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Characterizations of Intuitionistic Fuzzy Soft Ideals and Fuzzy Soft Filters Based On Lattice Operations

K Geetha[1], S.Shobana[2], K. Durga[3]

[1]Assistant Professor, Department of Mathematics,
Vivekanandha College of Arts And Sciences For Women [Autonomous],Elayampalayam,
Tiruchengode,Namakkal-637205,Tamilnadu,India.
[2]Research Scholar Department of Mathematics,
Vivekanandha College Of Arts And Sciences For Women [Autonomous] ,
Elayampalayam,Tiruchengode,Namakkal-637205, Tamilnadu,India.
[3]Research Scholar Department of Mathematics,
Vivekanandha College Of Arts And Sciences For Women [Autonomous] ,
Elayampalayam,Tiruchengode,Namakkal-637205, Tamilnadu,India.

ABSTRACT

The theory of intuitionistic fuzzy ideal and intuitionistic fuzzy filter on a lattice was introduced Thomas and Nair.In this paper, we introduce the notions of the intuitionistic fuzzy soft ideal and filter on a lattice, prime intuitionistic fuzzy soft ideal and filter ,interaction of prime intuitionistic fuzzy soft ideal and filter and then we evaluate their some characterizations and basic properties are investigated.

Keywords :Lattice, Intuitionistic Fuzzy Soft Set, Intuitionistic Fuzzy Soft Ideal, Intuitionistic Fuzzy Soft Filter.

I INTRODUCTION

The theory of soft sets was introduced by Molodstov in 1999 in order to deal with uncertainties.Soft set theory has a rich potential for applications in several directions.the concept of the Intuitionistic fuzzy was introduced in 1983 by K.Aanassov as an extension of Zadeh fuzzy set and the theory of lattices was defined by Richard Dedikind. Fuzzy set is to be used in many areas of daily life such as Engineering, Medicine, Meterology In fuzzy soft setting, for the same purposes, several authors introduced and investigated the concepts of some kinds of fuzzy soft ideals and fuzzy soft filters in different ways and on different structures. In fuzzy soft set theory, the non-membership degree of an element x can be viewed as $\nu_{(F,A)}$(x)=1−$\mu_{(F,A)}$(x) (using the standard strong negation on the real interval[0;1]), which is fixed. While, inintuitionistic fuzzy soft setting, the non-membership degree is a more-or-less independent degree: the only condition is that $\nu_{(F,A)}$(x) ≤1−$\mu_{(F,A)}$(x). Certainly, fuzzy soft sets are Atanassov's intuitionistic fuzzy soft sets by setting $\nu_{(F,A)}$(x)=1−$\mu_{(F,A)}$(x).After that, several intuitionistic fuzzy algebraic structures were studied by many authors. we present interesting characterizations of these notions of the intuitionistic fuzzy soft ideal and filter on a lattice, prime intuitionistic fuzzy soft ideal and filter ,interaction of prime intuitionistic fuzzy soft ideal and filter and then we evaluate their some characterizations and basic properties are investigated.Finally, we present some conclusions and we discuss future research in section 5.

2 Atanassovs intuitionistic fuzzy soft sets

The notion of fuzzy sets was first introduced by Zadeh[10]

Definition: 2.1

Let X be a nonempty set. A fuzzy soft set is characterized by a membership function $\mu_{(F,A)}: x[0,1]$ where$\mu_{(F,A)}(x)$ is interpreted as the degree of membership of the element x in the fuzzy soft subset (G,B) for any $x \in X$.

Definition: 2.2

Let X be a nonempty set. An intuitionistic fuzzy soft set (IFS, for short) (F,A) on X is an object of the form characterized by a membership function $\mu_{(F,A)}: x[0,1]$ and a non-membership function $\nu_{(F,A)}: x[0,1]$ which satisfy the condition $0 \leq \mu_{(F,A)} + \nu_{(F,A)}$ for any $x \in X$

The class of intuitionistic fuzzy soft sets on X is denoted by IFSs(X).

(i) $(F,A) \subseteq (G,B)\ if\ \mu_{(F,A)}(x) \leq \mu_{(G,B)}(x)\ and\ \nu_{(F,A)}(x) \geq \nu_{(G,B)}(x)\ for\ all\ x \in X$.

(ii) $(F,A) = (G,B)\ if\ \mu_{(F,A)}(x) = \mu_{(G,B)}(x)\ and\ \nu_{(F,A)}(x) = \nu_{(G,B)}(x)\ for\ all\ x \in X$

In the sequel, we need the following definition of level sets (which is also often called (α,β)-cuts) of an intuitionistic fuzzy set.

Definition: 2.3.

Let (F,A) be an intuitionistic fuzzy soft set on a set X. The (α;β)-cut of (F,A) is a crisp subset,
Where α,β∈[0,1]with α+β≤1.

Definition: 2.4.

Let A be an intuitionistic fuzzy soft set on a set X. The support of (F,A) is the crisp subset on X given by

Intuitionistic fuzzy soft lattices

The concept of intuitionistic fuzzy soft lattice was introduced by Thomas and Nair [30] as an intuitionistic fuzzy seton a crisp lattice stable by the supremum and the infimum of the binary operations $\curlywedge$ and $\curlyvee$.

Definition: 2.5

Let L be a lattice and be an IFS on L. Then (F,A) is called an intuitionistic fuzzy soft lattice if for all x;y∈L, the following conditions are satisfied:

(i) $\mu_{(F,A)}(x \curlyvee y) \geq \mu_{(F,A)}(x) \wedge \mu_{(F,A)}(y)$

(ii) $\mu_{(F,A)}(x \curlywedge y) \geq \mu_{(F,A)}(x) \wedge \mu_{(F,A)}(y)$

(iii) $\nu_{(F,A)}(x \curlyvee y) \leq \nu_{(F,A)}(x) \vee \nu_{(F,A)}(y)$

(iv) $\nu_{(F,A)}(x \curlywedge y) \leq \nu_{(F,A)}(x) \vee \nu_{(F,A)}(y)$

Definition: 2.6

Let L be a lattice and be an IFSs on L. Then I is called an intuitionistic fuzzy soft ideal on L (IFs-ideal, for short) if for all $x, y \in L$, the following conditions are satisfied,

(i) $\mu_I(x \curlyvee y) \geq \mu_I(x) \wedge \mu_I(y)$,

(ii) $\mu_I(x \curlywedge y) \geq \mu_I(x) \vee \mu_I(y)$,

(iii) $\nu_I(x \curlyvee y) \leq \nu_I(x) \vee \nu_I(y)$,

(iv) $\nu_I(x \curlywedge y) \leq \nu_I(x) \wedge \nu_I(y)$,

Definition: 2.7

Let L be a lattice and be an IFSs on L. Then F is called an intuitionistic fuzzy soft filter on L (IFs-filter, for short) if for all $x, y \in L$, the following conditions are satisfied,

(i) $\mu_F(x \curlyvee y) \geq \mu_F(x) \vee \mu_F(y)$,

(ii) $\mu_F(x \curlywedge y) \geq \mu_F(x) \wedge \mu_F(y)$,

(iii) $\nu_F(x \curlyvee y) \leq \nu_F(x) \wedge \nu_F(y)$,

(iv) $\nu_F(x \curlywedge y) \leq \nu_F(x) \vee \nu_F(y)$,

Proposition 2.1.

Let L be a lattice, L^d be its order-dual lattice and $A \in IFSs(L)$. Then it holds that (F,A) is an IF-ideal on L if and only if (F,A) is an IF-filter on L^d and conversely.

Proof:

Given L is lattice and $(F,A) \in IFSs(L)$ and (F,A) is an IF-ideal on L.

To prove: (F,A) is an IF-filter on L^d.

We have L is a lattice.

And $I_{(F,A)} =$ is an IFSs on L then $I_{(F,A)}$ is called an IF-ideal on L. if for all $x, y \in L$.

(i) $\mu_{I_{(F,A)}}(x \vee y) \geq \mu_{I_{(F,A)}}(x) \wedge \mu_{I_{(F,A)}}(y)$…………………(a)

Then is a IFSs on L then $F_{(F,A)}$is called an IF-filter on L.

$\mu_{I_{(F,A)}}(x \vee y) \geq \mu_{I_{(F,A)}}(x) \wedge \mu_{I_{(F,A)}}(y)$ we show that IF-filter on L^d. $\mu_{I_{(F,A)}}(x \vee y) \geq \mu_{I_{(F,A)}}(x) \vee \mu_{I_{(F,A)}}(y)$ on L.

$\mu_{F_{(F,A)}}(x \vee y) \leq \mu_{F_{(F,A)}}(x) \wedge \mu_{F_{(F,A)}}(y)$ on L^d…………………(b)

From equation (a) and (b)

$\mu_{I_{(F,A)}}(x \vee y) \geq \mu_{I_{(F,A)}}(x) \wedge \mu_{I_{(F,A)}}(y)$ then $\mu_{F_{(F,A)}}(x \vee y) \leq \mu_{F_{(F,A)}}(x) \wedge \mu_{F_{(F,A)}}(y)$

$\mu_{I_{(F,A)}}(x \vee y) = \mu_{F_{(F,A)}}(x \vee y)$ Similarly we prove (ii), (iii), and (iv).

Proposition 2.2.

Let L be a lattice, (F,A) and (F,B) are two intuitionistic fuzzy soft sets on L. Then it holds that

(i) If (F,A) and (F,B) are two IF-ideals on L, then (F,A) ∩ (F,B) is an IF-ideal on L,

(ii) If (F,A) and (F,B) are two IF-filters on L, then (F,A) ∩ (F,B) is an IF-filter on L,

Proof:

If (F,A) and (F,B) are two IF-ideals on L then,

To prove: (F,A) ∩ (F,B) is an IF-ideal on L

Let L be a lattice is a IFSs on L then $I_{((F,A)\cap(F,B))}$ is called IF-ideal on L. if for all $x, y \in L$.

(i) $\mu_{I_{((F,A)\cap(F,B))}}(x \vee y) \geq \mu_{I_{((F,A)\cap(F,B))}}(x) \wedge \mu_{I_{((F,A)\cap(F,B))}}(y)$

(ii) $\mu_{I_{((F,A)\cap(F,B))}}(x \wedge y) \geq \mu_{I_{((F,A)\cap(F,B))}}(x) \vee \mu_{I_{((F,A)\cap(F,B))}}(y)$

(iii) $\nu_{I_{((F,A)\cap(F,B))}}(x \vee y) \leq \nu_{I_{((F,A)\cap(F,B))}}(x) \vee \nu_{I_{((F,A)\cap(F,B))}}(y)$

(iv) $\nu_{I_{((F,A)\cap(F,B))}}(x \wedge y) \leq \nu_{I_{((F,A)\cap(F,B))}}(x) \wedge \nu_{I_{((F,A)\cap(F,B))}}(y)$

Hence $(F,A) \cap (F,B)$ is an IF-ideal on L.

3 Characterizations of IFS-ideals and IFS-filters on a lattice

In this section, we provide interesting characterizations of IFS-ideals and IFS-filters on a lattice.

3.1 Basic characterization of IFS-ideals and IFS-filters

In this subsection, we characterize the notion of IFS-ideals and IFS-filters on a lattice in terms of the lattice-operations. We start with the key results.

Theorem 3.1.

Let L be a lattice and (F,A) ∈ IFSs(L). Then for all x;y ∈ L, the following four statements hold,

(i)$\nu_{(F,A)}(x \wedge y) \leq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$ if only if $\left(x \leq y \Longrightarrow \nu_{(F,A)}(x) \leq \nu_{(F,A)}(y)\right)$,

(ii)$\nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$ if only if $\left(x \leq y \Longrightarrow \nu_{(F,A)}(x) \geq \nu_{(F,A)}(y)\right)$,

(iii) $\mu_{(F,A)}(x \wedge y) \geq \mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)$ if only if $\left(x \leq y \Longrightarrow \mu_{(F,A)}(x) \geq \mu_{(F,A)}(y)\right)$,

(iv) $\mu_{(F,A)}(x \vee y) \geq \mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)$ if only if $\left(x \leq y \Longrightarrow \mu_{(F,A)}(x) \leq \mu_{(F,A)}(y)\right)$,

Proof.

Let x,y ∈ L,

(i) Suppose that $\nu_{(F,A)}(x \wedge y) \leq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$. If $x \leq y$ then $(x \vee y) = y$.

Since $\nu_{(F,A)}(x \wedge y) \leq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$, if follows that $\mu_{(F,A)} = \nu_{(F,A)}(x \wedge y) \leq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$. Hence $\nu_{(F,A)}(x) \leq \nu_{(F,A)}(y)$.

Conversely, suppose that $x \leq y \Longrightarrow \nu_{(F,A)}(x) \leq \nu_{(F,A)}(y)$. Then it follows that $\nu_{(F,A)}(x) \leq \nu_{(F,A)}(x \wedge y)$ and $\nu_{(F,A)}(y) \leq \nu_{(F,A)}(x \wedge y)$. Hence $\nu_{(F,A)}(x \wedge y) \leq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$.

(ii)suppose that$\nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$. If $x \leq y$ then $(x \vee y) = x$. Since $\nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$, it follows $\nu_{(F,A)}(x) = \nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$. Hence, $\nu_{(F,A)}(x) \geq \nu_{(F,A)}(y)$.

Conversely, suppose that $\left(x \leq y \Longrightarrow \nu_{(F,A)}(x) \geq \nu_{(F,A)}(y)\right)$. Then it follows that $\nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(x)$ and $\nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(y)$. Hence, $\nu_{(F,A)}(x \vee y) \geq \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$.

(iii) similarly proof as (i).

(iv) similarly proof as (ii).

Corollary 3.1

Let L be a lattice and F be an IFs-filter on L. Then for any x,y ∈ L it holds that,

(i) If $x \leq y$, then $\mu_F(x) \leq \mu_F(y)$, (i.e., the map μ_F:L→[0;1] is antitone),

(ii) If $x \leq y$, then $\nu_F(x) \geq \nu_F(y)$ (i.e., the map ν_F:L→[0;1] is monotone).

Proof:

(i) If $x \leq y$ then $(x \wedge y) = x$. Since $\mu_F(x \wedge y) \leq \mu_F(x) \wedge \mu_F(y)$ it follows that $\mu_F(x) = \mu_F(x \wedge y) \leq \mu_F(x) \wedge \mu_F(y)$, $\mu_F(x) \geq \mu_F(x)$. Hence $\mu_F(x) \geq \mu_F(x)$.

Corollary 3.2

Let L be a lattice and I be an IFs-ideal on L. Then for any x,y ∈ L it holds that

(i) If $x \leq y$, then $\mu_I(x) \geq \mu_I(y)$, (i.e., the map μ_I:L→[0;1] is antitone),

(ii) If $x \leq y$, then $\nu_I(x) \leq \nu_I(y)$ (i.e., the map ν_I:L→[0;1] is monotone).

Theorem 3.3.

Let L be a lattice and F ∈ IFSs(L). Then it holds that F is an intuitionistic fuzzy soft filter on L if and only if the following conditions are satisfied,

(i) $\mu_F(x \wedge y) = \mu_F(x) \wedge \mu_F(y)$, for any $x, y \in L$,

(ii) $\nu_F(x \wedge y) = \nu_F(x) \vee \nu_F(y)$, for any $x, y \in L$,

Proof.

Suppose F is an filter is an Ifs-filter on L, then for any $x, y \in L$. It holds that $\mu_F(x \wedge y) \leq \mu_F(x) \wedge \mu_F(y)$ and $\nu_F(x \wedge y) \leq \nu_F(x) \vee \nu_F(y)$ since $x \leq x \wedge y$ and $y \leq x \wedge y$ it follows from corollary3.1 that,

$\mu_F(x) \geq \mu_F(x \wedge y)$ and $\mu_F(y) \geq \mu_F(x \wedge y)$ Hence, $\mu_F(x) \wedge \mu_F(y) \geq \mu_F(x \wedge y)$

Thus, $\mu_F(x \wedge y) = \mu_F(x) \wedge \mu_F(y)$ Also, since $x \leq x \wedge y$ and $y \leq x \wedge y$ we obtain from corollary3.1 that $\nu_F(x) \leq \nu_F(x \wedge y)$ and $\nu_F(y) \leq \nu_F(x \wedge y)$ hence $\nu_F(x \curlyvee y) \vee \nu_F(y) \leq \nu_F(x \wedge y)$ they $\nu_F(x \wedge y) = \nu_F(x) \vee \nu_F(y)$.

Conversely, Suppose that, $\mu_F(x \wedge y) = \mu_F(x) \wedge \mu_F(y)$ and $\nu_F(x \wedge y) = \nu_F(x) \vee \nu_F(y)$ for any x,y ∈ L. then it is easy to see that $\mu_F(x \wedge y) \geq \mu_F(x) \wedge \mu_F(y)$,$\nu_F(x \wedge y) \leq \nu_F(x) \vee \nu_F(y)$, for all x,y ∈ L,next we will show that, $\mu_F(x \wedge y) \geq \mu_F(x) \vee \mu_F(y)$ and $\nu_F(x \wedge y) \leq \nu_F(x) \wedge \nu_F(y)$ for any x,y ∈ L. let x,y ∈ L, since $x \wedge (x \curlyvee y) = x$ and $y \wedge (x \curlyvee y) = y$ then it holds that $\mu_F\big(x \wedge (x \curlyvee y)\big) = \mu_F(x)$ and $\mu_F\big(y \wedge (x \curlyvee y)\big) = \mu_F(y)$. From hypothesis (i) and (ii) $\mu_F(x) \wedge \mu_F(x \curlyvee y) = \mu_F(x)$ and $\mu_F(y) \wedge \mu_F(x \curlyvee y) = \mu_F(y)$ Hence, $\mu_F(x \curlyvee y) \geq \mu_F(x)$ and $\mu_F(x \curlyvee y) \geq \mu_F(y)$. Thus, $\mu_F(x \curlyvee y) \geq \mu_F(x) \vee \mu_F(y)$, for any x,y ∈ L. in the same way, we obtain that $\mu_F(x \curlyvee y) \leq \mu_F(x) \wedge \mu_F(y)$ for any x,y ∈ L. therefore, F is an IFS-ideal on L.

3.2 Characterizations of IFS-ideals and IFS-filters in terms of their level sets

In this subsection, we provide some interesting characterizations and properties of IFS-ideals and IFS-filters in terms of their level sets.

Proposition 3.1.

Let L be a lattice and (F,A) ∈ IFSs(L). The following statements hold

(i) If (F,A) is an IFs-filter, then its support Supp(F,A) is a filter on L.

(ii) If (F,A) is an IFs-ideal, then its support Supp(F,A) is an ideal on L.

Proof:

Let (F,A) ∈ L (i) suppose that (F,A) ∈ IFs(L) is an IFs-filter and we show that Supp(F,A) is an filter on L.

(a) Let $x \in Supp(F,A)$ and $y \le x$, then it holds that $\mu_{(F,A)}(x) > 0$ (or) ($\mu_{(F,A)} = 0$ and $\nu_{(F,A)} < 1$).

There are two cases to consider ($y \le x$ and $\mu_{(F,A)}(x) > 0$) or ($y \le x$ and $\mu_{(F,A)}(x) = 0$ and $\nu_{(F,A)}(x) < 1$).

First case: suppose that $y \le x$ and $\mu_{(F,A)} > 0$. Since $y \le x$, then it holds that $x \wedge y = x$. The implies that $\mu_{(F,A)}(x) = \mu_{(F,A)}(x \wedge y) > 0$. From theorem 3.3(i), it follows that $\mu_{(F,A)}(xy) = \mu_F(x) \wedge \mu_F(y) > 0$. Hence $\mu_{(F,A)}(y) > 0$. Thus $y \in Supp(F,A)$.

Second case: $y \le x$ and $\mu_{(F,A)} = 0$ and $\nu_{(F,A)}(x) < 1$. Since $y \le x$, then it holds that $x \wedge y = x$.this implies that $\mu_{(F,A)}(x) = \mu_{(F,A)}(x \wedge y) = 0$ and $\nu_{(F,A)}(x) = \nu_{(F,A)}(x \wedge y) < 1$. From theorem3.3 it follows that $\mu_{(F,A)}(x \wedge y) = \mu_F(x) \wedge \mu_F(y) = 0$, and $\nu_{(F,A)}(x \wedge y) = \nu_F(x) \vee \nu_F(y) < 1$. Hence, $\mu_{(F,A)}(y) > 0$ or $\mu_{(F,A)}(y) = 0$ and $\nu_{(F,A)}(y) < 1$. Thus $y \in Supp(F,A)$.

(b) Let $x, y \in Supp(F,A)$. We need to show that $x \wedge y \in Supp(F,A)$. There are four case to consider.

First case: $\mu_{(F,A)}(x) > 0$ and $\mu_{(F,A)}(y) > 0$. Since (F,A) is an IFs-ideal, then it follows from theorem3.3(i) that $\mu_{(F,A)}(x \wedge y) = \mu_F(x) \wedge \mu_F(y) > 0$ hence $x \wedge y \in Supp(F,A)$.

Second case: $\mu_{(F,A)}(x) = 0$ and $\nu_{(F,A)}(x) < 1$ and $\mu_{(F,A)}(y) = 0$ and $\nu_{(F,A)}(y) < 1$.

It follows from theorem3.3 that $\mu_{(F,A)}(x \wedge y) = 0$ and $\mu_{(F,A)}(x \wedge y) < 1$. Hence $x \wedge y \in Supp(F,A)$.

Third case: $\mu_{(F,A)}(x) > 0$ and $\mu_{(F,A)}(y) = 0$ and $\nu_{(F,A)}(y) < 1$. It follows from theorem3.3 that $\mu_{(F,A)}(x \wedge y) = 0$ and $\nu_{(F,A)}(x \wedge y) < 1$. Hence $x \wedge y \in Supp(F,A)$ The last case $\mu_{(F,A)}(x) = 0$ and $\nu_{(F,A)}(x) < 1$, and $\mu_{(F,A)}(y) > 0$ is analogous to the third case. Thus $Supp(F,A)$ is an filter on L.

Theorem: 3.4

Let L be a lattice and A∈ IFS(L). Then it holds that

(i) (F,A) is a prime IFS-filter if and only if their level sets are filters.

(i) (F,A) is a prime IFS-ideal if and only if their level sets are ideals.

Proof

Follows from proposition 2.1.

4 Prime IFS-ideals (resp.IFS-filters) on a lattice:

In this section,we introduce and characterize the prime IFS-ideals (resp.IFS-filters) on a lattice.

Definition 4.1

An IFS-ideals I on a lattice L is called a prime IFS-ideals if,for any x,y ∈ L

$\mu_I(x \wedge y) \leq \mu_I(x) \vee \mu_I(y)$ and $\nu_I(x \wedge y) \geq \nu_I(x) \wedge \nu_I(y)$

Definition 4.2

An IFS-filter F on a lattice L is called a prime IFS-filter if,for any x,y ∈L

$\mu_F(x \curlyvee y) \leq \mu_F(x) \vee \mu_F(y)$ and $\nu_F(x \curlyvee y) \geq \nu_F(x) \wedge \nu_F(y)$

A combination of theorem 3.2 and Definition 2.6 leads to the following characterization of prime IFS- ideals

proposition 4.1

let L be a lattice and I∈ IFSs (L). then it holds that I is a prime IFS-ideal on L if and only if the following conditions hold:

(i) $\mu_I(x \curlyvee y) = \mu_I(x) \wedge \mu_I(y)$,for any x,y ∈ L,

(ii) $\mu_I(x \wedge y) = \mu_I(x) \vee \mu_I(y)$,for any x,y ∈ L,

(iii) $\nu_I(x \curlyvee y) = \nu_I(x) \vee \nu_I(y)$,for any x,y $\in$ L,

(iv) $\nu_I(x \curlywedge y) = \nu_I(x) \wedge \nu_I(y)$,for any x,y $\in$ L,

similarly, Theorem 3.3 and Definition 2.7 lead to the following characterization of prime IFS-filters.

proposition 4.2

let L be a lattice and F$\in$ IFSs (L). then it holds that F is a prime IFS-filter on L if and only if the following conditions hold:

(i) $\mu_F(x \curlyvee y) = \mu_F(x) \vee \mu_F(y)$,for any x,y $\in$ L,

(ii) $\mu_F(x \curlywedge y) = \mu_F(x) \wedge \mu_F(y)$,for any x,y $\in$ L,

(iii) $\nu_F(x \curlyvee y) = \nu_F(x) \wedge \nu_F(y)$,for any x,y $\in$ L,

(iv) $\nu_F(x \curlywedge y) = \nu_F(x) \vee \nu_F(y)$,for any x,y $\in$ L,

the following proposition shows that the support of a prime IFS-ideal (resp.prime IFS-filter) on a lattice is a prime ideal (resp.prime filter) on that lattice.

proposition 4.3

Let L be a lattice and A$\in$ IFSs(L). Then it holds that

(i) If (F,A) is a prime IFS-filter, then its support Supp(F,A) is a prime filter on L.

(i) If (F,A) is a prime IFS-ideal, then its support Supp(F,A) is a prime ideal on L.

proof

(i) Suppose that (F,A) is a prime IFS-filter on a lattice L. from proposition 3.1 it holds that Supp(F,A) is an filter on L. Next, we prove that Supp(F,A).it then holds that $\mu_{(F,A)}(x \curlyvee y) > 0$ or ($\mu_{(F,A)}(x \curlyvee y) = 0$ and $\nu_{(F,A)}(x \curlyvee y) < 1$). we consider the following cases:

(a) If $\mu_{(F,A)}(x \curlyvee y) > 0$,then the fact that (F,A) is prime IFS-filter on L implies that

$\mu_{(F,A)}(x) \vee \mu_{(F,A)}(y) = \mu_{(F,A)}(x \curlyvee y) > 0$ this implies that either$\mu_{(F,A)}(x) > 0$ $\mu_{(F,A)}(Y) > 0$ .Hence ,either x$\in$ Supp(F,A) or y$\in$ Supp(F,A)

(b) If $\mu_{(F,A)}(x \curlyvee y)$ =0 and $\nu_{(F,A)}(x \curlyvee y)$ <1). then the fact that (F,A) is prime IFS-filter on L implies that

$\mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)$ = $\mu_{(F,A)}(x \curlyvee y) = 0$ and $\nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$ =$\nu_{(F,A)}(x \curlyvee y)$ <1).

These imply that ($\mu_{(F,A)}(x) = 0 \wedge \mu_{(F,A)}(y) = 0$) and $\nu_{(F,A)}(x) < 1 \vee \nu_{(F,A)}(y)$ <1

Hence, ($\mu_{(F,A)}(x) = 0$ and $\nu_{(F,A)}(x) < 1$ or ($\mu_{(F,A)}(y) = 0$) and $\nu_{(F,A)}(y)$ <1). thus, either x$\in$ Supp(F,A) and y$\in$ Supp(F,A).Therefore, Supp(F,A) is a prime filter on L.

(ii) Follows in the same way by using proposition 2.1 and (i)

In the same direction , we get the following theorem which provides a characterization of prime IFS-ideals(resp.prime IFS-filters) in terms of their level sets.

Theorem 4.1

Let L be a lattice and A$\in$ IFSs(L). Then it holds that

(i) (F,A) is a prime IFS-filter if and only if their level sets are prime filters.

(i) (F,A) is a prime IFS-ideal if and only if their level sets are prime ideals.

proof

(i) from theorem 3.4,(F,A) is an IFS-filter on L if and only if $(F,A)_{\alpha,\beta}$ is an filter on L, for any $\alpha, \beta \in$]0,1] satisfying $\alpha + \beta \leq 1$.it remains to show the primality. Suppose that (F,A) is a prime IFS-filter on L. Let x,y $\in$ L such that $(x \curlyvee y) \in (F,A)_{\alpha,\beta}$.Then from proposition 4.1 it follows that $\mu_{(F,A)}(x \curlyvee y)$=$\mu_{(F,A)}(x) \vee \mu_{(F,A)}(y) \geq \alpha$

and $\nu_{(F,A)}(x \curlyvee y)$ =$\nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y) \leq \beta$.These imply that either ($\mu_{(F,A)}(x) \geq \alpha$ and $\nu_{(F,A)}(x) \leq \beta$)or ($\mu_{(F,A)}(y) \geq \alpha$ and $\nu_{(F,A)}(y) \leq \beta$). Hence, either x$\in (F,A)_{\alpha,\beta}$ or y$\in (F,A)_{\alpha,\beta}$.Thus $(F,A)_{\alpha,\beta}$ is a prime filter, for any $\alpha, \beta \in$]0,1] satisfying $\alpha + \beta \leq 1$. conversely, suppose that $(F,A)_{\alpha,\beta}$ is a prime filter, for any $\alpha, \beta \in$]0,1] such that $\alpha + \beta \leq 1$ and (F,A) is not a prime IFS-filter on L.Then it holds that ther exist x,y $\in$ L such that $\mu_{(F,A)}(x \curlyvee y) > \mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)$ and $\nu_{(F,A)}(x \curlyvee y) < \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(y)$. These imply that $\mu_{(F,A)}(x \curlyvee y) > \mu_{(F,A)}(x)$ and $\mu_{(F,A)}(x \curlyvee y) > \mu_{(F,A)}(y)$) and $\nu_{(F,A)}(x \curlyvee y) < \nu_{(F,A)}(x)$ and $\nu_{(F,A)}(x \curlyvee y) < \nu_{(F,A)}(y)$). If we put $\mu_{(F,A)}(x \curlyvee y)$= α and $\nu_{(F,A)}(x \curlyvee y)$= β,then it follows that $\mu_{(F,A)}(x) < \alpha$ and $\nu_{(F,A)}(x) > \beta$) and $\mu_{(F,A)}(y) < \alpha$ and $\nu_{(F,A)}(y) > \beta$).Hence $(x \curlyvee y) \in (F,A)_{\alpha,\beta}$.x,y $\notin (F,A)_{\alpha,\beta}$.That is a contradiction with the fact that $(F,A)_{\alpha,\beta}$ is a prime filter on L for any $\alpha,\beta \in$]0,1]. Hence,(F,A) is a prime IFS-filter on L.(ii) Follows from proposition 2.1 and (i).

5 Interaction of prime IFS-ideals(resp. IFS-filters) with the basic set- operations and a lattices-homomorphism.

In this section, we study the Interaction of prime IFS-ideals(resp. IFS-filters) with the basic set- operations and a lattices- homomorphism.

5.1 Interaction of IFS-ideals(resp. IFS-filters) with the basic operations

In this subsection, we discuss the Interaction of IFS-ideals(resp. IFS-filters) with intersection, union, complement and two associated intuitionistic fuzzy soft sets.

proposition 5.1

Let $((F,A)_i)_{i\in I}$ be a family of IFS-sets on a lattice L.then it holds that

(i) If $(F,A)_i$ is a prime IFS- filter on L,for any $i \in I$,then $\bigcap_{i\in I}(F,A)_i$ is a prime IFS-filter on L.

(ii) If $(F,A)_i$ is a prime IFS- ideal on L,for any $i \in I$,then $\bigcap_{i\in I}(F,A)_i$ is a prime IFS-ideal on L.

proof

(i) suppose that for any $i \in I$,$(F,A)_i$ is a prime IFS-filter on L. From proposition 2.2,it follows that $\bigcap_{i\in I}(F,A)_i$is an IFS-filter on L. it remains to show that $\bigcap_{i\in I}(F,A)_i$is prime. Let $x,y \in L$ such that $(x \curlyvee y) \in \bigcap_{i\in I}(F,A)_i$.Then it follows that $(x \curlyvee y) \in (F,A)_i$,for any $i \in I$,$(F,A)_i$is a prime IFS-filter,it follows that $\mu_{(F,A)_i}(x \curlyvee y) \leq \mu_{(F,A)_i}(x) \vee \mu_{(F,A)_i}(y)$ and $\nu_{(F,A)_i}(x \curlyvee y) \geq \nu_{(F,A)_i}(x) \wedge \nu_{(F,A)_i}(y)$ for any $i \in I$. This implies that $\mu_{\bigcap_{i\in I}(F,A)_i}(x \curlyvee y) \leq \mu_{(F,A)_i}(x \curlyvee y) \leq \mu_{(F,A)_i}(x) \vee \mu_{(F,A)_i}(y)$ and $\nu_{\bigcap_{i\in I}(F,A)_i}(x \curlyvee y) \geq \nu_{(F,A)_i}(x \curlyvee y) \geq \nu_{(F,A)_i}(x) \wedge \nu_{(F,A)_i}(y)$ for any $i \in I$. hence

$\mu_{\bigcap_{i\in I}(F,A)_i}(x \curlyvee y) \leq \bigwedge_{i\in I} \mu_{(F,A)_i}(x) \vee \mu_{(F,A)_i}(y)$ and $\nu_{\bigcap_{i\in I}(F,A)_i}(x \curlyvee y) \geq \bigvee_{i\in I} \nu_{\bigcap_{i\in I}(F,A)_i}(x) \wedge \nu_{\bigcap_{i\in I}(F,A)_i}(y)$

thus,$\mu_{\bigcap_{i\in I}(F,A)_i}(x \curlyvee y) \leq \mu_{\bigcap_{i\in I}(F,A)_i}(x) \vee \mu_{\bigcap_{i\in I}(F,A)_i}(y)$ and $\nu_{\bigcap_{i\in I}(F,A)_i}(x \curlyvee y) \geq \nu_{\bigcap_{i\in I}(F,A)_i}(x) \wedge \nu_{\bigcap_{i\in I}(F,A)_i}(y)$

therefore, $\bigcap_{i\in I}(F,A)_i$is a prime IFS-filter on L.

(ii) Follows from proposition 2.1 and (i)

proposition 5.2

Let L be a lattice and (F,A) $\in$ IFSs(L).Then it holds that

(i) (F,A) is a prime IFS-filter if and only if (F,A)is a prime IFS-filter on L.

(ii) (F,A) is a prime IFS-ideal if and only if (F,A)is a prime IFS-ideal on L.

proof

(i) suppose that (F,A) is a prime IFS-filter on a lattice L. since (F,A)=$\{\langle x, \mu_{(F,A)}, 1-\mu_{(F,A)}(x) | x \in X\rangle$,then it easily to see that (F,A) is an IFS-filter on L.Next ,we show that (F,A) is prime .we have that

$$\mu_{(F,A)}(x \vee y) = \mu_{(F,A)}(x \vee y) = \mu_{(F,A)}(x) \vee \mu_{(F,A)}(y) = \mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)$$

And $\nu_{(F,A)}(x \vee y) = 1-\mu_{(F,A)}(x \vee y) = 1-\mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)$

$= 1-\mu_{(F,A)}(x) \wedge 1-\mu_{(F,A)}(x) = \nu_{(F,A)}(x) \wedge \nu_{(F,A)}(x)$. Thus,we can conclude that (F,A) is a prime IFS-filter on L. Conversely, suppose that (F,A) is a prime IFS-filter. By using the same method as above we get that (F,A) is a prime IFS-filter on L.

(ii) follows from proposition 2.1 and (i)

proposition 5.2

Let L be a lattice and (F,A) $\in$ IFSs(L).Then it holds that

(i) (F,A) is a prime IFS-filter if and only if (F,A)is a prime IFS-filter on L.

(ii) (F,A) is a prime IFS-ideal if and only if (F,A)is a prime IFS-ideal on L.

proof

The proof is similar as in proposition 5.3

Conclusion

In this paper, we have indicated intuitionistic fuzzy soft ideal and filter on a lattice in terms of the lattice operations and terms of their associated crisp set. As interesting kinds, we have introduced the notions of prime intuitionistic fuzzy soft ideal and filter on a lattice and investigated their different characterizations and properties. Moreoverr we intend to extend this work to other kinds of intuitionistic fuzzy soft ideal and filters.

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