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Enhancement of resonance decay instability of Langmuir wave in a dusty plasma

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ABSTRACT

The presence of highly charged dust grains in plasma exhibit charge fluctuations. The effect of these dust charge fluctuations is studied on the resonant decay instability (RDI) of a Langmuir wave into a Langmuir and an ion acoustic wave. The growth rate and threshold pump power (μ_{th}) required for the onset of RDI were evaluated based on existing dusty plasma parameters. The presence of dust charge fluctuations enhances RDI, thus decreasing the μ_{th} required for onset of RDI for a particular value of parameter δ_c (where δ_c is ratio between ion and electron density) for negatively charged dust grains.

Keywords: Dust grains, Langmuir wave, resonance decay instability, growth rate

I. INTRODUCTION

In recent years, research involving the parametric effects of electrostatic [1-5] and electromagnetic waves [6-7] with large amplitude has in focus for its profound applications in fusion devices [8] and laboratory plasmas [9-12]. Electrostatic waves in dusty plasmas characterized by low concentration of suspended dust grains have been exploited prominently in recent times [13-17]. Chow et al. [16-17] modelled electrostatic ion cyclotron (EIC) instability in complex plasma utilizing Vlasov theory for analysis and results were in accordance with Barkan et al. [15].

The dust has been noted to influence a three-wave parametric process in unmagnetized [18-20] and magnetized plasmas [21]. Konar and V.K Jain [22] studied the resonance decay of the Langmuir wave in a plasma cylinder by taking into account damping of decay waves. This paper presents the analysis of resonant decay instability (RDI) of a Langmuir wave in cylindrical plasma with immersed dust grains.

The entire process can be explained as: A high frequency electrostatic pump wave (ω_0, k_0) with large amplitude couples with a plasma mode of low frequency at (ω_s, k_s) to give sideband $(\omega_1 = \omega_s - \omega_0, k_1 = k_s - k_0)$ of high frequency. This sideband now combines with (ω_0, k_0) wave providing a low frequency pondermotive force at (ω_s, k_s) which enhances the original perturbation (ω_s, k_s). Section II gives complete instability analysis using fluid treatment while section III contains results and discussions. Conclusion part is stated in Sec. IV.

II. INSTABILITY ANALYSIS

Consider a cylinder of radius 'r_0' filled with homogeneous dusty plasma that is infinite in Z-direction. In equilibrium, the temperature, densities, charge, and mass of the three species ions, electrons and dust grains in the plasma cylinder are represented by (T_i , n_{i0}, e, m_i) , (T_e, n_{e0}, -e, m_e) and (T_d , n_{d0},-Q_{d0}, m_d) respectively. We assume potential variations of the three waves of the form

$$\phi = \phi(r)\exp\left[-i\left(\omega_s t - l\theta - k_{sz} z\right)\right],$$

$\phi_j = \phi_j(r)\exp\left[-i\left(\omega_j t - l_j\theta - k_{jz} z\right)\right]$, j=0,1, which vanish at the boundary of the plasma cylinder, i.e., $\phi = \phi_j = 0$ at r=r_0. The following condition should be satisfied by decay waves for resonant interaction to take place: $\omega_1 = \omega_s - \omega_0$, $k_{1z} = k_{sz} - k_{0z}$ and $l_1 = l - l_0$.

The mode structure equation for the localized Langmuir pump wave can be written as

$$\frac{\partial^2 \phi_0}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_0}{\partial r} + \left(k_{0\perp}^2 - \frac{l_0^2}{r^2}\right)\phi_0 = 0, \qquad (1)$$

where

$k_{0\perp}^2 = \dfrac{\omega_0^2 - \omega_{pe}^2}{V_{the}^2} - k_{0z}^2$, $\omega_{pe}\left(= \sqrt{\dfrac{4\pi n_{e0} e^2}{m_e}}\right)$ and V_{the} are

electron plasma frequency and electron thermal velocity respectively.

Perturbed densities of electron, ion and dust are given by

$$n_{e1} = \frac{n_{e0} e\left(\phi + \phi_p\right)}{m_e V_{the}^2} \tag{2}$$

$$n_{i1} = \frac{n_{i0} e k_s^2 \phi (1 - i\alpha)}{m_i \omega_s^2}, \tag{3}$$

$$n_{d1} = -\frac{n_{d0} Q_{d0} k_s^2 \phi}{m_d \omega_s^2}. \tag{4}$$

where

$$\alpha = \frac{2\sqrt{\pi}\omega_s^3}{k_s^2 k_{sz} V_{thi}^3} \exp\left[\frac{-\omega_s^2}{k_{sz}^2 V_{thi}^2}\right], \quad \phi_p = \frac{eE_0.E_1}{2m_e \omega_0 \omega_1}$$ and V_{thi} is ion thermal velocity.

We obtain dust charge fluctuation by following Jana *et al*.[14] i.e.,

$$Q_{d1} = \frac{|I_{e0}|}{i(\omega_s + i\eta_c)}\left[\frac{ek_s^2 \phi (1 - i\alpha)}{m_i \omega_s^2} - \frac{e(\phi + \phi_p)}{m_e V_{the}^2}\right] \tag{5}$$

where η_c is the dust charging rate given as

$$\eta_c = 0.79a\left(\frac{\omega_{pi}}{\lambda_{Di}}\right)\left(\frac{1}{\delta_c}\right)\left(\frac{m_i}{m_e}\frac{T_i}{T_e}\right)^{\frac{1}{2}} \sim 10^{-2}\omega_{pe}\left(\frac{a}{\lambda_{De}}\right)\frac{1}{\delta_c},$$ $\delta_c = n_{i0}/n_{e0}$, λ_{Di}, λ_{De} are ion and electron debye length respectively while ‘a’ is dust grain size.

In Eq. (5), we assume that the dust charging time $\left(\eta_c^{-1}\right)$ is nearly equal to wave period $\left(\omega_s^{-1}\right)$.

Substituting perturbed densities in the Poisson’s equation $\nabla^2\phi = 4\pi[n_{e1}e - n_{i1}e + n_{d0}Q_{d1} + Q_{d0}n_{d1}]$, yields

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\left[k_{\perp d}^2-\frac{l^2}{r^2}\right]\phi=\frac{-e\omega_s^2\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)}\right]\nabla\phi_0.\nabla\phi_1}{2m_e\omega_0\omega_1C_s^2\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)\delta_c}\right]}, \tag{6}$$

where

$$k_{\perp d}^2=\frac{\omega_s^2\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)}\right](1+i\alpha)}{C_s^2\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)\delta_c}\right]}-k_{sz}^2, \tag{7}$$

$C_s^2=\frac{T_e}{m_i}$ and $\beta_c=\frac{|I_{e0}|n_{d0}}{en_{e0}}=0.397\left(1-\frac{1}{\delta_c}\right)\left(\frac{a}{v_{te}}\right)\omega_{pi}^2\left(\frac{m_i}{m_e}\right)$ is the coupling parameter.

Similarly for the side band , we get

$$\frac{\partial^2\phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_1}{\partial r}+\left[k_{\perp 1d}^2-\frac{l_1^2}{r^2}\right]\phi_1=$$
$$\frac{e\omega_{pe}^2\omega_1\left[\phi\nabla^2\phi_0^*+\nabla\phi_0^*.\nabla\phi\right]\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)}\right]}{m_e\omega_0V_{the}^2\left[\omega_1^2-\omega_{pe}^2\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)}\right]-\omega_{pd}^2\right]} \tag{8}$$

where

$$k_{\perp 1d}^2=\frac{\left(\omega_1^2-\omega_{pd}^2\right)(1+i\alpha_1)}{V_{the}^2\left[1+\frac{i\beta_c}{(\omega_s+i\eta_c)}\right]}-\frac{\omega_{pe}^2}{V_{the}^2}-k_{1z}^2. \tag{9}$$

where $\alpha_1=\frac{2\sqrt{\pi}\omega_1^3}{k_1^2k_{1z}V_{the}^3}\exp\left[\frac{-\omega_1^2}{k_{1z}^2V_{the}^2}\right].$

When no pump wave is present, Eqs. (6) and (8) gives linear mode structure equations of Ion-acoustic and Langmuir waves respectively which can be written as

$$\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\left[k_{\perp m}^2-\frac{l^2}{r^2}\right]\phi=0 \quad (10)$$

$$\frac{\partial^2 \phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial \phi_1}{\partial r}+\left[k_{\perp 1n}^2-\frac{l_1^2}{r^2}\right]\phi_1=0 \quad (11)$$

Combining Eqs.(6), (8), (10) and (11) using the perturbation technique, we get

$$\left(k_{\perp d}^2-k_{\perp m}^2\right)\left(k_{\perp 1d}^2-k_{\perp 1n}^2\right)=\mu, \quad (12)$$

where $\mu=\dfrac{R\int_0^{r_o}(\nabla\phi_0.\nabla\phi_{1n})\phi_m^* r dr\int_0^{r_o}\left[\phi_m\nabla^2\phi_0^*+\nabla\phi_0^*.\nabla\phi_m\right]\phi_{1n}^* r dr}{\left[\omega_1^2-\omega_{pe}^2\left(1+\dfrac{i\beta_c}{(\omega_s+i\eta_c)}\right)-\omega_{pd}^2\right]}$,

and $R=\dfrac{-e^2\omega_{pe}^2\omega_s^2\left[1+\dfrac{i\beta_c}{(\omega_s+i\eta_c)}\right]^2}{2m_e^2\omega_0^2V_{the}^2C_s^2\left[1+\dfrac{i\beta_c}{(\omega_s+i\eta_c)\delta_c}\right]}$.

Rewriting Eq. (12), we get

$$\varepsilon(\omega_s,k_{sz})\varepsilon_1(\omega_1,k_{1z})=\mu, \quad (13)$$

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where

$$\varepsilon(\omega_s, k_{sz}) = k_{\perp d}^2 - k_{\perp m}^2$$

and

$$\varepsilon_1(\omega_1, k_{1z}) = k_{\perp 1d}^2 - k_{\perp 1n}^2.$$

In resonant decay process, we look for solutions of Eq. (13) when $\varepsilon(\omega_s, k_{sz})$ and $\varepsilon_1(\omega_1, k_{1z})$ are simultaneously equal to zero. In the presence of the pump, we express $\omega_s = \omega_r + i\gamma$ and $\omega_1 = \omega_{1r} + i\gamma$ where $\omega_{1r} = \omega_r - \omega_0$. ω_r is the real frequency of ion acoustic mode and γ the growth rate.

Thus, we get

$$k_{\perp d}^2 = k_{\perp m}^2 + \frac{2\omega_r i\gamma\left[1 + \dfrac{i\beta_c}{\omega_s + i\eta_c}\right]}{C_s^2\left[1 + \dfrac{i\beta_c}{(\omega_s + i\eta_c)\delta_c}\right]},$$

$$k_{1\perp d}^2 = k_{\perp n}^2 + \frac{2\omega_{1r} i\gamma}{V_{the}^2\left[1 + \dfrac{i\beta_c}{\omega_s + i\eta_c}\right]}. \qquad (14)$$

Using Eq.(12) and (14), we obtain

$$(\gamma + \Gamma_L)(\gamma + \Gamma_{L1}) = \frac{\mu V_{the}^2 C_s^2\left[1 + \dfrac{i\beta_c}{(\omega_s + i\eta_c)\delta_c}\right]}{4\omega_r\omega_0}, \qquad (15)$$

where

$\Gamma_L\left(= \frac{-\alpha\omega_r}{2}\right)$ and $\Gamma_{L1}\left(= \frac{-\alpha_1\omega_{1r}}{2}\right)$ are linear damping rates of two decay modes.

Substituting γ =0 in equation (15), we get

$$\mu_{th} = \frac{4\omega_r \omega_0 \Gamma_L \Gamma_{L1}}{V_{the}^2 C_s^2 \left[1 + \frac{i\beta_c}{(\omega_s + i\eta_c)\delta_c}\right]} \tag{16}$$

where μ_{th} gives the threshold value of the pump amplitude for the onset of resonant parametric instability.

For pump amplitude much greater than threshold, we get growth rate by substituting $\Gamma_L = \Gamma_{L1} = 0$ in equation (15) i.e.,

$$\gamma_0^2 = \frac{\mu V_{the}^2 C_s^2 \left[1 + \frac{i\beta_c}{(\omega_s + i\eta_c)\delta_c}\right]}{4\omega_r \omega_0}. \tag{17}$$

Eq. (17) shows that γ_0 is proportional to the amplitude of the pump.

Now we will discuss two cases of interest:

Case I: In the presence of dust charge fluctuations i.e., dust charging rate η is finite.

Case II: In the absence of dust charge fluctuations i.e., $Q_{d_1} = 0$ when dust charging rate $\eta \rightarrow \infty$.

We obtain the dispersion relation of Konar *et al.* [22] (cf. pages 502 and 503), in the absence of dust grains in plasma cylinder by substituting δ_c=1and β_c=0.

III. RESULTS

The resonant decay instability (RDI) of a Langmuir wave in dusty plasma cylinder is studied using the typical dusty plasma parameters n_{i0}=1.24x10^{8}cm^{-3}, $n_{d0} = 2.0 \times 10^4 cm^{-3}$, T_i=0.1eV, $m_i/m_e \approx$ 7.16x10^4 (Potassium), r_0=1.0 cm, a=10^{-4}cm, $\omega_0 = 1.3 \times 10^8 rad/\sec.$, k_{sz}=40.0cm^{-1} and k_{sz}=2k_{z1}. We vary the parameter $\delta_c (= n_{io}/n_{eo})$ from 1 to 5.

In Figs.1 and 2, we depict the variation of normalized growth rate $\left(\gamma_0 / \omega_{pi}\right)$ and μ_{th} with δ_c for two different cases i.e., with and without dust charge fluctuations using Eqs. (17) and (16) respectively for the above mentioned parameters. It can be seen from Fig.1, that $\left(\gamma_0 / \omega_{pi}\right)$ decreases exponentially with δ_c , both in the presence and absence of dust charge fluctuations. However, the decrease is more drastic in absence of dust charge fluctuations. Thus dust charge fluctuations reduce the suppression of RDI. Similarly from Fig. (2), we can conclude that the threshold pump power required for the onset of RDI for a particular value of δ_c is less in the presence of dust charge fluctuations. Both the results are consistent with the experimental observations [16] and theoretical predictions [17].

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Fig. 1: Normalised growth rate $\frac{\gamma_0}{\omega_{pi}}$ of the resonance decay instability as a function of $\delta_c\left(=n_{i0}/n_{e0}\right)$ for the parameters given in text and $\left(\frac{T_e}{T_i}=1.0\right)$ with and without dust charge fluctuations.

Fig. 2: Threshold of the resonance decay instability (μ_{th}) as a function of $\delta_c\left(=n_{i0}/n_{e0}\right)$ for the same parameters as in Fig.1 and $\left(\frac{T_e}{T_i}=1.0\right)$ with and without dust charge fluctuations.

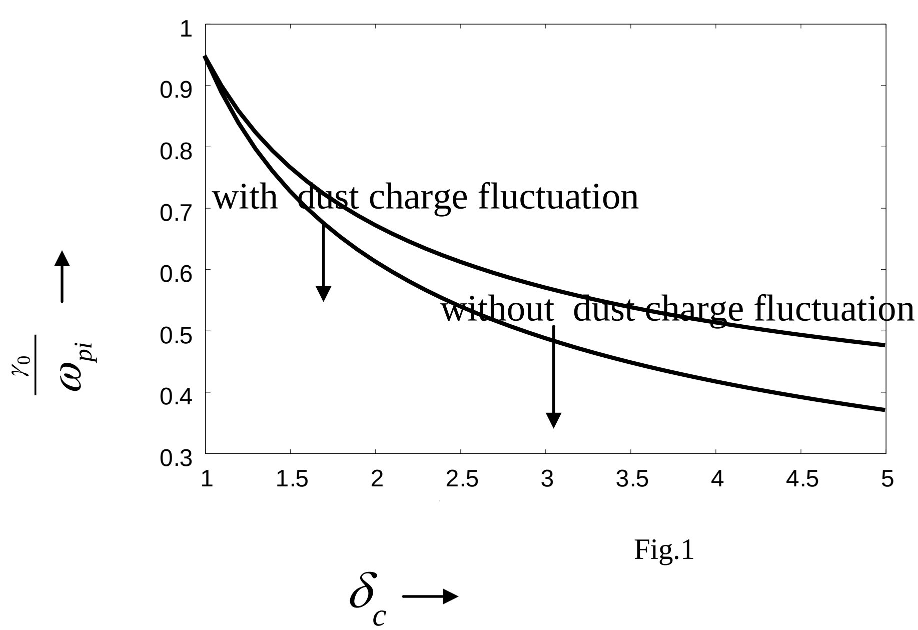

Fig.1

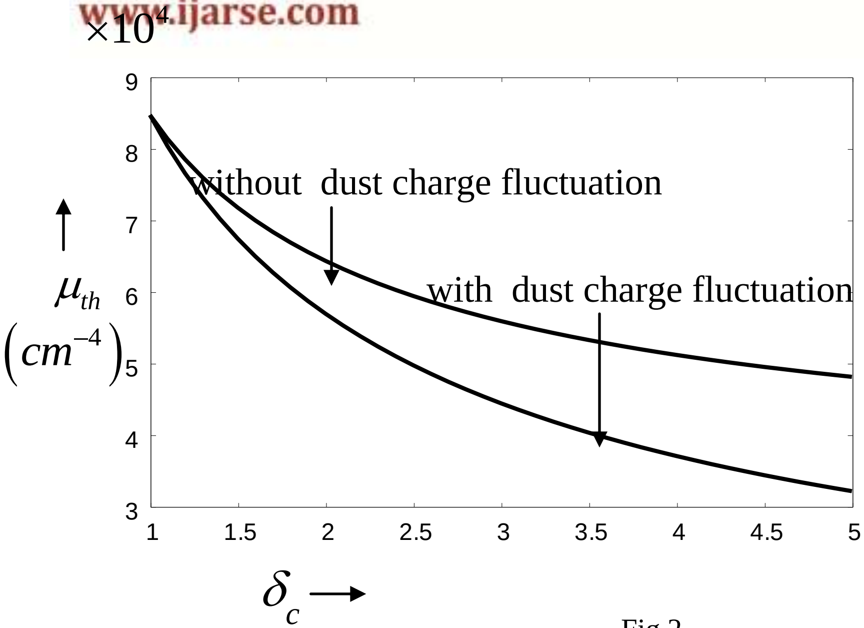
×10^4
9
8
7
6
5
4
3
1
1.5
2
2.5
3
3.5
4
4.5
5
without dust charge fluctuation
with dust charge fluctuation
μ_{th}
(cm^{-4})
δ_c

Fig.2