)$$

$$\phi_1(t) = Q_{10}(t)[s]\phi_0(t) + Q_{16}(t)$$

$$\phi_2(t) = Q_{23}(t)[s]\phi_3(t) + Q_{27}(t)$$

$$\phi_3(t) = Q_{34}(t)[s]\phi_4(t) + Q_{3,10}(t)[s]\phi_{10}(t)$$

$$\phi_4(t) = Q_{43}(t)[s]\phi_3(t) + Q_{48}(t)$$

$$\phi_5(t) = Q_{50}(t)[s]\phi_0(t) + Q_{59}(t)$$

$$\phi_{10}(t) = Q_{10,5}(t)[s]\phi_5(t) + Q_{10,12}(t)[s]\phi_{12}(t) + Q_{10,13}(t)$$

$$\phi_{12}(t) = Q_{12,3}(t)[s]\phi_3(t) + Q_{12,11}(t) \qquad (3)$$

Taking LST of above relation (3) and solving for $\tilde{\phi}_0(s)$ we have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \qquad (4)$$

The reliability of system model can be obtained by taking Laplace inverse transform of (4). The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1}\ (5)$$

Where

$$N_1 = [\mu_0 + p_{01}\mu_1 + p_{02}\mu_2][(1-p_{34}p_{43}) - p_{3,10}p_{10,12}p_{12,3}] + p_{02}p_{23}p_{3,10}[p_{10,5}\mu_5 + p_{10,12}\mu_{12}]$$
$$+ p_{02}p_{23}[\mu_3 + p_{34}\mu_4 + p_{3,10}\mu_{10}]$$

$$D_1 = [1-p_{01}p_{10}][1-p_{34}p_{43} - p_{3,10}p_{10,12}p_{12,3}] - p_{02}p_{23}p_{02}p_{3,10}p_{50}p_{10,5}$$

5. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in upstate at instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are as follows:

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$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12.6}(t)[c]A_2(t)$$

$$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{24.7}(t)[c]A_4(t)$$

$$A_3(t) = M_3(t) + q_{34}(t)[c]A_4(t) + q_{3,10}(t)[c]A_{10}(t)$$

$$A_4(t) = M_4(t) + q_{43}(t)[c]A_3(t) + q_{4,10.8}(t)[c]A_1(t)$$

$$A_5(t) = M_5(t) + q_{50}(t)[c]A_0(t) + q_{51.9}(t)[c]A_1(t)$$

$$A_{10}(t) = M_{10}(t) + q_{10,1.13,15}(t)[c]A_1(t) + q_{10,4.13,14}(t)[c]A_4(t) + q_{10,5}(t)[c]A_5(t) + q_{10,12}(t)[c]A_{12}(t)$$

$$A_{12}(t) = M_{12}(t) + q_{12,3}(t)[c]A_3(t) + q_{12,4.11}(t)[c]A_4(t) \qquad (6)$$

$M_i(t)$ be the probability that system is up initially in state $S_i \in E$ is up at time t without visiting any other regenerative state, we have

$M_0(t) = \bar{Z}_1(t)\bar{S}_2(t)$, $M_1(t) = \bar{G}_1(t)\bar{Z}_2(t)$, $M_2(t) = \bar{Z}_1(t)\bar{G}_2(t)$, $M_3(t) = \bar{Z}_1(t)\bar{S}_{d2}(t)$,

$M_4(t) = \bar{G}_1(t)\bar{Z}_{d2}(t)$, $M_5(t) = \bar{Z}_1(t)\bar{F}_{d2}(t)$, $M_{10}(t) = \bar{Z}_1(t)\bar{H}_{d2}(t)$, $M_{12}(t) = \bar{Z}_1(t)\bar{G}_{d2}(t)$

Taking LST of above relation (6) and solving for $A_0^*(s)$ the steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA_0^*(s) = \frac{N_2}{D_2} \qquad (7)$$

Where

$$N_2 = [1 - p_{34}p_{43}][p_{10,5} + p_{10,1.13,15}][\mu_0 + p_{01}\mu_1] + [1 - p_{34}p_{43}][p_{10,1.13,15} + p_{10,5}p_{51.9}][p_{02}\mu_1 - p_{12.6}\mu_0]$$
$$+ [1 - p_{01}p_{10}][1 - p_{34}p_{43}][\{p_{10,5} + p_{10,1.13,15}\}\mu_2 + p_{10,5}\mu_5 + \mu_{10} + p_{10,12}\mu_{12} + [1 - p_{01}p_{10}]\{\{1 - p_{24.7}$$
$$p_{4,10.8}\}\mu_3 + \{1 - p_{23}p_{3,10}\}\mu_4] + [1 - p_{01}p_{10}][p_{4,10.8}\mu_3 - p_{3,10}\mu_4][p_{10,12}p_{12,3} - p_{23}p_{10,4.13,14}$$
$$- p_{23}p_{10,12}]$$

$$D_2 = [1 - p_{34}p_{43}][p_{10,5} + p_{10,1.13,15}][\mu_0 + p_{01}\mu_1'] + [1 - p_{34}p_{43}][p_{10,1.13,15} + p_{10,5}p_{51.9}][p_{02}\mu_1' - p_{12.6}\mu_0]$$
$$+ [1 - p_{01}p_{10}][1 - p_{34}p_{43}][\{p_{10,5} + p_{10,1.13,15}\}\mu_2' + p_{10,5}\mu_5' + \mu_{10}' + p_{10,12}\mu_{12}' + [1 - p_{01}p_{10}]\{\{1 - p_{24.7}$$
$$p_{4,10.8}\}\mu_3 + \{1 - p_{23}p_{3,10}\}\mu_4'] + [1 - p_{01}p_{10}][p_{4,10.8}\mu_3 - p_{3,10}\mu_4'][p_{10,12}p_{12,3} - p_{23}p_{10,4.13,14}$$
$$- p_{23}p_{10,12}]$$

6. SPECIAL CASE-WEIBULL DISTRIBUTION

In the following the values of different performance measures are obtained assuming all the random variables as Weibull distributed with common shape parameter (η) and different scale parameters as follows:

$z_1(t) = \lambda_1 \eta t^{\eta-1} \exp(-\lambda_1 t^{\eta})$, $z_2(t) = \lambda_2 \eta t^{\eta-1} \exp(-\lambda_2 t^{\eta})$, $z_{d2}(t) = \lambda_2^d \eta t^{\eta-1} \exp(-\lambda_2^d t^{\eta})$,

$s_2(t) = \mu_2^c \eta t^{\eta-1} \exp(-\mu_2^c t^{\eta})$, $s_{d2}(t) = \mu_2^d \eta t^{\eta-1} \exp(-\mu_2^d t^{\eta})$, $g_1(t) = \beta_1 \eta t^{\eta-1} \exp(-\beta_1 t^{\eta})$,

$g_2(t)=\beta_2\eta t^{\eta-1}\exp(-\beta_2 t^{\eta})$, $\quad f_{d2}(t)=\gamma_2^d\eta t^{\eta-1}\exp(-\gamma_2^d t^{\eta})$, $\quad h_{d2}(t)=\alpha_2^d\eta t^{\eta-1}\exp(-\alpha_2^d t^{\eta})$,

$g_{d2}(t)=\beta_2^d\eta t^{\eta-1}\exp(-\beta_2^d t^{\eta})$

where $t\geq 0$ and $\eta,\lambda_1,\lambda_2,\lambda_2^d,\mu_2^c,\mu_2^d,\beta_1,\beta_2,\gamma_2^d,\beta_2^d,\alpha_2^d>0$ respectively.

We can obtain the following result

$\text{MTSF}=\dfrac{N_1}{D_1}$, $\qquad$ Availability $(A_0)=\dfrac{N_2}{D_2}$

$$N_1=\Gamma(1+\frac{1}{\eta}).\frac{\begin{array}{l}[\{(\beta_1+\lambda_2)(\lambda_1+\beta_2)\}^{\frac{1}{\eta}}+(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}\{\lambda_1(\lambda_1+\beta_2)^{\frac{1}{\eta}}+\mu_2^c(\beta_1+\lambda_2)^{\frac{1}{\eta}}\}][(\lambda_1+\alpha_2^d)\\(\lambda_1+\beta_2^d)\{\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)\}-a^d\alpha_2^d\mu_2^d\beta_2^d(\beta_1+\lambda_2^d)]\{(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)\\(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}-1}(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}}+\mu_2^c\beta_2\{(\lambda_1+\mu_2^c)(\lambda_1+\beta_2)\}^{\frac{1}{\eta}-1}(\beta_1+\lambda_2)^{\frac{1}{\eta}}[\alpha_2^d\\\mu_2^d\{b^d(\lambda_1+\beta_2^d)^{\frac{1}{\eta}}+a^d(\lambda_1+\gamma_2^d)\}\{(\lambda_1+\mu_2^d)(\lambda_1+\alpha_2^d)\}^{\frac{1}{\eta}-1}(\beta_1+\lambda_2)^{\frac{1}{\eta}}+\{((\beta_1+\lambda_2^d)\\(\lambda_1+\alpha_2^d))^{\frac{1}{\eta}}+(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}(\lambda_1(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}}+\mu_2^d(\lambda_1+\beta_2^d)^{\frac{1}{\eta}})\}\{(\lambda_1+\gamma_2^d)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}\end{array}}{[(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\lambda_1+\beta_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)]^{\frac{1}{\eta}}}$$

$$D_1=\Gamma(1+\frac{1}{\eta}).\frac{\begin{array}{l}[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\{\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)\}-a^d\alpha_2^d\mu_2^d\beta_2^d\\(\beta_1+\lambda_2^d)](\lambda_1+\beta_2)(\lambda_1+\gamma_2^d)-b^d\alpha_2^d\mu_2^c\beta_2\mu_2^d\gamma_2^d(\beta_1+\lambda_2)(\beta_1+\lambda_2^d)(\lambda_1+\beta_2^d)\end{array}}{\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\lambda_1+\beta_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)}$$

$$N_2=\Gamma(1+\frac{1}{\eta}).\frac{\begin{array}{l}[\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]b^d\{(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)\}^{\frac{1}{\eta}-1}\{(\lambda_1+\beta_2)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}[\{(\beta_1+\lambda_2)^{\frac{1}{\eta}}\\+\lambda_1(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}\}\{(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)\}+\lambda_1(\lambda_1+\gamma_2^d+\alpha_2^d)\{\mu_2^c(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}-\lambda_2(\beta_1\\+\lambda_2^d)^{\frac{1}{\eta}-1}\}]\{(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)\}^{\frac{1}{\eta}-1}+[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)][\{(\lambda_1\\+\gamma_2^d)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}\{b^d(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}}+(\lambda_1+\beta_2)^{\frac{1}{\eta}}\}+\alpha_2^d(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}(\lambda_1+\beta_2)^{\frac{1}{\eta}}\{b^d\\(\lambda_1+\beta_2^d)^{\frac{1}{\eta}}+a^d(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}}\}]\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)\}^{\frac{1}{\eta}-1}+[\lambda_1\lambda_2+\mu_2^c\\(\beta_1+\lambda_2)]\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}-1}(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}}[[\{\lambda_1\beta_1+\\(\beta_1+\lambda_2^d)\beta_2\}(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}+\{\lambda_1\beta_2+\lambda_1(\lambda_1+\mu_2^d)\}(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\\+\{\lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}-\mu_2^d(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}\}a^d\{\alpha_2^d\beta_2^d(\lambda_1+\beta_2)-\beta_2(\lambda_1+\beta_2^d)(\lambda_1+\alpha_2^d)\}]\end{array}}{[(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\lambda_1+\beta_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)]^{\frac{1}{\eta}}}$$

$$D_2=\Gamma\left(1+\frac{1}{\eta}\right).\frac{\begin{array}{l}[\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]b^d\{(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)(\lambda_1+\alpha_2^d)\}^{\frac{1}{\eta}-1}\{\beta_2\alpha_2^d\beta_2^d\gamma_2^d\\ (\lambda_1+\beta_2)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}[\{(\beta_1(\beta_1+\lambda_2))^{\frac{1}{\eta}}+\lambda_1(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}\{\beta_1^{\frac{1}{\eta}}+\lambda_2(\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\}\}(\lambda_1\\ +\gamma_2^d)(\lambda_1+\alpha_2^d)+\lambda_1(\lambda_1+\gamma_2^d+\alpha_2^d)\{\mu_2^c(\beta_1^{\frac{1}{\eta}}+\lambda_2(\beta_1+\lambda_2)^{\frac{1}{\eta}-1})(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}-\lambda_2(\beta_1)^{\frac{1}{\eta}}\\ (\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\}]+[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)][\{\alpha_2^d\gamma_2^d(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}}\\ (\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}[b^d\{\beta_2^{\frac{1}{\eta}}+\lambda_1(\lambda_1+\beta_2)^{\frac{1}{\eta}-1}\}(\lambda_1+\alpha_2^d)\{\beta_2^d(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}+a^d\alpha_2^d\{(\beta_2^d)^{\frac{1}{\eta}}\\ +\lambda_1(\lambda_1+\beta_2^d)^{\frac{1}{\eta}-1}\}\{\beta_2(\lambda_1+\beta_2)^{\frac{1}{\eta}}\}]+\{\beta_2(\lambda_1+\beta_2)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}}[b^d\alpha_2^d\{(\gamma_2^d)^{\frac{1}{\eta}}+\lambda_1\\ (\lambda_1+\gamma_2^d)^{\frac{1}{\eta}-1}\}\{\alpha_2^d\beta_2^d\}^{\frac{1}{\eta}}(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}+\{(\alpha_2^d\beta_2^d\gamma_2^d)^{\frac{1}{\eta}}+\lambda_1(\lambda_1+\alpha_2^d)^{\frac{1}{\eta}-1}[(\beta_2^d\gamma_2^d)^{\frac{1}{\eta}}\\ +(a^d\gamma_2^d+b^d\beta_2^d)(\alpha_2^d)^{\frac{1}{\eta}}]\}(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}}]](\beta_1)^{\frac{1}{\eta}}\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\beta_1\\ +\lambda_2^d)\}^{\frac{1}{\eta}-1}+[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)]\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)\}^{\frac{1}{\eta}-1}\\ \{\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}}[(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2^d)[\lambda_1\beta_1+(\beta_1+\lambda_2^d)\beta_2(\beta_1)^{\frac{1}{\eta}}(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\\ +\{\lambda_1+\beta_2+\mu_2^d\}\lambda_1\{\beta_1^{\frac{1}{\eta}}+\lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}]+\{\lambda_2^d\beta_1^{\frac{1}{\eta}}(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}-\mu_2^d\\ \{\beta_1^{\frac{1}{\eta}}+\lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}\}a^d\{\alpha_2^d\beta_2^d(\lambda_1+\beta_2)-\beta_2(\lambda_1+\alpha_2^d)(\lambda_1+\beta_2)\}]\end{array}}{\begin{array}{l}[\beta_1\beta_2\alpha_2^d\beta_2^d\gamma_2^d(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\lambda_1+\beta_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)\\ (\lambda_1+\alpha_2^d)]^{\frac{1}{\eta}}\end{array}}$$

7. CONCLUSION

A stochastic model of a dissimilar unit cold standby system is developed by using the theory of semi-Markov regenerative processes. The idea of assigning priority to the main unit is explored. The expressions are derived for system reliability and availability using the tools of mathematical transforms.Assuming the general probability distribution initially for all the random variables, the results are also given for Weibull distribution. Which in turns, allowedfor the scope of further numerical illustration of the study.

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