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Reliability and Availability Analysis of a Dissimilar Unit Cold Standby System withMaximum Redundancy Time Limit

R.K. Bhardwaj, Komaldeep Kaur

Department of Statistics, Punjabi University Patiala-147002, India

ABSTRACT

A probabilistic model of a cold standby system is developed. There are two dissimilar units in the system. Initially the system starts operating taking original unit in active and duplicate in passive mode. The passive unit fails after crossing a pre-specified time limit and become degraded after repair. The theory of renewal processes is explored to derive expressions for reliability and availability of the model. An assumed data set is considered for numerical analysis of the results. For a special case all the random variables are assumed to follow Weibull distribution.

Keywords- *Availability, Reliability, Redundancy Time, Standby System, Stochastic Processes.*

1. INTRODUCTION

The reliability and availability are the vital characteristics of a working system.There are various ways to improve theseessentials attributes. To serve this cause some redundancy techniques are discussed in the related studies. The cold standby redundancy is one such techniqueused commonly in the literature [1-6]. Most of the studies considered identical units for the standby [7-14]. This consideration seems costlier as the standard unit always has budgets higher than a duplicate unit.

Therefore, keeping this fact in view in this paper a dissimilar unit cold standby system models is considered.To save cost a duplicate unit is taken as standby instead of a standard one. As the original unit fails it goes under repair and becomes as new after each repair. The duplicate standby unit can fails after exceeding a pre-specified maximum time limit, termed as maximum redundancy time limit.There is a single repair facility in the system,accountable for all remedial actions. The duplicate unit becomes degraded after the first repair and gets replaced by new one at further failure. The replacement takes some random amount of time andthe original unit is given the first choice for operation as well as repair upon the duplicate unit.All the random variables are statistically independent and follow general probability distribution with distinct distribution functions. The inter-switching of original and duplicate unit is perfect and prompt. The paper utilized the concepts of semi-Markov regenerative processes of renewal theory [15-17]. The expressions are derived for reliability and availability of the system.An assumed rough data set is taken for the numerical exploration of results.For the same all the random variable are assumed to follow the Weibull distribution with common shape parameter.

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2. NOTATIONS

$E / \overline{E}$	: The set of regenerative/ Non-regenerative states
$No_1/No_2/Do_2$	: Original unit/ Duplicate unit/ Degraded unit in operation.
Cs_2/DCs_2	: Duplicate unit/ Degraded unit in cold-standby mode.
F_{ur_i}/F_{UR_i}	: Failed unit i= 1, 2 under repair /under repair continuously from previous state.
F_{wr_2}/F_{WR_2}	: Duplicate failed unit waiting for repair / waiting for repair continuously from previous state.
DF_{urp_2}/DF_{URp_2}	: Degraded failed unit under replacement / under replacement continuously from previous state.
DF_{wrp_2}/DF_{WRp_2}	: Failed unit waiting for replacement / waiting for replacement continuously from previous state.
$z_i(t)/Z_i(t)$	: pdf / cdf of failure time of i= 1, 2 unit.
$z_{d2}(t)/Z_{d2}(t)$	: pdf / cdf of failure time of degraded unit.
$s_2(t)/S_2(t)$	: pdf / cdf of maximum redundancy time of duplicate unit.
$s_{d2}(t)/S_{d2}(t)$	: pdf / cdf of maximum redundancy time of degraded unit.
$g_i(t)/G_i(t)$	: pdf / cdf of repair time of i=1, 2 unit.
$f_{d2}(t)/F_{d2}(t)$	: pdf / cdf of replacement time of degraded unit.
$q_{ij}(t)/Q_{ij}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative State S_j or failed state S_j without visiting any other regenerative state in $(0,t]$.
$q_{ij.kr}(t)/Q_{ij.kr}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative state S_j or failed state S_j visiting state S_k, S_r once in $(0,t]$.
$\mu_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	: Probability that server busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states
$[s]/[c]$	: Symbol for Laplace-Stietjes convolution/Laplace convolution
$\sim /*$	: Symbol for Laplace- stietjes Transform (LST)/Laplace transform (LT)
$'(desh)$	: Symbol used to represent alternative result

3. THE MODEL DEVELOPMENT

The following are possible transition states of the system model

The regenerative states:

$S_0=\left(No_1,Cs_2\right), S_1=\left(F_{ur_1},No_2\right), \quad S_2=\left(No_1,F_{ur_2}\right), \quad S_3=\left(No_1,DCs_2\right), S_4=\left(F_{ur_1},Do_2\right),$

$S_5=\left(No_1,DF_{urp_2}\right)$

The non-regenerative states:

$S_6=\left(F_{UR_1},F_{wr_2}\right), S_7=\left(F_{ur_1},F_{wr_2}\right), S_8=\left(F_{UR_1},DF_{wrp_2}\right), S_9=\left(F_{ur_1},DF_{wrp_2}\right)$

3.1 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements:-

$$p_{ij}=Q_{ij}(\infty)=\int_0^\infty q_{ij}(t)dt \quad (1)$$

$$p_{01}=\int_0^\infty z_1(t)\bar{S}_2(t)dt, \quad p_{02}=\int_0^\infty s_2(t)\bar{Z}_1(t)dt, \quad p_{10}=\int_0^\infty g_1(t)\bar{Z}_2(t)dt, \quad p_{16}=\int_0^\infty z_2(t)\bar{G}_1(t)dt,$$

$$p_{12.6}=p_{16}p_{62}, \quad p_{23}=\int_0^\infty g_2(t)\bar{Z}_1(t)dt, \quad p_{27}=\int_0^\infty z_1(t)\bar{G}_2(t)dt, \quad p_{34}=\int_0^\infty z_1(t)\bar{S}_{d2}(t)dt,$$

$$p_{45.8}=p_{48}p_{85}, \quad p_{35}=\int_0^\infty s_{d2}(t)\bar{Z}_1(t)dt, \quad p_{43}=\int_0^\infty g_1(t)\bar{Z}_{d2}(t)dt, \quad p_{48}=\int_0^\infty z_{d2}(t)\bar{G}_1(t)dt,$$

$$p_{50}=\int_0^\infty f_{d2}(t)\bar{Z}_1(t)dt, \quad p_{59}=\int_0^\infty z_1(t)\bar{F}_{d2}(t)dt, \quad p_{62}=\int_0^\infty g_1(t)dt, \quad p_{72}=\int_0^\infty g_1(t)dt,$$

$$p_{85}=\int_0^\infty g_1(t)dt, \quad p_{95}=\int_0^\infty g_1(t)dt,$$

For these Transition Probabilities, it can be verified that

$$p_{01}+p_{02}=p_{10}+p_{16}=p_{10}+p_{12.6}=p_{23}+p_{27}=p_{34}+p_{35}=p_{43}+p_{48}=p_{43}+p_{45.8}=p_{50}+p_{59}$$

$$p_{62}=p_{72}=p_{85}=p_{95}=1$$

The Mean sojourn time μ_i in state S_i are given by:

$$\mu_i=E(t)=\int_0^\infty P(T_i>t)dt \quad (2)$$

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$\mu_0 = \int_0^\infty \bar{Z}_1(t)\bar{S}_2(t)dt, \quad \mu_1 = \int_0^\infty \bar{G}_1(t)\bar{Z}_2(t), \quad \mu_2 = \int_0^\infty \bar{G}_2(t)\bar{Z}_1 dt, \quad \mu_3 = \int_0^\infty \bar{S}_{d2}(t)\bar{Z}_1 dt,$

$\mu_4 = \int_0^\infty \bar{G}_1(t)\bar{Z}_{d2} dt, \quad \mu_5 = \int_0^\infty \bar{Z}_1(t)\bar{F}_{d2}(t)dt$

4. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t)[s]\phi_1(t) + Q_{02}(t)[s]\phi_2(t)$

$\phi_1(t) = Q_{10}(t)[s]\phi_0(t) + Q_{16}(t)$

$\phi_2(t) = Q_{23}(t)[s]\phi_3(t) + Q_{27}(t)$

$\phi_3(t) = Q_{34}(t)[s]\phi_4(t) + Q_{35}(t)[s]\phi_5(t)$

$\phi_4(t) = Q_{43}(t)[s]\phi_3(t) + Q_{48}(t)$

$\phi_5(t) = Q_{50}(t)[s]\phi_0(t) + Q_{59}(t)$ (3)

Taking Laplace transform of above equations and solving for $\tilde{\phi}_0(s)$ we get

$$\tilde{\phi}_0(s) = \frac{\tilde{Q}_{23}\tilde{Q}_{02}[\tilde{Q}_{34}\tilde{Q}_{48} + \tilde{Q}_{35}\tilde{Q}_{59}] + [1-\tilde{Q}_{34}\tilde{Q}_{43}][\tilde{Q}_{01}\tilde{Q}_{16} + \tilde{Q}_{02}\tilde{Q}_{27}]}{[1-\tilde{Q}_{34}\tilde{Q}_{43}][1-\tilde{Q}_{01}\tilde{Q}_{10}] - \tilde{Q}_{35}\tilde{Q}_{23}\tilde{Q}_{02}\tilde{Q}_{50}}$$

Here the argument s is omitted for brevity.Further, we can have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \quad (4)$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (4). The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \to 0} \frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad (5)$$

Where

$N_1 = [1 - p_{34}p_{43}][\mu_0 + p_{01}\mu_1 + p_{02}\mu_2] + p_{02}p_{23}[\mu_3 + p_{34}\mu_4 + p_{35}\mu_5]$

$D_1 = [1 - p_{34}p_{43}][1 - p_{01}p_{10}] - p_{35}p_{23}p_{02}p_{50}$

5. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state S_i at t=0. Let $M_i(t)$ be the probability that system is up initially in state $S_i \in E$ is up at time t without visiting any other regenerative state, then we have

$$M_0(t) = \bar{Z}_1(t)\bar{S}_2(t), \qquad M_1(t) = \bar{G}_1(t)\bar{Z}_2(t),$$

$$M_2(t) = \bar{Z}_1(t)\bar{G}_2(t), \ M_3(t) = \bar{Z}_1(t)\bar{S}_{d2}(t),$$

$$M_4(t) = \bar{G}_1(t)\bar{Z}_{d2}(t), \ M_5(t) = \bar{Z}_1(t)\bar{F}_{d2}(t)$$

The recursive relations for $A_i(t)$ are as obtained as follows:

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12.6}(t)[c]A_2(t)$$

$$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{27}(t)[c]A_7(t)$$

$$A_3(t) = M_3(t) + q_{34}(t)[c]A_4(t) + q_{35}(t)[c]A_5(t)$$

$$A_4(t) = M_4(t) + q_{43}(t)[c]A_3(t) + q_{45.8}(t)[c]A_5(t)$$

$$A_5(t) = M_5(t) + q_{50}(t)[c]A_0(t) + q_{59}(t)[c]A_9(t)$$

$$A_7(t) = q_{72}(t)[c]A_2(t)$$

$$A_9(t) = q_{95}(t)[c]A_5(t) \quad (6)$$

Taking LST of above relation (6) and solving for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{[1-q_{27}^*q_{72}^*][1-q_{34}^*q_{43}^*][1-q_{59}^*q_{95}^*][M_0^* + q_{01}^*M_1^*] + [q_{01}^*q_{12.6}^* + q_{02}^*][1-q_{59}^*q_{95}^*][\{1-q_{34}^*q_{43}^*\}M_2^* + q_{23}^*\{M_3^* + q_{34}^*M_4^*\}] + q_{23}^*[q_{01}^*q_{12.6}^* + q_{02}^*][q_{34}^*q_{45.8}^* + q_{35}^*]M_5^*}{-q_{23}^*q_{50}^*[q_{34}^*q_{45.8}^* + q_{35}^*][q_{01}^*q_{12.6}^* + q_{02}^*] + [1-q_{27}^*q_{72}^*][1-q_{34}^*q_{43}^*][1-q_{59}^*q_{95}^*][1-q_{01}^*q_{10}^*]}$$

Then the steady state availability of the system is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \quad (7)$$

Where

$$N_2 = p_{23}p_{50}[1-p_{34}p_{43}][\mu_0 + p_{01}\mu_1] + [1-p_{01}p_{10}][1-p_{34}p_{43}][p_{50}\mu_2 + p_{23}\mu_5] + p_{23}p_{50}[1-p_{01}p_{10}]\{\mu_3 + p_{34}\mu_4\}$$

$$D_2 = p_{23}p_{50}[1-p_{34}p_{43}][\mu_0 + p_{01}\mu_1^{'}]+[1-p_{01}p_{10}][1-p_{34}p_{43}][p_{50}\{\mu_2 + p_{27}\mu_7\} + p_{23}\{\mu_5 + p_{59}\mu_9\}]+p_{23}p_{50}$$
$$[1-p_{01}p_{10}]\{\mu_3 + p_{34}\mu_4^{'}\}$$

6. SPECIAL CASE (WEIBULL DISTRIBUTION)

Let us assume that all the random variables follows Weibull distribution as follows: $z_1(t) = \lambda_1 \eta t^{\eta-1}\exp(-\lambda_1 t^{\eta})$, $z_2(t) = \lambda_2 \eta t^{\eta-1}\exp(-\lambda_2 t^{\eta})$, $z_{d2}(t) = \lambda_2^d \eta t^{\eta-1}\exp(-\lambda_2^d t^{\eta})$, $s_2(t) = \mu_2^c \eta t^{\eta-1}\exp(-\mu_2^c t^{\eta})$, $s_{d2}(t) = \mu_2^d \eta t^{\eta-1}\exp(-\mu_2^d t^{\eta})$, $g_1(t) = \beta_1 \eta t^{\eta-1}\exp(-\beta_1 t^{\eta})$, $g_2(t) = \beta_2 \eta t^{\eta-1}\exp(-\beta_2 t^{\eta})$, $f_{d2}(t) = \gamma_2^d \eta t^{\eta-1}\exp(-\gamma_2^d t^{\eta})$ *where* $t \geq 0$ and $\eta, \lambda_1, \lambda_2, \lambda_2^d, \mu_2^c, \mu_2^d, \beta_1, \beta_2, \gamma_2^d > 0$ respectively.Further, let us assume the following values for different parameters: Failure rate of original unit (λ_1) = 0.01 to 0.09 per unit time, Failure rate of duplicate unit (λ_2) = 0.15 per unit time,Failure rate of degraded unit (λ_2^d) = 0.21 per unit time,Maximum redundancy time of duplicate unit (μ_2^c) = 0.8 per unit time,Maximum redundancy time of degraded unit (μ_2^d) = 1.0 per unit time,Repair rate of original unit (β_1) = 2.1 per unit time, Repair rate of duplicate unit (β_2) = 2.6 per unit time, Replacement rate of degraded unit (γ_2^d) = 2.0 per unit time,

Now we consider the following cases:

Case-1: When shape parameter η=0.5

In this case the probability densities of Weibull distribution becomeasfollows:

$$z_1(t) = \frac{\lambda_1}{2\sqrt{t}}\exp(-\lambda_1\sqrt{t}), \quad z_2(t) = \frac{\lambda_2}{2\sqrt{t}}\exp(-\lambda_2\sqrt{t}), \quad z_{d2}(t) = \frac{\lambda_2^d}{2\sqrt{t}}\exp(-\lambda_2^d\sqrt{t}), \quad s_2(t) = \frac{\mu_2^c}{2\sqrt{t}}\exp(-\mu_2^c\sqrt{t}),$$

$$s_{d2}(t) = \frac{\mu_2^d}{2\sqrt{t}}\exp(-\mu_2^d\sqrt{t}), \quad g_1(t) = \frac{\beta_1}{2\sqrt{t}}\exp(-\beta_1\sqrt{t}), \quad g_2(t) = \frac{\beta_2}{2\sqrt{t}}\exp(-\beta_2\sqrt{t}) \quad f_{d2}(t) = \frac{\gamma_2^d}{2\sqrt{t}}\exp(-\gamma_2^d\sqrt{t})$$

where $t \geq 0$ and $\eta, \lambda_1, \lambda_2, \lambda_2^d, \mu_2^c, \mu_2^d, \beta_1, \beta_2, \gamma_2^d > 0$ respectively.

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Table 1: Effect of various parameters on MTSF, Availability and Profit with respect to λ_1

	λ_1	Parameters							
			λ_2	λ_2^d	μ_2^c	μ_2^d	β_1	β_2	γ_2^d
	0.01	561.75	524.83	525.28	389.40	470.04	578.89	599.65	609.25
	0.02	279.42	260.92	261.26	193.98	234.04	287.83	298.21	302.85
	0.03	185.34	172.98	173.28	128.86	155.39	190.84	197.77	200.76
	0.04	138.32	129.03	129.31	96.31	116.08	142.37	147.56	149.73
	0.01	0.99919	0.99913	0.99913	0.99883	0.99903	0.99949	0.99924	0.99925
	0.02	0.99837	0.99825	0.99825	0.99765	0.99805	0.99896	0.99847	0.99849
	0.03	0.99753	0.99735	0.99735	0.99646	0.99705	0.99843	0.99768	0.99772
	0.04	0.99668	0.99644	0.99644	0.99525	0.99603	0.99789	0.99688	0.99693

Case-2: When shape parameter η=1

In this case the Weibull distribution reduces to Exponential distribution and we get the following:

$z_1(t) = \lambda_1 \exp(-\lambda_1 t)$, $z_2(t) = \lambda_2 \exp(-\lambda_2 t)$, $z_{d2}(t) = \lambda_2^d \exp(-\lambda_2^d t)$, $s_2(t) = \mu_2^c \exp(-\mu_2^c t)$,

$s_{d2}(t) = \mu_2^d \exp(-\mu_2^d t)$, $g_1(t) = \beta_1 \exp(-\beta_1 t)$, $g_2(t) = \beta_2 \exp(-\beta_2 t)$, $f_{d2}(t) = \gamma_2^d \exp(-\gamma_2^d t)$

where $t \geq 0$ and $\eta, \lambda_1, \lambda_2, \lambda_2^d, \mu_2^c, \mu_2^d, \beta_1, \beta_2, \gamma_2^d > 0$ respectively.

Table 2: Effect of various parameters on MTSF, Availability and Profit with respect to λ_1

	λ_1	Parameters							
			λ_2	λ_2^d	μ_2^c	μ_2^d	β_1	β_2	γ_2^d
	0.01	300.51	280.76	280.99	262.09	276.43	309.66	318.16	325.21
	0.02	151.16	141.15	141.32	131.75	139.05	155.69	160.05	163.52
	0.03	101.38	94.62	94.77	88.30	93.26	104.37	107.34	109.63
	0.04	76.49	71.35	71.49	66.58	70.37	78.71	80.99	82.68

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	0.01	0.99841	0.99830	0.99830	0.99818	0.99827	0.99876	0.99850	0.99853
	0.02	0.99684	0.99662	0.99662	0.99638	0.99656	0.99752	0.99701	0.99708
	0.03	0.99528	0.99495	0.99495	0.99459	0.99487	0.99629	0.99554	0.99564
	0.04	0.99373	0.99329	0.99329	0.99282	0.99319	0.99507	0.99408	0.99421

Case-3: When shape parameter η=2

In this case the Weibull distribution reduces to Rayleigh distribution and we get the following:

$z_1(t) = 2\lambda_1 t\exp(-\lambda_1 t^2)$, $z_2(t) = 2\lambda_2 t\exp(-\lambda_2 t^2)$, $z_{d2}(t) = 2\lambda_2^d t\exp(-\lambda_2^d t^2)$, $s_2(t) = 2\mu_2^c t\exp(-\mu_2^c t^2)$,

$s_{d2}(t) = 2\mu_2^d t\exp(-\mu_2^d t^2)$, $g_1(t) = 2\beta_1 t\exp(-\beta_1 t^2)$, $g_2(t) = 2\beta_2 t\exp(-\beta_2 t^2)$, $f_{d2}(t) = 2\gamma_2^d t\exp(-\gamma_2^d t^2)$,

where $t \geq 0$ and $\eta, \lambda_1, \lambda_2, \lambda_2^d, \mu_2^c, \mu_2^d, \beta_1, \beta_2, \gamma_2^d > 0$ respectively.

Table 3: Effect of various parameters on MTSF, Availability and Profit with respect to λ_1

	λ_1	Parameters							
			λ_2	λ_2^d	μ_2^c	μ_2^d	β_1	β_2	γ_2^d
	0.01	332.33	310.50	310.75	318.29	323.71	342.51	353.88	363.97
	0.02	167.99	156.87	157.06	160.64	163.50	173.08	178.89	183.89
	0.03	113.21	105.67	105.83	108.09	110.10	116.61	120.57	123.87
	0.04	85.82	80.07	80.22	81.82	83.40	88.37	91.41	93.86
	0.01	0.99792	0.99778	0.99778	0.99783	0.99787	0.99819	0.99805	0.99810
	0.02	0.99589	0.99560	0.99560	0.99570	0.99577	0.99641	0.99614	0.99624
	0.03	0.99389	0.99346	0.99347	0.99361	0.99372	0.99467	0.99426	0.99442
	0.04	0.99194	0.99137	0.99138	0.99155	0.99170	0.99295	0.99242	0.99264

7. DISCUSSION AND CONCLUSION

The numerical effect of change in values of different parameters on the reliability and availability is shown in Tables 1-3. Table 1 gives thevalues of MTSF and availability when the shape parameter of Weibull distribution η=0.5. Whereas Table 2 and Table 3 gives the values of MTSF and availability,w.r.t. the failure rate of the original unit, when the shape parameter of Weibull distribution as η=1 and η=2 respectively. All the three cases exhibit similar results. Both the performance measures declines with increasing values of λ_1, λ_2, λ_2^d, μ_2^c and μ_2^d but the trend declines with increasing values of β_1, β_2 and γ_2^d .Therefore the numerical results reveal thatunder the stated framework a cold standby system can be made more reliable and available for use by keeping:

a. the failure rate of both units under control b. maximum redundancy times lower

c. repair times of both the units higher and d. faster replacement rate of failed unit.

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