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On Busy Period and Number of Remedial Actions of Server in a Cold Standby System with Maximum Redundancy Time

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ABSTRACT:

This paper develops a theoretical model of a cold standby system with the possibility of standby failure. The semi-Markov regenerative point processes are used to compute the busy period and expected number of system restorative actions. The system has one original unit in operation and another duplicate as standby. The original unit is given priority for operation as well as rectification over the duplicate unit. Allthe random variables are independent and follow general probability distribution. For a particular case Weibull distribution is considered and results are obtained.

Keywords-*Busy period, Redundancy, Remedial actions, Standby system and Semi-Markov processes.*

1. INTRODUCTION

A cold standby system is one in which few units (or a unit) are in active working mode whereas few other units (or a unit) are kept passive in redundant mode for contingency [1-2]. These systems are widely considered in the past studies such as [3-4] discussed cold standby system models with possible standby failure, [5-7] studied cold standby systems with repairable server, [8] analyzed a standby system with neglected failures, [9] described a 2-out-of-3 redundant system and [10] discussed a standby system with two stage repair and waiting time.In all such systems the standby unit has to replace the operating unit at its failure and in the meantime the failed unit has to be repaired or replaced by new one so as to keep the system working [11]. These remedial tasks are to be performed by the server. So in a repairable system the role of service facility or the server is of the essence. In fact, the time for which the server remain engaged in remedial activities as well as the total number of such tasks performed in given duration increasesthe system running costs. Hence need to be studied essentially. Another important factor contributing to system cost is the price tag of standby unit. The cold standby unit being not of often use can be compromised learnedly.Therefore in view of these facts, in the current paper a dissimilar unit cold standby system model is developed. The system consists oftwo unit one original unit in operation and a duplicate instandby. The unit in standby can fail after crossing the maximum redundancy time limit. The duplicate becomes degraded. If the degraded unit fails it is replaced by new one. The original unit is given operation as well as repair priorities upon the duplicate one [12]. All the random variables are independently follow general distribution. The switch is perfect and instantaneous. The semi-Markov regenerative point processes [13-14] are used to compute the busy period and expected number of system restorative actions.For a particular case all the random variables are assumed to follow Weibull distribution.

2. NOTATIONS

$E / \overline{E}$	: The set of regenerative/ Non-regenerative states
$No_1 / No_2 / Do_2$	: Original unit/ Duplicate unit/ Degraded unit in operation.
Cs_2 / DCs_2	: Duplicate unit/ Degraded unit in cold-standby mode.
F_{ur_i} / F_{UR_i}	: Failed unit i= 1, 2 under repair /under repair continuously from previous state.
F_{wr_2} / F_{WR_2}	: Duplicate failed unit waiting for repair / waiting for repair continuously from previous state.
DF_{urp_2} / DF_{URp_2}	: Degraded failed unit under replacement / under replacement continuously from previous state.
DF_{wrp_2} / DF_{WRp_2}	: Failed unit waiting for replacement / waiting for replacement continuously from previous state.
$z_i(t) / Z_i(t)$	: pdf / cdf of failure time of i= 1, 2 unit.
$z_{d2}(t) / Z_{d2}(t)$	: pdf / cdf of failure time of degraded unit.
$s_2(t) / S_2(t)$	: pdf / cdf of maximum redundancy time of duplicate unit.
$s_{d2}(t) / S_{d2}(t)$	: pdf / cdf of maximum redundancy time of degraded unit.
$g_i(t) / G_i(t)$	: pdf / cdf of repair time of i=1, 2 unit.
$f_{d2}(t) / F_{d2}(t)$	: pdf / cdf of replacement time of degraded unit.
$q_{ij}(t) / Q_{ij}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative State S_j or failed state S_j without visiting any other regenerative state in $(0,t]$.
$q_{ij.kr}(t) / Q_{ij.kr}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative state S_j or failed state S_j visiting state S_k, S_r once in $(0,t]$.
$\mu_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$[s]/[c]$	: Laplace Stieltjes convolution / Laplace convolution.
$\sim / *$	: Laplace Stieltjes Transform (LST) / Laplace Transform (LT).

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3. THE MODEL

The state transition diagram along with all possible states and transition time distributions is given in Fig. 1

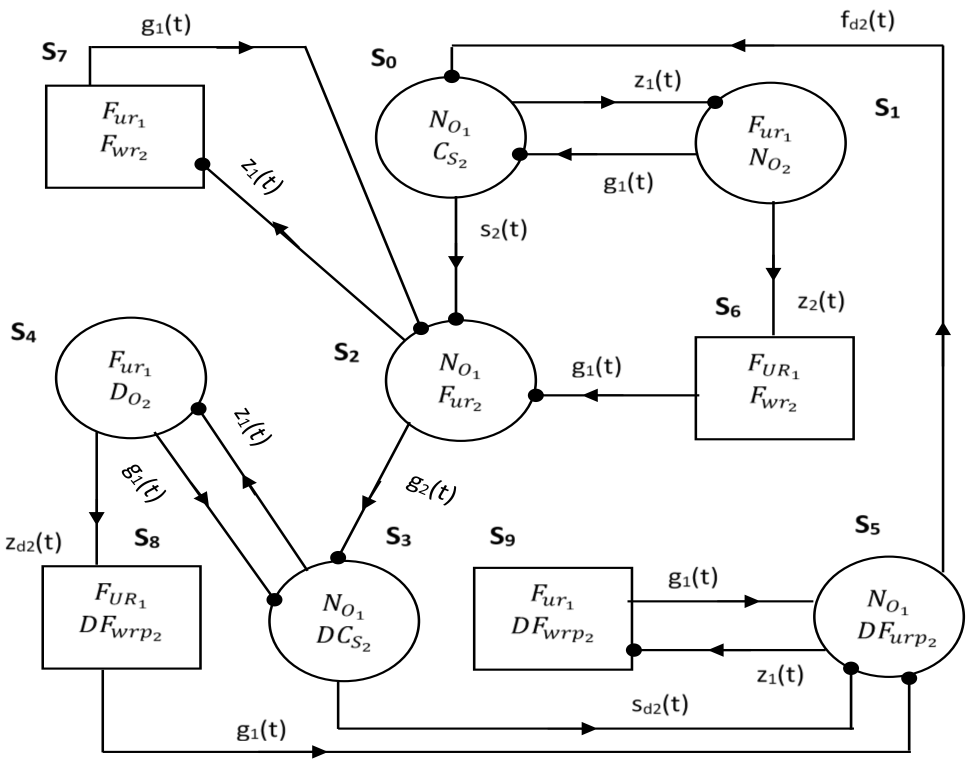

Figure-1: System State Transition Diagram

4. BUSY PERIOD EVALUATION

Let $W_{i\in E}^{R_1/R_2/Rp_2^d}(t)$ is the probability that the server is busy due to repair of unit 1/ repair of unit 2/ replacement of degraded unit in the regenerative state up to time ‘t’ without making any transition to any other regenerative state or returning to the same state via one or more non- regenerative states.

$$W_1^{R_1}(t)=\overline{G}_1(t)\overline{Z}_2(t)+(z_2(t)[c]1)\overline{G}_1(t),\quad W_2^{R_2}(t)=\overline{Z}_1(t)\overline{G}_2(t),\quad W_4^{R_1}(t)=\overline{G}_1(t)\overline{Z}_{d2}(t)+(z_{d2}(t)[c]1)\overline{G}_1(t),$$

$$W_5^{Rp_2^d}(t)=\overline{Z}_1(t)\overline{F}_{d2}(t),\ W_7^{R_1}(t)=\overline{G}_1(t),\quad W_9^{R_1}(t)=\overline{G}_1(t)$$

Further, let $B_i(t)$ be the probability that the server is busy at an instant t given that the system entered state S_i at time t=0. The recursive relations for $B_i(t)$ are as follows:

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$$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$$

$$B_1(t) = W_1^{R_1}(t) + q_{10}(t)[c]B_0(t) + q_{12.6}(t)][c]B_2(t)$$

$$B_2(t) = W_2^{R_1}(t) + q_{23}(t)[c]B_3(t) + q_{27}(t)[c]B_7(t)$$

$$B_3(t) = q_{34}(t)[c]B_4(t) + q_{35}(t)[c]B_5(t)$$

$$B_4(t) = W_4^{R_1}(t) + q_{43}(t)[c]B_3(t) + q_{45.8}(t)[c]B_5(t)$$

$$B_5(t) = W_5^{Rp_2^d}(t) + q_{50}(t)[c]B_0(t) + q_{59}(t)[c]B_9(t)$$

$$B_7(t) = W_7^{R_1}(t) + q_{72}(t)[c]B_2(t)$$

$$B_9(t) = W_9^{R_1}(t) + q_{95}(t)[c]B_5(t) \qquad (1)$$

Taking LT of relation (1), and solving for $B_0^*(s)$, the time for which server is busy is given by

$$B_0(\infty) = \lim_{s \to 0} sB_0^*(s) = \frac{N_3}{D_2} \qquad (2)$$

$$N_3 = p_{01}p_{23}p_{50}[1 - p_{34}p_{43}]W_1^{R_1^*}(0) + p_{23}p_{34}p_{50}[1 - p_{01}p_{10}]W_4^{R_1^*}(0) + [1 - p_{01}p_{10}][1 - p_{34}p_{43}][p_{50}\{W_2^{R_2^*}(0)$$
$$+ p_{27}W_7^{R_1^*}(0)\} + p_{23}\{W_5^{Rp_2^{d^*}}(0) + p_{59}W_9^{R_1^*}(0)\}]$$

$$D_2 = p_{23}p_{50}[1 - p_{34}p_{43}][\mu_0 + p_{01}\mu_1'] + [1 - p_{01}p_{10}][1 - p_{34}p_{43}][p_{50}\{\mu_2 + p_{27}\mu_7\} + p_{23}\{\mu_5 + p_{59}\mu_9\}]$$
$$+ p_{23}p_{50}[1 - p_{01}p_{10}]\{\mu_3 + p_{34}\mu_4'\}$$

5. EXPECTED NUMBER OF REPAIRS OF THE ORIGINAL UNIT

Let $R_i^1(t)$ be the expected number of repairs of the failed original unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^1(t)$ are as follows:

$$R_0^1(t) = Q_{01}(t)[c]R_1^1(t) + Q_{02}(t)[c]R_2^1(t)$$

$$R_1^1(t) = Q_{10}(t)[c][1 + R_0^1(t)] + Q_{12.6}(t)][c][1 + R_2^1(t)]$$

$$R_2^1(t) = Q_{23}(t)[c]R_3^1(t) + Q_{27}(t)[c]R_7^1(t)$$

$$R_3^1(t) = Q_{34}(t)[c]R_4^1(t) + Q_{35}(t)[c]R_5^1(t)$$

$$R_4^1(t) = Q_{43}(t)[c][1 + R_3^1(t)] + Q_{45.8}(t)[c][1 + R_5^1(t)]$$

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$$R_5^1(t) = Q_{50}(t)[c]R_0^1(t) + Q_{59}(t)[c]R_9^1(t)$$

$$R_7^1(t) = Q_{72}(t)[c][1 + R_2^1(t)]$$

$$R_9^1(t) = Q_{95}(t)[c][1 + R_5^1(t)] \quad (3)$$

Taking LST of above relation (3) and solving for $\tilde{R}_0^1(s)$. The expected number of repairs per unit time to failed original unit is given by

$$R_0^1(\infty) = \lim_{s \to 0} s\tilde{R}_0^1(s) = \frac{N_4^{R_1}}{D_2} \quad (4)$$

$$N_4^{R_1} = p_{01}p_{23}p_{50}[1 - p_{34}p_{43}] + p_{23}p_{34}p_{50}[1 - p_{01}p_{10}] + [1 - p_{01}p_{10}][1 - p_{34}p_{43}][p_{23}p_{59} + p_{27}p_{50}]$$

$$D_2 = p_{23}p_{50}[1 - p_{34}p_{43}][\mu_0 + p_{01}\mu_1^{'}] + [1 - p_{01}p_{10}][1 - p_{34}p_{43}][p_{50}\{\mu_2 + p_{27}\mu_7\} + p_{23}\{\mu_5 + p_{59}\mu_9\}]$$
$$+ p_{23}p_{50}[1 - p_{01}p_{10}]\{\mu_3 + p_{34}\mu_4^{'}\}$$

6. EXPECTED NUMBER OF REPAIRS OF THE DUPLICATE UNIT

Let $R_i^2(t)$ be the expected number of repairs of the failed duplicate unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^2(t)$ are as follows:

$$R_0^2(t) = Q_{01}(t)[c]R_1^2(t) + Q_{02}(t)[c]R_2^2(t)$$

$$R_1^2(t) = Q_{10}(t)[c]R_0^2(t) + Q_{12.6}(t)][c]R_2^2(t)$$

$$R_2^2(t) = Q_{23}(t)[c][1 + R_3^2(t)] + Q_{27}(t)[c]R_7^2(t)$$

$$R_3^2(t) = Q_{34}(t)[c]R_4^2(t) + Q_{35}(t)[c]R_5^2(t)$$

$$R_4^2(t) = Q_{43}(t)[c]R_3^2(t) + Q_{45.8}(t)[c]R_5^2(t)$$

$$R_5^2(t) = Q_{50}(t)[c]R_0^2(t) + Q_{59}(t)[c]R_9^2(t)$$

$$R_7^2(t) = Q_{72}(t)[c]R_2^2(t)$$

$$R_9^2(t) = Q_{95}(t)[c]R_5^2(t) \quad (5)$$

Taking LST of above relation (5) and solving for $\tilde{R}_0^2(s)$. The expected number of repairs per unit time to failed duplicate unit is given by

$$R_0^2(\infty) = \lim_{s \to 0} s\tilde{R}_0^2(s) = \frac{N_4^{R_2}}{D_2} \qquad (6)$$

$$N_4^{R_2} = p_{23}[1 - p_{01}p_{10}][1 - p_{34}p_{43}]p_{50}$$

$$D_2 = p_{23}p_{50}[1 - p_{34}p_{43}][\mu_0 + p_{01}\mu_1^{'}] + [1 - p_{01}p_{10}][1 - p_{34}p_{43}][p_{50}\{\mu_2 + p_{27}\mu_7\} + p_{23}\{\mu_5 + p_{59}\mu_9\}]$$
$$+ p_{23}p_{50}[1 - p_{01}p_{10}]\{\mu_3 + p_{34}\mu_4^{'}\}$$

7. EXPECTED NUMBER OF REPLACEMENTS OF THE DEGRADED UNIT

Let $R_i^{C_2^d}(t)$ be the expected number of replacements of the failed degraded unit by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^{C_2^d}(t)$ are as follows:

$$R_0^{C_2^d}(t) = Q_{01}(t)[c]R_1^{C_2^d}(t) + Q_{02}(t)[c]R_2^{C_2^d}(t)$$

$$R_1^{C_2^d}(t) = Q_{10}(t)[c]R_0^{C_2^d}(t) + Q_{12.6}(t)][c]R_2^{C_2^d}(t)$$

$$R_2^{C_2^d}(t) = Q_{23}(t)[c]R_3^{C_2^d}(t) + Q_{27}(t)[c]R_7^{C_2^d}(t)$$

$$R_3^{C_2^d}(t) = Q_{34}(t)[c]R_4^{C_2^d}(t) + Q_{35}(t)[c]R_5^{C_2^d}(t)$$

$$R_4^{C_2^d}(t) = Q_{43}(t)[c]R_3^{C_2^d}(t) + Q_{45.8}(t)[c]R_5^{C_2^d}(t)$$

$$R_5^{C_2^d}(t) = Q_{50}(t)[c][1 + R_0^{C_2^d}(t)] + Q_{59}(t)[c]R_9^{C_2^d}(t)$$

$$R_7^{C_2^d}(t) = Q_{72}(t)[c]R_2^{C_2^d}(t)$$

$$R_9^{C_2^d}(t) = Q_{95}(t)[c]R_5^{C_2^d}(t) \qquad (7)$$

Taking LST of above relation (7) and solving for $\tilde{R}_0^{C_2^d}(t)$. The expected number of replacements per unit time to failed degraded unit is given by

$$R_0^{C_2^d}(\infty) = \lim_{s \to 0} s\tilde{R}_0^{C_2^d}(s) = \frac{N_4^{C_2^d}}{D_2} \qquad (8)$$

$$N_4^{C_2^d} = p_{23}p_{50}[1-p_{01}p_{10}][1-p_{34}p_{43}]$$

$$\begin{aligned} D_2 = {} & p_{23}p_{50}[1-p_{34}p_{43}][\mu_0 + p_{01}\mu_1^{'}] + [1-p_{01}p_{10}][1-p_{34}p_{43}][p_{50}\{\mu_2 + p_{27}\mu_7\} + p_{23}\{\mu_5 + p_{59}\mu_9\}] \\ & + p_{23}p_{50}[1-p_{01}p_{10}]\{\mu_3 + p_{34}\mu_4^{'}\} \end{aligned}$$

8. PARTICULAR CASE:

Let us assume the random variables follows Weibull distribution with common shape parameter (η) and different scale parameters as given below.

$z_1(t) = \lambda_1\eta t^{\eta-1}\exp(-\lambda_1 t^{\eta})$, $z_2(t) = \lambda_2\eta t^{\eta-1}\exp(-\lambda_2 t^{\eta})$, $z_{d2}(t) = \lambda_2^d\eta t^{\eta-1}\exp(-\lambda_2^d t^{\eta})$,

$s_2(t) = \mu_2^c\eta t^{\eta-1}\exp(-\mu_2^c t^{\eta})$, $s_{d2}(t) = \mu_2^d\eta t^{\eta-1}\exp(-\mu_2^d t^{\eta})$, $g_1(t) = \beta_1\eta t^{\eta-1}\exp(-\beta_1 t^{\eta})$,

$g_2(t) = \beta_2\eta t^{\eta-1}\exp(-\beta_2 t^{\eta})$, $f_{d2}(t) = \gamma_2^d\eta t^{\eta-1}\exp(-\gamma_2^d t^{\eta})$

where $t \geq 0$ and $\eta, \lambda_1, \lambda_2, \lambda_2^d, \mu_2^c, \mu_2^d, \beta_1, \beta_2, \gamma_2^d > 0$ respectively.

For Weibull distribution, we obtained the following result

$$N_3 = \Gamma(1+\frac{1}{\eta}).\frac{\begin{gathered}\{\lambda_1[\lambda_1\lambda_2^d + \mu_2^d(\beta_1+\lambda_2^d)](\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\{\beta_1\gamma_2^d[(\beta_1)^{\frac{1}{\eta}} + \lambda_2(\beta_1+\lambda_2)^{\frac{1}{\eta}-1}] + (\beta_2+\gamma_2^d)[\lambda_1\lambda_2+\mu_2^c \\ (\beta_1+\lambda_2)](\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\} + \beta_2\lambda_1\gamma_2^d[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)]\{(\beta_1)^{\frac{1}{\eta}} + \lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}(\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\} \\ [(\lambda_1+\beta_2)(\lambda_1+\gamma_2^d)]^{\frac{1}{\eta}-1} + [\gamma_2^d\{(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}-1} + \beta_2\{(\lambda_1+\beta_2)\}^{\frac{1}{\eta}-1}](\beta_1)^{\frac{1}{\eta}}\{(\beta_1+\lambda_2)(\beta_1+\lambda_2^d)\}^{\frac{1}{\eta}-1} \\ [\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]\end{gathered}}{[\beta_1(\beta_1+\lambda_2)(\beta_1+\lambda_2^d)(\lambda_1+\beta_2)(\lambda_1+\gamma_2^d)]^{\frac{1}{\eta}}(\lambda_1+\mu_2^c)(\lambda_1+\mu_2^d)}$$

$$N_4^{R_1} = \frac{\begin{gathered}\lambda_1[\lambda_1\lambda_2^d + \mu_2^d(\beta_1+\lambda_2^d)][\beta_2\gamma_2^d(\beta_1+\lambda_2) + \{\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)\}(\beta_2+\gamma_2^d)] + \lambda_1\beta_2\gamma_2^d[\lambda_1\lambda_2+\mu_2^c(\beta_1 \\ +\lambda_2)](\beta_1+\lambda_2^d)\end{gathered}}{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)}$$

$$N_4^{R_2} = \frac{\beta_2\gamma_2^d[\lambda_1\lambda_2 + \mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d + \mu_2^d(\beta_1+\lambda_2^d)]}{(\beta_2+\lambda_1)(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\lambda_1+\gamma_2^d)(\beta_1+\lambda_2^d)},$$

$$N_4^{C_2^d} = \frac{\beta_2\gamma_2^d[\lambda_1\lambda_2 + \mu_2^c(\beta_1+\lambda_2)][\lambda_1\lambda_2^d + \mu_2^d(\beta_1+\lambda_2^d)]}{(\beta_2+\lambda_1)(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\mu_2^d)(\lambda_1+\gamma_2^d)(\beta_1+\lambda_2^d)} \text{ and}$$

$$D_2 = \Gamma(1+\frac{1}{\eta}).\frac{\begin{aligned}&\{\beta_2\gamma_2^d[\{\beta_1(\beta_1+\lambda_2)\}^{\frac{1}{\eta}} + \lambda_1(\lambda_1+\mu_2^c)^{\frac{1}{\eta}-1}\{(\beta_1)^{\frac{1}{\eta}} + \lambda_2(\beta_1+\lambda_2)^{\frac{1}{\eta}-1}\}][(\lambda_1+\beta_2)(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}-1}\\&+[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\gamma_2^d(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}-1}\{(\beta_1)^{\frac{1}{\eta}} + \lambda_1(\beta_2+\lambda_1)^{\frac{1}{\eta}-1}\} + \beta_2(\lambda_1+\beta_2)^{\frac{1}{\eta}-1}\{(\beta_1)^{\frac{1}{\eta}}\\&+\lambda_1(\lambda_1+\gamma_2^d)^{\frac{1}{\eta}-1}\}]\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)\}^{\frac{1}{\eta}-1}\}[\lambda_1\lambda_2^d+\mu_2^d(\beta_1+\lambda_2^d)]\{(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)\}^{\frac{1}{\eta}-1}\\&+\beta_2\gamma_2^d[\lambda_1\lambda_2+\mu_2^c(\beta_1+\lambda_2)][\{\beta_1(\beta_1+\lambda_2^d)\}^{\frac{1}{\eta}} + \lambda_1(\lambda_1+\mu_2^d)^{\frac{1}{\eta}-1}\{(\beta_1)^{\frac{1}{\eta}} + \lambda_2^d(\beta_1+\lambda_2^d)^{\frac{1}{\eta}-1}\}]\\&\{(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\gamma_2^d)\}^{\frac{1}{\eta}-1}\end{aligned}}{[\beta_1(\lambda_1+\mu_2^c)(\beta_1+\lambda_2)(\lambda_1+\beta_2)(\lambda_1+\mu_2^d)(\beta_1+\lambda_2^d)(\lambda_1+\gamma_2^d)]^{\frac{1}{\eta}}}$$

9. CONCLUSION

The research paper presents a theoretical analysis of the busy period and expected number of system restorative tasks of a dissimilar unit cold standby system. The stochastic model is developed using the theory of semi-Markov processes. The model is investigated by identifying various regenerative points at different possible state transition epochs. To trace practicality of the study a special case of Weibull distribution is discussed at the end and results are derived.

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