

Study of Centrality Dependence of Transverse Momentum Spectra of Pions and the Freeze-Out Parameters at $\sqrt{s_{NN}}=7.7, 9.2, 62.4, 130, 200, 2760$ GeV: A USTFM Approach

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ABSTRACT

The mid-rapidity transverse momentum spectra of charged pions (π^+ , π^-) produced at different energies have been studied using a Unified Statistical Thermal Freeze-out Model (USTFM). We have also studied the effect of different collision centralities on the mid-rapidity production of pions. The calculated results are found to be in good agreement with experimental data. The theoretical fits of transverse momentum spectra using the model calculations provide the thermal freeze-out conditions in terms of the temperature and collective flow parameters for charged pion species. The system reveals the presence of significant collective flow and a well defined temperature in the system thus indicating the formation of thermally equilibrated hydrodynamic system at various energies. The model incorporates longitudinal as well as a transverse hydrodynamic flow. The contributions from heavier decay resonances have also been taken into account. We have also imposed the criteria of exact strangeness conservation in the system.

keywords: Freeze-out parameters, Hydrodynamic flow, transverse momentum distribution, exact strangeness conservation.

1. INTRODUCTION:

The yields of charged pions are an important indicator of the multi-particle production phenomenon in the ultra-relativistic nucleus-nucleus collisions. The study of ultra-relativistic nuclear collisions in general allows us to learn how baryon numbers initially

carried by the nucleons only, before the nuclear collision, are distributed in the final state [1] at the thermo-chemical freeze-out after the collision. The variation of the net proton yield (i.e. $p - \bar{p}$) with rapidity in such experiments can throw light on the collision scenario. At RHIC the mid rapidity net proton yield decreases gradually with increasing energy. Thus at RHIC energies the nuclear collisions start exhibiting some transparency [2-3]

As the mean free paths for different hadrons, due to expansion increases, the process of decoupling of the hadrons from the rest of the system takes place and the hadron spectra are frozen in time. The hadrons with smaller cross-sections stop interacting with the surrounding matter earlier and hence decouple earlier. Hence a so called sequential thermal kinetic freeze-out of different hadronic species occurs. Following this, the hadrons freely stream out to the detectors. The freeze-out conditions of given hadronic specie are thus directly reflected in its transverse momentum and rapidity spectra [4]. It is believed that the produced hadrons also carry information about the collision dynamics and the subsequent space-time evolution of the system. Hence precise measurements of the transverse momentum distributions of identified hadrons along with the rapidity spectra are essential for the understanding of the dynamics and properties of the created matter up to the final freeze-out [5]. The transverse momentum distributions are believed to be encoded with the information about the collective transverse and longitudinal expansions and the thermal temperature at freeze-out. The particle production in p-p collisions are very important as these can serve as a baseline for understanding the particle production mechanism and extraction of the signals of QGP formation in heavy ion collisions [6]. The value of chemical potential is always lower in p-p collisions than in heavy ion collisions due to the lower stopping power in p-p collisions. As one goes to LHC energies, the stopping reduces much further giving rise to nearly zero net baryon density at mid-rapidity and thus the value of the chemical potential at mid-rapidity essentially reduces to zero. Thus at LHC, we believe the p-p collisions to be completely transparent.

II.THE MODEL:

The hot and dense matter produced in relativistic heavy ion collisions may evolve through the following scenario: Pre-equilibrium, thermal (or chemical) equilibrium of partons, possible formation of QGP or a QGP-hadron gas mixed state, a gas of hot interacting hadrons, a chemical freeze-out state when the produced hadrons no longer strongly interact with each other inelastically so that the particle number changing processes stop (except the resonance decay processes). After this the particles may continue to interact through elastic

processes where a certain fraction of the thermal energy gets converted into a collective hydrodynamic flow thereby leading to a decrease in the local thermal temperatures. This would then lead to reshaping of the spectra of hadrons till a final stage where a hydrodynamic freeze-out would occur [7]. In our model it is assumed that a (near) simultaneous freeze-out of all hadronic species occurs. In order to obtain the particle spectra in the overall rest frame of the hadronic fireball we first define the invariant cross-section for a given hadron in the local rest frame of the expanding hadronic fluid element. As the invariant cross-sections will have same shape in all Lorentz frames hence for a given hadronic specie we can write:

$$E d^3n/d^3p = E' d^3n/d^3p' \quad (1)$$

The primed quantities on the RHS refer to the invariant spectra of a given pionic species, in the local hadronic fluid element while the unprimed quantities on the LHS refer to the invariant spectra of the same hadronic specie but in the overall rest frame of the hadronic fireball, formed in the ultra-relativistic nuclear collisions. Further we can write :

$$E' = m_T \cosh(y') \quad (2)$$

Where $m_T = \sqrt{p_T^2 + m^2}$ is the transverse mass and the y' are the transverse momentum and rapidity of the given hadron with mass m in the rest frame of local hadronic fluid element.

We now use the quantum distribution function to write:

$$\frac{d^3n}{d^3p'} \sim \frac{1}{e^{\frac{(E' - \mu)}{T}} \pm 1}$$

We can replace the primed quantities on the RHS in Eqn. (1) by the unprimed quantities by using the Lorentz transformations $p_T' = \gamma_T(p_T - \beta_T E)$ and $y' = y - y_0$, where y_0 is the rapidity of the local hadronic fluid element in the overall rest frame of the hadronic fireball. We have assumed that the longitudinal expansion of the hadronic fluid follows the simple behaviour which allows us to use $y_0 = cz$, where z represents the z coordinate of the fluid element along the collision axis in the rest frame of the hadronic fireball with $c=1$. Thus writing $y_0 = z$, we obtain the following expression for the longitudinal velocity component of the hadronic fluid element:

$$\beta_z(z) = 1 - 2/\exp(2z) + 1 \quad (3)$$

In works [8, 9] it has been clearly shown that there is a strong evidence of increasing baryon chemical potential along the collision axis. It is therefore possible to reproduce the variation of the antibaryon to baryon ratio with rapidity in accordance with the RHIC data

only when we incorporate this effect [7]. Hence in the present model we incorporate this effect by writing:

$$\mu = a + by_0^2 \quad (4)$$

Where y_0 is the rapidity of the hadronic fluid element along the beam axis. Further as in many earlier works it is assumed here also that the transverse (or radial) velocity component of the hadronic fluid varies with the transverse coordinate r as [7] :

$$\beta_T(r) = \beta_T^s(r/R)^n \quad (5)$$

where R is the transverse radius of the fireball and n is an index to fix the profile of $\beta_T(r)$ in the radial direction. In order to reproduce the correct observed rapidity distributions of hadrons it is necessary in the present model to assume that the transverse radius of the fireball decreases with the z coordinate of the system as :

$$R = r_0 \exp(-|z|)^2 / \sigma^2 \quad (6)$$

Consequently at the freeze-out we don't have a spherically symmetric system but rather a system which with varying transverse radius follows a Gaussian profile along the rapidity (or beam) axis. The other parameter viz. the fluid's surface transverse expansion velocity β_T^s is fixed in the model by using the parameterization :

$$\beta_T^s = \beta_T^0 [1 - \beta_z^2(z)] \quad (7)$$

The above relation (or restriction) is also required to ensure that the net particle velocity β must satisfy:

$$\beta = \sqrt{(\beta_T^2 + \beta_z^2)} < 1 \quad (8)$$

We finally perform an integral over the physical volume of the system to obtain the net pionic yield (π^+, π^-).

The effects of resonance decays are also taken into account in this analysis. The spectrum of a given decay product of a given hadron in the rest frame of the fireball can be written as:

$$\frac{d^3 n^{decay}}{d^3 p} = \frac{1}{2pE} \frac{m_h}{p^*} \int_{E_-}^{E_+} dE_h E_h \left(\frac{d^3 n_h}{d^3 p_h} \right)$$

Where the subscript h stands for the decaying (parent) hadron. According to the two body decay kinematics, the momentum and energy of the produced hadron in the rest frame of decaying hadron are given by the expressions

$$p^* = (E^* - m^2), \quad E^* = \frac{m_h^2 - m_j^2 + m^2}{2m_h}$$

Using Boltzmann distribution for a massive decaying hadron in the local rest frame of the hadronic fluid element leads to the following final expression for the invariant cross section of the produced hadron:

$$\begin{aligned} \frac{E' d^3 N}{d^3 p'} &= \frac{1}{2p'} \left\{ \frac{m_h}{p^*} \right\} g_h \exp(-\alpha \theta E' E^*) \\ &\times \left\{ \frac{\alpha}{\theta} [E' E^* \sinh(\alpha \theta p' p^*) - p' p^* \cosh(\alpha \theta p' p^*)] \right. \\ &\quad \left. + T^2 \sinh(\alpha \theta p' p^*) \right\} \end{aligned}$$

Where

$$\alpha = \frac{m_h}{m^2}$$

and

$$\theta = \frac{1}{T}$$

In the present work we have incorporated both the single as well as double decay contributions to the invariant cross section. The present analysis contributes to the theoretical understanding of heavy decay contributions and their importance to fit the experimental data using our Unified Statistical Freeze-out Model (USTFM).

Results: In figure 1 we have shown the transverse momentum spectra of π^+ and π^- theoretical curves fits well with experimental data points obtained from the most central Au + Au collisions at $\sqrt{s_{NN}} = 7.7 \text{ GeV}$ at RHIC. The parameter values used to obtain these curves are same. We have used, $a = 24.5 \text{ MeV}$, $b = 11.0 \text{ MeV}$, $c = 1 \text{ fm}^{-1}$.

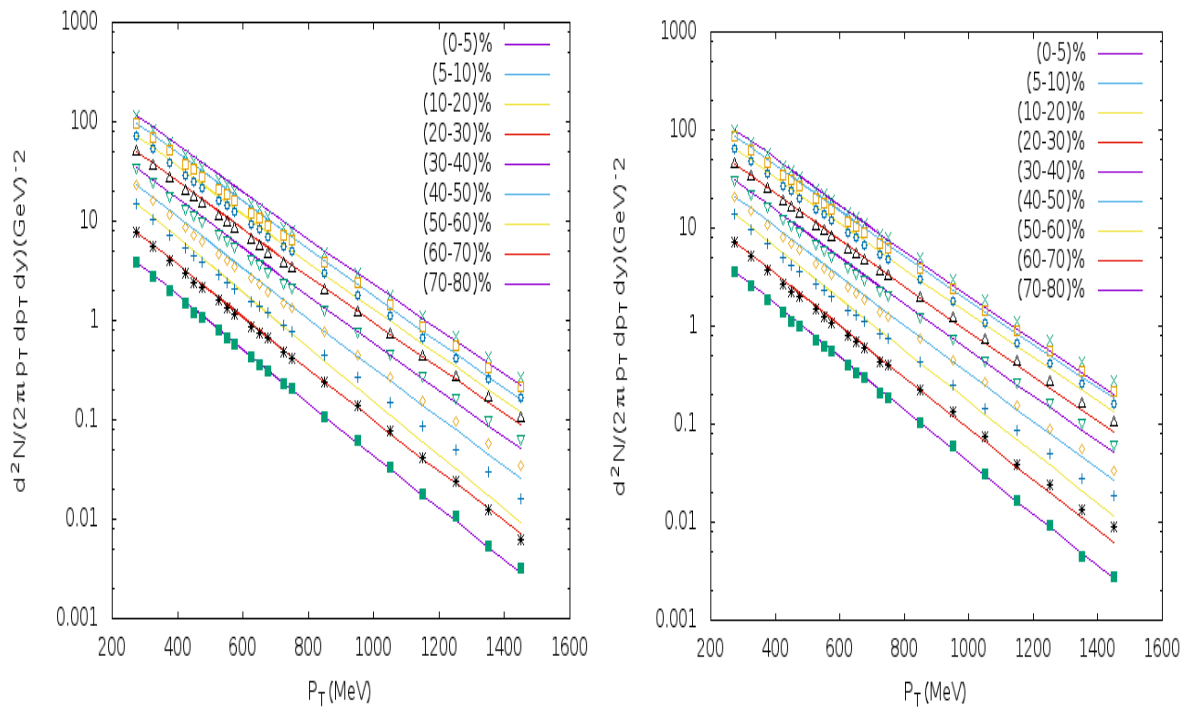


Fig1: Transverse momentum spectrum of π^- and π^+ for various centralities at $\sqrt{s_{NN}} = 7.7$ GeV

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(0-5)	0.83	64.0	1.1 ± 0.1	1.54
(5-10)	0.83	62.0	1.1 ± 0.2	2.09
(10-20)	0.82	62.0	1.0 ± 0.1	2.04
(20-30)	0.82	63.0	1.0 ± 0.2	1.06
(30-40)	0.81	61.0	1.1 ± 0.0	0.84
(40-50)	0.81	58.0	1.1 ± 0.1	1.48
(50-60)	0.78	60.0	1.0 ± 0.2	0.47
(60-70)	0.83	56.0	1.0 ± 0.1	0.37
(70-80)	0.83	54.0	1.2 ± 0.1	0.31

Table1: Freeze-out parameters for π^- for various centralities at $\sqrt{s_{NN}} = 7.7$ GeV

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(0-5)	0.83	64.0	1.0 ± 0.2	1.20
(5-10)	0.83	65.0	1.1 ± 0.1	0.89

(10-20)	0.83	65.0	1.0 ± 0.2	0.98
(20-30)	0.82	65.0	1.0 ± 0.1	1.07
(30-40)	0.82	64.0	1.0 ± 0.0	1.20
(40-50)	0.80	64.0	1.0 ± 0.1	1.58
(50-60)	0.78	63.0	1.0 ± 0.2	2.07
(60-70)	0.78	63.0	1.0 ± 0.1	0.99
(70-80)	0.77	63.0	1.0 ± 0.2	0.29

Table2: Freeze-out parameters for π^+ for various centralities at $\sqrt{s_{NN}}=7.7$ GeV

In figure 2 we have shown the transverse momentum spectra of π^+ and π^- theoretical curves fits well with experimental data points obtained from the most central Au + Au collisions at $\sqrt{s_{NN}}=9.2$ GeV at RHIC. The parameter values used to obtain these curves are $a = 23.0$ MeV, $b = 10.4$ MeV, $c = 1$ fm⁻¹

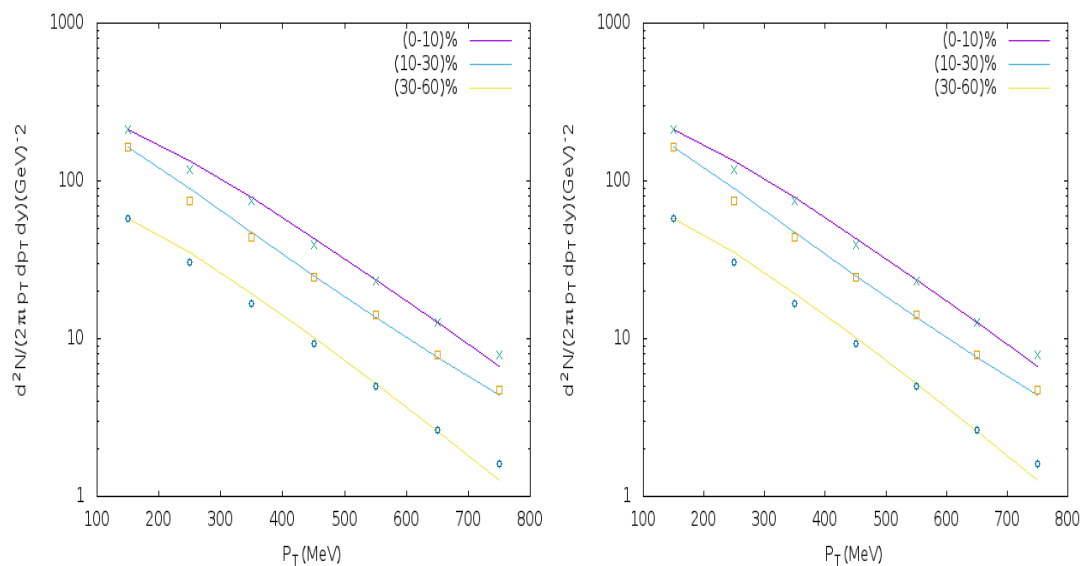


Fig 2: Transverse momentum spectrum of π^- and π^+ for various centralities at $\sqrt{s_{NN}} = 9.2$ GeV

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(0-10)	0.82	59.36	1.1 ± 0.2	0.73
(10-30)	0.80	60.20	1.0 ± 0.1	1.22
(30-60)	0.78	60.70	1.2 ± 0.1	1.38

Table3: Freeze-out parameters for π^- for various centralities at $\sqrt{s_{NN}}=9.2\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(0-10)	0.81	59.40	1.0 ± 0.0	0.61
(10-30)	0.80	60.10	1.1 ± 0.1	1.09
(30-60)	0.78	60.6	1.2 ± 0.1	1.35

Table4: Freeze-out parameters for π^+ for various centralities at $\sqrt{s_{NN}}=9.2\text{GeV}$

In figure 3 we have shown the transverse momentum spectra of π^+ and π^- theoretical curves fits with experimental data points obtained from the most central Au + Au collisions at $\sqrt{s_{NN}}=62.4\text{GeV}$ at RHIC. The parameter values used to obtain these curves are same. We have used , $a = 23.0\text{MeV}$, $b = 10.4\text{ MeV}$, $c = 1\text{ fm}^{-1}$

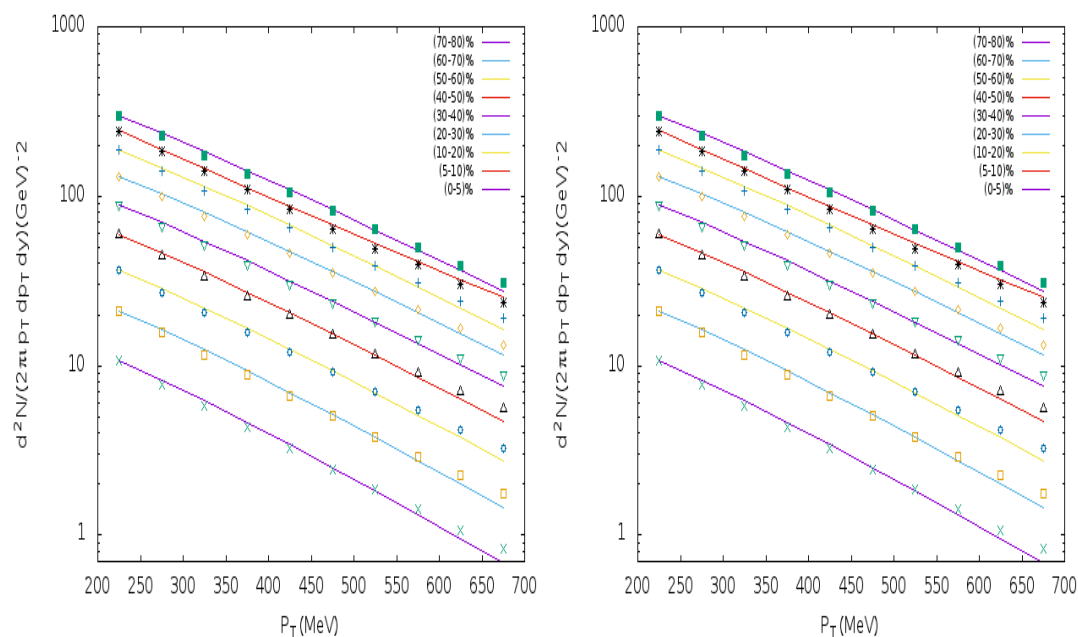


Fig3: Tran

verse momentum spectrum of π^- and π^+ for various centralities at $\sqrt{s_{NN}}=62.4\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(70-80)	0.76	62.1	1.1 ± 0.1	0.69
(60-70)	0.77	61.8	1.1 ± 0.2	0.47
(50-60)	0.78	61.4	1.0 ± 0.2	0.42

(40-50)	0.78	60.0	1.0±0.1	0.54
(30-40)	0.79	59.9	1.0±0.0	0.50
(20-30)	0.80	59.7	1.0±0.1	0.37
(10-20)	0.81	59.6	1.0±0.2	0.49
(5-10)	0.82	59.2	1.0±0.1	0.28
(0-5)	0.82	59.0	1.0±0.1	0.27

Table5: Freeze-out parameters for π^- for various centralities at $\sqrt{s_{NN}}=62.4\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(70-80)	0.75	62.4	1.1±0.1	0.73
(60-70)	0.77	62.0	1.1±0.2	0.62
(50-60)	0.78	61.8	1.0±0.1	0.67
(40-50)	0.79	61.4	1.0±0.2	0.76
(30-40)	0.80	60.7	1.1±0.0	0.56
(20-30)	0.80	60.4	1.1±0.1	0.54
(10-20)	0.81	60.1	1.0±0.2	0.50
(5-10)	0.82	59.8	1.0±0.1	0.28
(0-5)	0.83	59.4	1.2±0.1	0.38

Table6: Freeze-out parameters for π^+ for various centralities at $\sqrt{s_{NN}}=62.4\text{GeV}$

In figure 4 we have shown the transverse momentum spectra of π^+ and π^- theoretical curves fits with experimental data points obtained from the most central Au + Au collisions at $\sqrt{s_{NN}}=130.0\text{GeV}$ at RHIC. The parameter values used to obtain these curves are same. We have used $a = 22.5\text{ MeV}$, $b = 9.6\text{ MeV}$, $c = 1\text{ fm}^{-1}$.

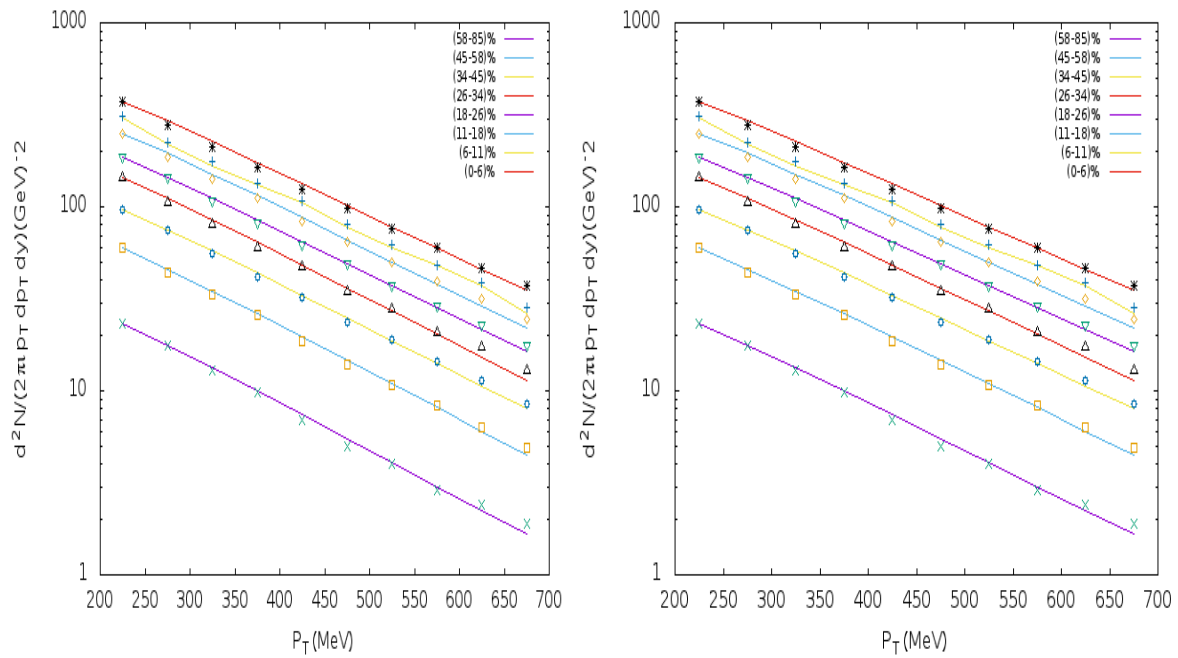


Fig4: Transverse momentum spectrum of π^- and π^+ for various centralities at $\sqrt{s_{NN}}=130.0\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(58-85)	0.81	61.4	1.0 ± 0.1	0.25
(45-58)	0.79	60.0	1.0 ± 0.2	0.28
(34-45)	0.81	60.6	1.1 ± 0.0	0.27
(26-34)	0.80	59.4	1.0 ± 0.0	0.43
(18-26)	0.79	58.8	1.2 ± 0.1	0.38
(11-18)	0.77	59.1	1.0 ± 0.1	0.80
(6-11)	0.81	61.1	1.2 ± 0.0	0.49
(0-6)	0.78	61.4	1.1 ± 0.1	1.36

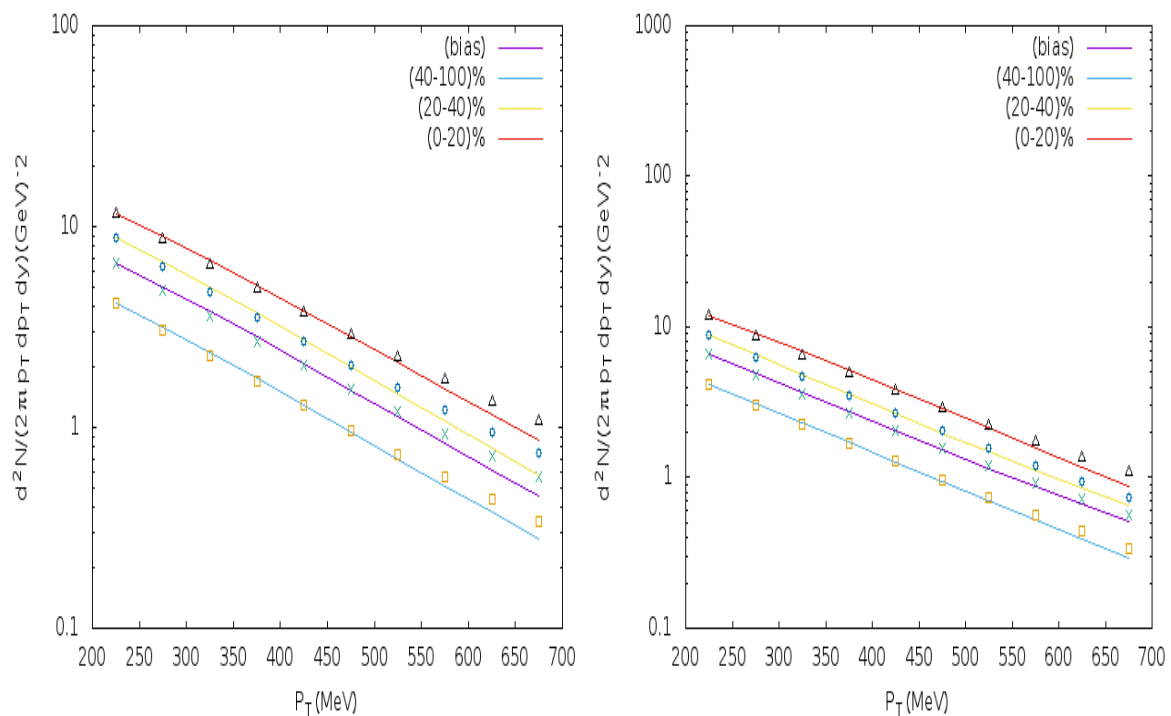
Table7: Freeze-out parameters for π^- for various centralities at $\sqrt{s_{NN}}=130.0\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(58-85)	0.80	60.3	1 ± 0.0	0.30
(45-58)	0.77	59.8	1 ± 0.1	0.48

(34-45)	0.79	59.6	1 ± 0.1	0.27
(26-34)	0.81	61.0	1 ± 0.0	0.34
(18-26)	0.78	61.2	1 ± 0.1	0.60
(11-18)	0.81	58.8	1 ± 0.2	0.59
(6-11)	0.80	59.9	1 ± 0.1	0.45
(0-6)	0.79	58.2	1 ± 0.0	0.66

Table8: Freeze-out parameters for π^+ for various centralities at $\sqrt{s_{NN}}=130.0\text{GeV}$

In figure 5 we have shown the transverse momentum spectra of π^+ and π^- theoretical curves fits with experimental data points obtained from the most central d+ Au collisions at $\sqrt{s_{NN}}=200.0\text{GeV}$ at RHIC. The parameter values used to obtain these curves are same. We have use $a = 21.5\text{ MeV}$, $b = 8.0\text{ MeV}$, $c = 1\text{ fm}^{-1}$.

Fig5: Transverse momentum spectrum of π^- and π^+ for various centralities at $\sqrt{s_{NN}}=200.0\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(bias)	0.81	61.4	1.0 ± 0.2	0.58
(40-100)	0.80	60.2	1.0 ± 0.0	0.45

(20-40)	0.79	62.0	1.0 ± 0.0	0.76
(0-20)	0.77	59.9	1.0 ± 0.1	0.43

Table9: Freeze-out parameters for π^- for various centralities at $\sqrt{s_{NN}}=200.0\text{GeV}$

Centrality%	β_T^0	T(MeV)	n	χ^2/dof
(bias)	0.79	61.2	1 ± 0.1	0.55
(40-100)	0.81	60.0	1 ± 0.1	0.42
(20-40)	0.80	59.8	1 ± 0.0	0.44
(0-20)	0.78	59.6	1 ± 0.1	0.45

Table10: Freeze-out parameters for π^+ for various centralities at $\sqrt{s_{NN}}=200.0\text{GeV}$

and In figure 6 we have shown the transverse momentum spectra of π^+ and π^- theoretical curves fits with experimental data points obtained from the most central

Pb + Pb collisions at $\sqrt{s_{NN}}=2760.0\text{GeV}$. The parameter values used to obtain these curves are same. We have use $a = 18.0\text{ MeV}$, $b = 7.5\text{ MeV}$, $c = 1\text{ fm}^{-1}$.

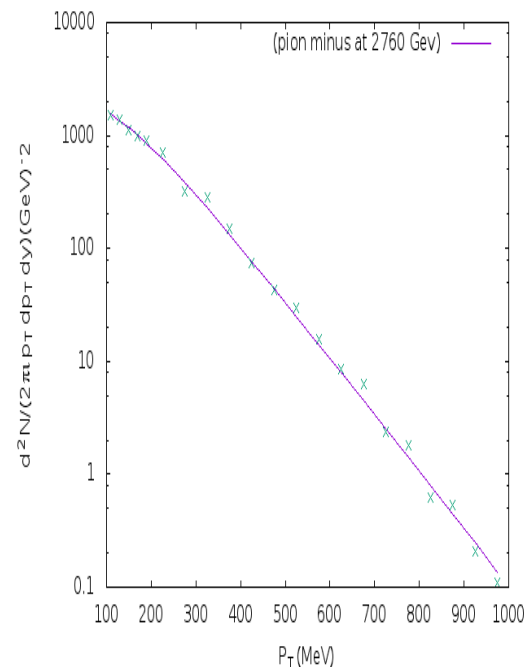
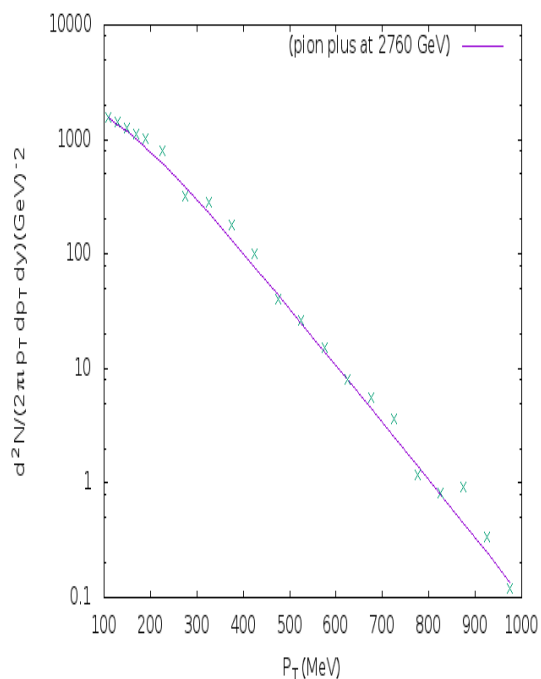


Fig 6: Transverse momentum spectrum of π^- and π^+ at $\sqrt{s_{NN}}=2760$ GeV

Particle	β_T^0	T(MeV)	n	χ^2/dof
π^-	0.74	60.2	1.1 ± 0.2	1.54
π^+	0.75	61.0	1.0 ± 0.1	1.87

Table11: Freeze-out parameters for π^- and π^+ for various centralities at $\sqrt{s_{NN}}=200.0$ GeV

Summary and conclusion: We have suitably modified the existing thermal model to describe the transverse momentum spectra of charged pions in a single thermal freeze-out model. This is achieved by incorporating a longitudinal as well as a transverse flow. The transverse momentum spectra for π^+ and π^- at $\sqrt{s_{NN}} = 7.7, 9.2, 62.4, 130.0, 200.0, 2760.0$ GeV are fitted quite well by using our Unified Statistical Thermal Freeze-out Model (USTFM), it is found that a better fit is obtained by using our model. We have incorporated the effects of transverse as well as longitudinal hydrodynamic flow in the produced system. A detailed description of our model is available in the references [10-15].

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